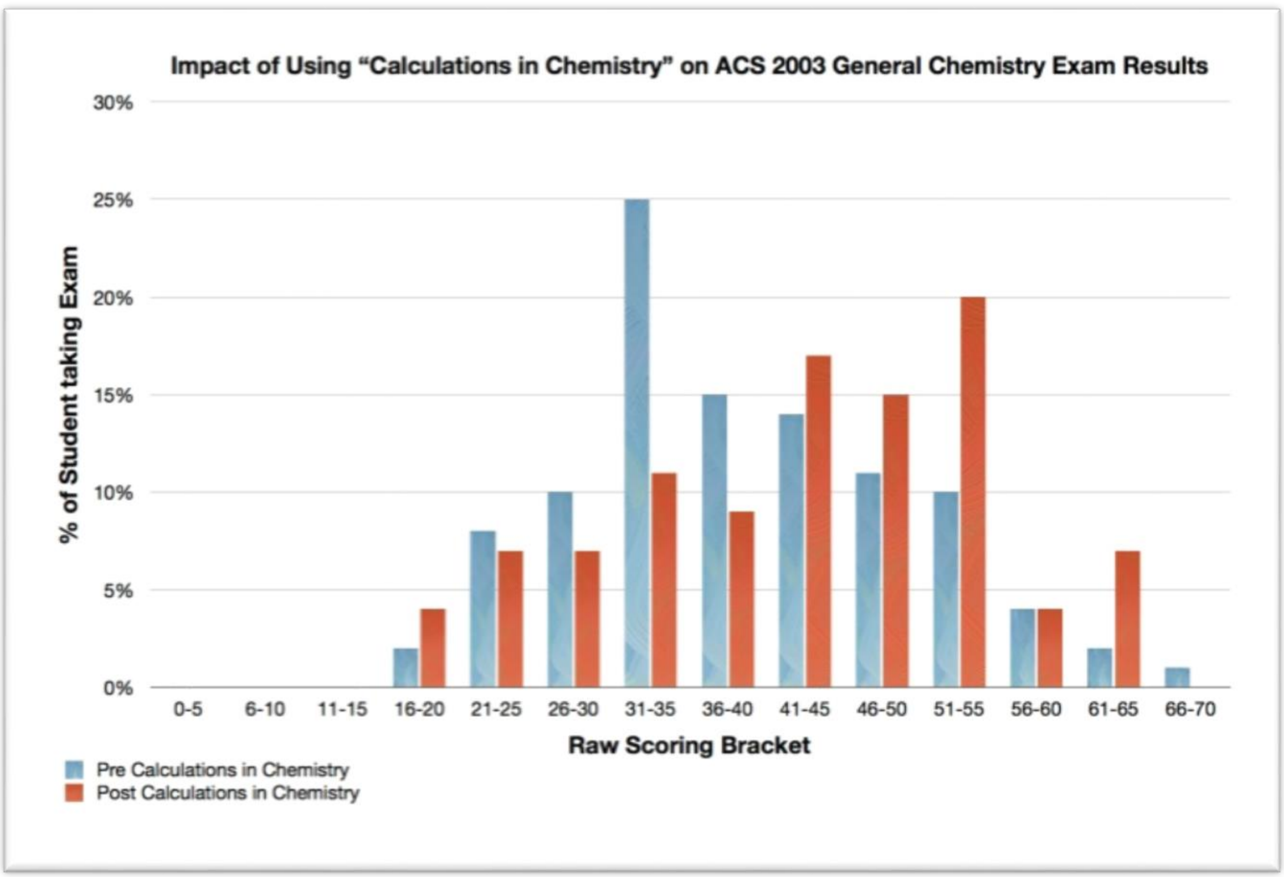
Helping Students Minimize Errors in Chemistry Calculations

Summary

This PDF contains

- Why science says students need to be taught algorithms to solve problems,
- A description of "prep step" algorithms that simplify chemistry calculations,
- Faculty reports on improved results using a "math review plus prep steps,"
- Samples of math review and "prep step" problems which can be done in class, and
- A link to free homework tutorials that review math and practice the prep steps.

The Science and Its Results



-- From Craig, P., "Building Student Confidence with Chemistry Computation" at https://confchem.ccce.divched.org/content/2017fallconfchemp5

What Science Says

According to cognitive scientists, when solving calculations in the sciences, students are most successful when they

- First review the math that is a component of the science, and
- Are systematically taught effective *algorithms*: Structured step-by-step procedures that solve different types of problems.

(For details on this cognitive science, click here -- and check your PDF downloads.)

Impact

To apply the recommendations of science without taking class time for math, our group designed "homework tutorials" that include a review of needed math before each chemistry topic. In addition, we developed, tested, modified, and re-tested about 20 "chem specific" problem-solving algorithms that are generally not taught in chemistry textbooks. Most of these involve "preparation steps" we advise students to take before substituting data into conversions or equations.

Published as a paperback and eBook as *Calculations In Chemistry*, the tutorials have been used in "Intro to Chem" courses as a text and in AP/General Chemistry as a source of "fundamentals review and homework."

In three recent academic publications, college instructors using the tutorials have reported substantially improved student results. In the most recent (2017) paper, in General Chemistry an increase in ACS Exam scores was reported from the 42^{nd} to the 60^{th} percentile (graphed above). Links to all of the evaluation papers are here (check PDFs).

The papers found the tutorials helped students over a range of math backgrounds. In addition to those studies, in reviews of the tutorials by over 20 student-users on Amazon, comments included:

> "If you struggle with math, this book is a MUST-HAVE."
>
> "Finally, I'm passing Chem after many failed attempts. Solid A's on tests now because I use this book."

If you have students whose math background is not what it should be, these lessons may help.

A link to all of the Amazon reviews is here.

To summarize: We believe the evidence is that if the recommendations of cognitive science are implemented, student achievement in chemistry improves substantially.

For all AP and Chem I instructors, a free copy of the complete tutorials in an eBook format is available (details here).

Preparation for AP Word Problems

From June 1 to September 17, access to the first six chapters of the tutorial eBook is *free* to all students (detail here). For AP classes, we suggest assignment of these chapters as homework as part of a "start of school review of Chem I fundamentals."

The tutorials review exponential math, the metric system, and significant figures in Chapters 1-3. Chapter 4 reviews the math of conversions, including problems similar to these:

1. Label these conversion factors as *legal* or *illegal*.

   a $\frac{1\text{ mL}}{1\text{ cm}^3}$ b. $\frac{10^3\text{ mL}}{1\text{ L}}$ c. $\frac{10^3\text{ kilohertz}}{1\text{ hertz}}$ d. $\frac{1\text{ g}}{10^2\text{ cg}}$

2. **?** gigagrams = 42 centigrams • $\frac{\text{gram}}{\_\_\_\_}$ • ________ **=**

2.. **?** mg = 4.2 kg • ________ • $\frac{\_\_\_\_}{\text{g}}$ **=**

3. **?** milliseconds = 0.0025 hours

4. How many fluid ounces equal 0.500 liters? Use the conversion: 12.0 fl. oz. = 355 mL.

5. **?** $\frac{\text{ng}}{\text{mL}}$ = $\frac{4.7 \times 10^{-2}\text{ mg}}{\text{L}}$

The PDF you are reading is also available in a Word format (here) so that sample problems and procedures may be "cut and pasted" into in-class slides and/or practice problems. The above problems differ from those in the homework tutorials and may be used for conversion review and practice.

After students practice the Chapter 4 mechanics of conversions, they are ready to add special rules for "word problems."

Word Problems -- General Rules

Chapter 5 covers the initial "prep step" algorithms. For AP students who had had substantial practice with dimensional analysis in Chem I/Honors, those "prep steps" can nearly always be learned quickly by assigning tutorial Chapters 1-5 as homework.

IF, however, AP sections include students who have had more limited practice with conversions, OR if a year has elapsed since their Intro course, we recommend both assigning tutorial Chapters 1 to 5 as homework *and* covering the "prep steps" *in-class* in two mini-lessons:

- The "general" prep steps can be taught after students complete the review of conversion math in homework Chapter 4.
- The "chem specific" prep steps can be taught at any point after "how to calculate a substance molar mass" is reviewed.

"Lesson plans" for the mini-lessons are suggested below, including six sample problems with answers that can be "cut and pasted" into slides or worksheets.

Word Problems With*out* Chemical Substances

Permit us to begin with a problem using contrived units -- to focus on the concept of arranging conversions, rather than using units that may suggest equation use.

Problem 1:

> If there are 3 floogles per 10 schmoos, 5 floogles/gom, and 3 goms have a mass of 25 gnarfs, how many gnarfs are in 4.2 schmoos? (Assume whole numbers are exact.)

In class, you might suggest students try this problem on their paper, wait just a few minutes, and then solicit a few answers. Some students may have difficulty, but this may "establish a need to know."

Then ask that they try the problem again, this time using the following rules to simplify problem solving.

Rules for Word Problems

The general **rule**:

> **To solve word problems, get rid of the words.**

By translating words into numbers and their units, most of the *initial* word problems in chemistry can be solved by chaining conversions -- as done in the conversions above.

The specific rules for word-problems begin with:

Rule 1: The first time you read a *word* problem, look only for the *unit* of the *answer*.

Rule 2: Write: " WANTED: ? ", followed by the *unit* of the *answer*, then an = sign.

"On your paper, apply Rules 1 and 2 to our problem."

> If there are 3 floogles per 10 schmoos, 5 floogles/gom, and 3 goms have a mass of 25 gnarfs, how many gnarfs are in 4.2 schmoos? (Assume the whole numbers are exact.)

In the eBook homework tutorials, we ask students (and here we ask you, to sample what students are asked to do) to do this:

> On your computer screen, scroll or size your window to **hide** the text that is *below* the following STOP. Next, on your paper, answer the question above. Then scroll below the STOP to *check* your answer.

STOP * * *

When introducing these rules in class, after allowing time to answer on their paper, we recommend asking for the solution line-by-line to write on the board. Calling on different students for each line may encourage their attention.

Solicit:

WANTED: ? gnarfs =

"Now apply these rules."

Rule 3. On the next line down, write "DATA:"

Rule 4. Read the problem a second time.

- **Each time you find a number, *stop*. Write the number on a line under DATA:**
- **After the number, write its *unit*.**

Rule 5. If two DATA quantities can be listed as an *equality*, do so.

- **If two quantities are proportional *or* equal, on a new line, write them as equal (=)**
- **Convert any p*er* or / in a unit into an equal sign (=).**
- **If *no* number is shown after a *per* or / , write *per* or / as " = 1 "**

For the problem above, on your paper, apply *just* those rules.

STOP * * *

To illustrate and allow for error correction, after a minute or two, solicit just these first two DATA lines:

WANTED: ? gnarfs =

DATA: 3 floogles = 10 schmoos ("write *per* as **= "**)

5 floogles = 1 gom ("write *per X* as **= 1** *X* ")

then ask that they finish the DATA, applying Rules 1-5.

STOP * * *

WANTED: ? gnarfs =

DATA: 3 floogles **=** 10 schmoos

5 floogles **=** 1 gom

3 goms **=** 25 gnarfs

4.2 schmoos

Our homework tutorials go over in detail over a dozen different ways that words and units can represent equalities and proportions, but Rule 5 covers most cases.

To SOLVE, these three rules help with all conversion calculations.

Rule 6. Skip a line, write SOLVE: On the line below, write the quantity you listed after WANTED: above.

Rule 7. If the WANTED unit does not have a denominator, after the equal sign, write the one item in your DATA that is *not* part of an equality. This is your "*given*" quantity.

Rule 8. After the *given*, write an empty conversion factor to multiply by. (• ___________) . Then write the given *unit* where it will cancel.

"Apply Rules 6 to 8 to the sample problem."

STOP * * *

SOLVE:

$$\textbf{? gnarfs} = 4.2 \text{ schmoos} \bullet \frac{\rule{2cm}{0.4pt}}{\text{schmoos}}$$

By the laws of physical dimensions, a base unit without a denominator must be derived *from* a base unit without a denominator. This law tells students which value in their DATA to choose as their "*given* quantity" to begin conversions. For students, picking the *given* can be confusing in a complex word problem. Rule 7 (with some tweaks we cover later) helps.

In addition, for problems that can be solved by conversions, if the WANTED unit is without a denominator, at most *one* DATA item will *not* be listed in an equality. That's the *given*.

In problems where *more* than one DATA item is not listed as part of an equality, either A) an equality was missed, or B) as we cover later, this is an indication of a problem for which an equation is needed.

Rule 8 *starts* the next conversion – putting one unit where it *must* be. This indicates which DATA equality should be chosen to finish the conversion, *and* arranges it right-side up – another helpful step in avoiding math mistakes.

Now display

Rule 9: Finish by chaining conversions from your DATA. Include fundamental equalities (such as metric unit definitions) as needed.

Rule 10. Cancel units that cancel as you go. When the unit on the right side of the = sign matches the WANTED unit, stop conversions and do the math.

"Apply Rules 9 and 10 to our problem. Try the math at the end without a calculator."

STOP * * *

After a minute, ask how they completed the first conversion factor. Then ask what must be written next.

$$? \ \textbf{gnarfs} = 4.2\ \cancel{\text{schmoos}} \cdot \frac{3\ \text{floogles}}{10\ \cancel{\text{schmoos}}} \cdot \frac{\qquad\qquad}{\text{floogles}}$$

Only one DATA equality uses schmoos, so it must be used for the first conversion factor.

Since floogles was not the WANTED unit, it is placed in the next conversion where it will cancel.

Permit another minute for students to finish the problem if needed.

STOP * * *

$$? \ \textbf{gnarfs} = 4.2\ \cancel{\text{schmoos}} \cdot \frac{3\ \cancel{\text{floogles}}}{10\ \cancel{\text{schmoos}}} \cdot \frac{1\ \cancel{\text{gom}}}{5\ \cancel{\text{floogles}}} \cdot \frac{25\ \text{gnarfs}}{3\ \cancel{\text{goms}}} = \frac{4.2 \cdot \cancel{3} \cdot \cancel{25}^{\,1}}{{}_{2}\cancel{10} \cdot \cancel{5} \cdot \cancel{3}}\ \text{gn.} = \textbf{2.1 gnarfs}$$

The arithmetic step is an opportunity to demonstrate how to simplify with mental math.

Writing the WANTED and DATA "prep steps" takes time but will simplify arranging conversions to arrive at the right answer.

With a bit of practice, students won't need to look at the rules, the steps become automatic, and using the steps, most will get complex calculations right nearly every time.

These steps simplify but do not trivialize the science. Conversions can be described as "multiplying a given quantity by 1" or as "applying a proportion to a given quantity." In either view, a scientific relationship (that students must identify, calculate, or recall) is applied to a known quantity -- one factor at a time.

According to cognitive science, solving "one step at a time" avoids overload in the "working memory" where the brain solve problems. In addition, stepwise procedures tend to open slots in working memory for the "context elements" of a problem. When context elements are present in working memory during the steps of processing information as students solve a problem, the context elements tend to form neural connections with the new knowledge. As different types of problems are solved, those wired connections become the brain's conceptual framework in a discipline.

Problem 2:

For additional practice in the "prep steps," here's a problem of a type science majors are likely to encounter in introductory college courses -- made easier by WANTED DATA SOLVE.

Signals from earth to objects in space travel at the speed of light (3.0×10^8 m/s). If a signal from Earth to a satellite headed for Mars takes 2.00 minutes to reach the satellite, how many kilometers from Earth is the satellite? (Try the arithmetic at the end without a calculator.)

"To solve, fill in this chart on your paper:"

WANTED:

DATA:

SOLVE:

?

STOP * * *

Have students work without the rules for a minute or two. Self-testing to recall what was done earlier is "retrieval practice" that builds long-term memory.

Then consider putting up this abbreviated list.

Rule 2: Write: "WANTED: ? ", followed by the *unit* of the *answer* and an = sign.

Rule 4. Read the problem a second time.

- **Each time you find a number, *stop*. Write the number on a line under "DATA:"**
- **After the number, write its *unit*.**

Rule 5. Watch for data you can list as *equalities*.

- **If two quantities are proportional *or* equal, write them as equal (=)**
- **Convert each p*er* or / in a unit to an equal sign (=).**
- **If *no* number is shown after a *per* or / , write *per* or / as " = 1 "**

Rule 7. SOLVE starting

$$\textbf{? WANTED} = \#\ \textit{given} \bullet \frac{\underline{\qquad\qquad}}{\textbf{given } \textit{unit}}$$

STOP * * *

After another minute , ask for just WANTED and DATA.

WANTED: **?** km =

DATA: 3.0×10^8 m = 1 s

2.00 min.

STOP * * *

After another minute, for one factor at a time, ask for the SOLVE conversions.

SOLVE:

WANT a unit without a denominator? Pick as *given* the DATA w/out a denominator.

$$? \text{ km} = 2.00 \textbf{ min.} \bullet \frac{\_\_\_\_}{\textbf{min.}}$$

$$? \text{ km} = 2.00 \text{ min.} \bullet \frac{60 \textbf{ s}}{1 \text{ min.}} \bullet \frac{\_\_\_\_}{\textbf{s}}$$

$$? \text{ km} = 2.00 \text{ min.} \bullet \frac{60 \text{ s}}{1 \text{ min}} \bullet \frac{3.0 \times 10^8 \textbf{ m}}{1 \text{ s}} \bullet \frac{\_\_\_\_}{\textbf{m}}$$

$$? \textbf{ km} = 2.00 \text{ min.} \bullet \frac{60 \text{ s}}{1 \text{ min}} \bullet \frac{3.0 \times 10^8 \text{ m}}{1 \text{ s}} \bullet \frac{1 \textbf{ km}}{10^3 \text{ m}} = \frac{2.00 \bullet 60 \bullet 3.0 \bullet 10^5}{1} \text{ km} = 360 \times 10^5 \text{ km}$$

$$\mathbf{= 3.6 \times 10^7 \ km}$$

which is a bit more than 22 million miles. The Earth-Mars closest orbital separation is 5.46×10^7 km.

For visual clarity in this document, we will often avoid showing the unit cancellation, but students may be advised to mark the cancellations on their papers.

In the Chapter 5 of the tutorials, we provide 16 practice problems similar to the above with gradually increasing difficulty. Those may supply ideas for more in-class problems if needed.

Word Problems -- Chemistry Rules

Let's add three *special* rules for chemistry. These can be introduced at *any* point after students review A) how to calculate the molar mass of a substance, and B) Avogadro's number.

Rule 2b. If the chemical substance formula is known, write the formula after the unit.

<u>Example</u>: 18.0 g $\mathbf{H_2O}$

In calculations, the formula showing what is being measured is an inseparable part of the unit and will be crucial when sequencing steps in stoichiometry.

Rule 5b. The Grams Prompt

In your WANTED *or* DATA, if you see grams or prefix-grams (such as kg or mg) of a substance with a known formula,

- calculate the molar mass of that formula; then
- write the molar mass as an *equality* in your DATA.

<u>Example</u>: 18.0 g H_2O = **1** mol H_2O

Rule 5c: The Avogadro Prompt

If your WANTED or DATA

- includes both a count of *invisible* particles (such as atoms or molecules) and units used to measure *visible* amounts (such as grams or milliliters), **or**
- contains any value that includes a 2-digit power of 10 (10^{xx} or 10^{-xx}):

in your DATA write:

1 mol (formula) = 6.02×10^{23} (particles)(formula)

<u>Example</u>: 1 mol H_2O = 6.02×10^{23} molecules H_2O

Apply these new rules plus WANTED DATA SOLVE to

Problem 3:

If 2 moles of NaOH per 1 mole of H_2SO_4 are required for a neutralization reaction, how many grams of NaOH is needed to neutralize 3.01×10^{22} molecules of H_2SO_4?

Fill in this chart on your paper. Watch for prompts!

WANTED:

DATA:

SOLVE:

STOP * * *

After a few minutes, ask *only* for their DATA table – which should look like this, but without the (parentheticals):

WANTED:	? **g NaOH** =	
DATA:	2 moles NaOH **=** 1 mole H_2SO_4	
	$3.01 \times \mathbf{10^{22}}$ molecules H_2SO_4	
	40.0 g NaOH **=** 1 mol NaOH	(grams prompt in WANTED)
	6.02×10^{23} molecules H_2SO_4 **=** 1 mol H_2SO_4	(Avogadro prompt)

In nearly all problems involving *grams* of a known *substance*, the molar mass will be needed to find its moles. Experienced problem solvers know to look for this, but the grams prompt is methodical – and provides training wheels for beginners.

Note that if *writing* the WANTED unit is skipped, the need for the NaOH molar mass remains, but is less evident.

Similar logic applies to the Avogadro prompt.

Most important, the prompts get needed relationships into the DATA before conversions begin. This avoids having to calculate a molar mass in the middle of arranging conversions, "interrupting a train of thought" sequencing steps in the brain's limited working memory.

After another minute or two, ask for the START of their SOLVE conversions.

STOP * * *

SOLVE:

$$\textbf{? g NaOH} = 3.01 \times 10^{22} \text{ molec. } H_2SO_4 \bullet \frac{\qquad\qquad}{\text{molec. } H_2SO_4}$$

STOP * * *

SOLVE:

$$\textbf{? g NaOH} = 3.01 \times 10^{22} \text{ molec. } H_2SO_4 \bullet \frac{1 \text{ mol } H_2SO_4}{6.02 \times 10^{23} \text{ molec. } H_2SO_4} \bullet \frac{2 \text{ mol NaOH}}{1 \text{ mol } H_2SO_4} \bullet \frac{40.0 \textbf{ g NaOH}}{1 \text{ mol NaOH}} =$$

$$= \mathbf{40.0 \times 10^{-1} \text{ g NaOH} = 4.00 \text{ g NaOH}}$$

By first listing the needed DATA relationships, solving for the unit WANTED is simplified.

For a second problem using "chem-specific" prompts:

Problem 4:

> Aluminum metal (Al) has a density of 2.70 g/mL. How many moles of Al are contained in 125 mL of Al? (Try the math without a calculator.)

Will students entering AP find this easy, or tough?

Cognitive studies tell us that *you*, after years of doing these, see the big conceptual picture and instinctively know several ways to solve. Novice learners, however, lack your hard-earned long-term memory. They need stepwise procedures (algorithms) that avoid overload in their working memory -- and get the right answer every time.

STOP * * *

WANTED: ? mol Al = =

DATA: 2.70 **g Al** = 1 mL Al

125 mL Al

27.0 g Al = 1 mol Al (g Al in DATA)

STOP * * *

SOLVE:

$$\textbf{? mol Al} = 125 \text{ mL Al} \bullet \frac{\rule{2cm}{0pt}}{\text{mL Al}}$$

$$\textbf{? mol Al} = 125 \text{ mL Al} \bullet \frac{{\color{red}1}\ \cancel{2.7}\text{ g Al}}{1 \text{ mL Al}} \bullet \frac{1\ \textbf{mol}\ \text{Al}}{{\color{red}10}\ \cancel{27.0}\text{ g Al}} = \textbf{12.5 mol Al}$$

Preparing for Majors in Sciences and Engineering

To prepare students for the tough calculations they will encounter in chemistry, physics, pre-med, and engineering majors, we set out to help instructors teach students to solve problems at the following level:

Problem 5:

> If a drop of water contains 0.050 mL and one million molecules evaporate per second, how many hours would it take for exactly 3 drops to evaporate?
>
> (Recall 1.00 g H_2O = 1 mL H_2O)

After practicing the WANTED, DATA, SOLVE and PROMPT steps, nearly all Honors level students have been able to solve this and similar problems nearly every time.

(Though on this one, they may need some reassurance in the middle.)

"Try the problem."

STOP * * *

Solicit these lines for the board:

WANTED: ? hr =

DATA: 0.050 mL H_2O = 1 drop H_2O

10^6 molecules = 1 s

3 drops (exact)

1.00 **g H_2O** = 1 mL H_2O

18.0 g H_2O = 1 mole H_2O (g of formula = grams prompt)

1 mole H_2O = 6.02 x 10^{23} H_2O (*visible* drops + *molecules* = Avo. prompt)

This may be a good point to pause, let them make any needed DATA adjustments, and finish.

STOP * * *

SOLVE:

$$? \text{ hr} = 3 \text{ drops} \bullet \frac{0.050 \text{ mL } H_2O}{1 \text{ drop}} \bullet \frac{1.00 \text{ g } H_2O}{1 \text{ mL } H_2O} \bullet \frac{1 \text{ mol } H_2O}{18.0 \text{ g } H_2O} \bullet \frac{6.02 \times 10^{23} \text{ molec.}}{1 \text{ mol } H_2O} \bullet \frac{1 \text{ s}}{10^{6} \text{ molec.}} \bullet \frac{1 \text{ min}}{60 \text{ s}} \bullet \frac{1 \text{ hr}}{60 \text{ min}} =$$

= 1.4×10^{12} hours

Which is a long time, you might say to students, because very tiny liquid water molecules actually evaporate much faster than one million per second.

Most Chem I students, after working through the rules gradually, calculate a correct answer. That's builds confidence in aspiring AP students and science majors.

Got time for one more prompt?

Molarity

Calculations involving solution *concentration* are simplified using WANTED DATA SOLVE and

Rule 11: The M Prompt

When reading for DATA, if you see # M (formula) *or* # molar (formula) , in your DATA write

 # mol (formula) = 1 L (formula) solution

Example: See " 0.12 M KCl "? In DATA write: 0.12 mol KCl = 1 L KCl soln.

On your paper, apply Rules 1-11 to this problem:

Problem 6:

If 2.80 grams of NaOH are dissolved in a 0.200 M NaOH solution, what is the solution volume in milliliters?

STOP * * *

WANTED: **?** mL NaOH soln. =

DATA: 2.80 **g NaOH**

0.200 mol NaOH = 1 L NaOH soln. (M prompt)

40.0 g NaOH = 1 mol NaOH (grams prompt)

If needed, adjust your data, then SOLVE.

STOP * * *

SOLVE:

$$? \text{ mL NaOH soln} = 2.80 \text{ g NaOH} \cdot \frac{1 \text{ mol NaOH}}{40.0 \text{ g NaOH}} \cdot \frac{1 \text{ L NaOH soln.}}{0.200 \text{ mol NaOH}} \cdot \frac{1 \text{ mL}}{10^{-3} \text{ L}} =$$

= 3.5×10^2 mL NaOH soln

Units arrange conversions. Prompts identify needed conversions and their numbers. The prompts minimize interruption while chaining conversions. The step-by-step methodical process leads to fewer math errors.

Additional Chem Specific Algorithms

If you obtain a free copy of our complete eBook lessons (here), you will find:

- Prompts for stoichiometry in Lesson 9.2
- Prompts to apply when solving for "ratio unit" measures (including molarity and molar mass and density) in Lesson 10.2
- A review of the math of percentages in Lesson 12.3 and fractional units in Lesson 16.3
- Using the WANTED unit to solve gas-laws (Lesson 16.4), and to achieve consistent units for both gas (Lesson 17.1) and heat equations (Lesson 18.6)
- Checking powers and roots of exponential notation –without a calculator (Lesson 21.1)
- The math of base 10 logarithms, pH prompts (Lesson 23.1), natural logs for first-order kinetics (Lesson 24.3), and radioactive decay prompts (Lesson 24.4)

A detailed table of contents for the 24 chapter free-to-instructors eBook is here (check PDFs).

A table of contents for a 1,200 tutorial page (3 volumes of paperbacks) covering nearly all the calculation topics in AP (plus Nernst, colligative properties, etc.) is here.

Hope this helps!

-- Eric (rick) Nelson (EANelson [AT] ChemReview.Net)