Introduction to Chemistry Calculations

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Module 25 – Nuclear Chemistry

Module 25 – Nuclear Chemistry 549
Lesson 25A: The Nucleus -- Review 549
Lesson 25B: Radioactive Decay Reactions 554
Lesson 25C: Fission and Fusion 558
Lesson 25D: Fractions and Percentages..561
Lesson 25E: Natural Logarithms 566
Lesson 25F: Radioactive Half-Life Calculations 576

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Module 25 – Nuclear Chemistry

Chemistry and Nuclear Reactions

The rules for nuclear reactions are quite different from those that govern chemistry. For example,

- During *chemical* reactions, the nucleus is not changed in any of the reacting atoms. Since the number of protons in the nucleus determines the identity of the atom, the number and kind of atoms must remain the same during chemical reactions. In *nuclear* reactions, nuclei can combine or divide to form different atoms.
- In nuclear reactions, the amount of energy added or released per atom is nearly always much higher in chemical reactions.
- On earth, chemical reactions occur with relative ease. In every biological organism, millions of chemical reactions occur each minute. Nuclear reactions occur naturally during radioactive decay, and may occur when cosmic rays enter our atmosphere, but on earth, compared to chemical reactions, nuclear reactions are rare. (In a star, nuclear reactions are common.)

Because the rules are so different, the detailed study of nuclear reactions is generally assigned to physics rather than chemistry. We make an exception for three nuclear reactions: radioactive decay, fission, and fusion. In **nuclear chemistry,** radioactive decay is a powerful tool in the study of che*mical reactions.* Together, decay, fission, and fusion explain how the atoms that we study in chemistry came to be formed.

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Lesson 25A: The Nucleus – Review

Pretest: If you recall the rules for the structure of the nucleus, try the problems in the practice set at the end of this lesson. If you can do those problems, you may skip this lesson.

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A Model for the Nucleus

Science has only a partial understanding of the nature of the atomic nucleus. When understandings in science are limited, *models* are developed that may be simplified, incomplete, or even speculative, but allow us to predict how systems will behave.

To explain the nuclear reactions that are important in chemistry, the model for the nucleus that we utilize in chemistry is simplified compared to the models of physics, but this simplified model can nearly always predict the impact of the structure of the nucleus on processes of interest in chemistry and biology.

Our chemistry model for the atom and its nucleus was introduced in Lesson 6B. To briefly review:

1. **Nuclear Structure**

   Atoms are composed three **subatomic particles.** In standard chemistry, our primary focus is on electrons. In *nuclear* chemistry, our focus is on protons and neutrons in the nucleus of the atom.

   - **Protons**
     - Protons have a **+1** electrical charge (1 unit of positive charge). Each proton has an mass of approximately 1.0 amu (atomic mass units) which is equivalent to 1.0 grams per mole.
     - The number of protons is the **atomic number** of a nucleus or atom. The number of protons determines the *name* (and thus the *symbol*) of a nucleus or atom. The number of protons determines the **nuclear charge** of an atom's nucleus.
     - The number of protons is a major factor in the atom's behavior.
     - The number of protons in an atom is never changed by *chemical* reactions, but can change during *nuclear* reactions.
   - **Neutrons**
     - Neutrons have an electrical charge of zero. A neutron has about the same mass as a proton: 1.0 amu.
     - Neutrons are believed to act as the glue of the nucleus: the particles that keep the repelling protons from flying apart.
     - Neutrons, like protons, are never gained or lost in *chemical* reactions, but the number of neutrons and protons in an atom can change in a *nuclear* reaction.
     - Unlike the number of protons, the number of neutrons in an atom has little to no influence on the types of *chemical* reactions that substances containing that atom will undergo. However, nuclei with the same number of protons but different numbers of neutrons will undergo different nuclear reactions.

       In addition, atoms with the same numbers of protons but differing numbers of neutrons in their nucleus will have different masses. As a result, some *physical* properties of the particles of a substance, such as their densities and the relative speeds at which the particles move, may differ measurably if its atoms have differing numbers of neutrons.

2. **The Nucleus**

   All of the protons and neutrons in an atom are found in the nucleus at the center of the atom. The diameter of a nucleus is roughly 100,000 times smaller than the effective diameter of most atoms. However, the nucleus contains all of an atom's positive charge and nearly all of its mass. Electrons are located outside the nucleus and are much lighter than protons and neutrons.

3. Types of Nuclei

Only certain combinations of protons and neutrons form a nucleus that is stable. In a nuclear reaction, if a combination of protons and neutrons is formed that is unstable, the nucleus will decay. In terms of stability, nuclei can be divided into three types.

- **Stable** nuclei are combinations of protons and neutrons that do not change in a planetary environment such as Earth over many billions of years.
- **Radioactive** nuclei are *somewhat* stable. Some radioactive nuclei exist for only a few seconds, and others exist on average for several billion years, but they fall apart (**decay**) at a constant and characteristic rate.
- **Unstable** nuclei, if formed in nuclear reactions, decay within a few seconds.

Nuclei that exist in the earth's crust include all of the stable nuclei plus some radioactive nuclei. All atoms with between one and 82 protons [except technetium (Tc) with 43 protons] have at least one nucleus found in the earth's crust that is stable. Atoms with 83 to 92 protons exist in the earth's crust but are always radioactive. Atoms with 93 or more protons exist on earth only when they are created in manmade nuclear reactions.

Radioactive atoms comprise a very small percentage of the matter on earth. Over 99.99% of the earth's atoms have stable nuclei that have not changed since their atoms came together to form the earth billions of years ago.

4. Terminology

Protons and neutrons are termed the **nucleons**. The combination of a certain number of protons and neutrons is called a **nuclide**. The set of nuclides that have the same number of protons (so they are the same atom) but differing numbers of neutrons are called the **isotopes** of the atom.

Isotopes

Some atoms have only one stable nuclide; others have as many as 10 stable isotopes.

Examples: All atoms with 1 proton are called hydrogen. Two kinds of hydrogen nuclei are stable: those with

- 1 proton; and
- 1 proton and **1** neutron, an isotope that is often referred to as **deuterium** or **heavy hydrogen**.

Most hydrogen atoms found on earth are the isotope containing one proton and no neutrons: only 1 H atom in about 6,400 contains a deuterium nucleus. However, deuterium can be separated from the majority isotope, and it has many important uses in chemistry.

An isotope of hydrogen consisting of one proton and *two* neutrons, called **tritium**, is not found in the earth's crust, but it can be isolated in measurable amounts from products in nuclear reactors. Unlike deuterium, tritium is radioactive. Half of the atoms in a sample of tritium will decay in 12 years.

Nuclide Symbols

Each nuclide has a **mass number** which is the *sum* of its number of protons and neutrons.

Mass Number of a nucleus = **Protons + Neutrons**

Example: All nuclei with 6 protons are carbon. If a carbon nucleus has 8 neutrons, the mass number of the carbon isotope is 14.

A nuclide can be identified in two ways,

- by its number of protons and number of neutrons, or
- by its **nuclide symbol** (also termed its **isotope symbol**).

The nuclide symbol for an atom has two required parts: the *atom symbol* and the *mass number.* The mass number is written as a ^{super}script in front of the atom symbol.

Example: The three isotopes of hydrogen can be represented as

- 1 proton + no neutrons *or* as $^{1}\mathbf{H}$ (a nuclide named "hydrogen-1");
- 1 proton + 1 neutron *or* as $^{2}\mathbf{H}$ (termed hydrogen-2 or deuterium); and
- 1 proton + 2 neutrons *or* as $^{3}\mathbf{H}$ (called hydrogen-3 or tritium).

Uranium has two isotopes that are important commercially and historically.

- $^{238}\mathbf{U}$, the most common naturally occuring isotope, contains 92 protons and 146 neutrons.
- $^{235}\mathbf{U}$, the isotope that is *split* in atomic bombs and nuclear power plants, contains 92 protons and 143 neutrons.

Using a table of atoms that includes atomic numbers, try this question.

Q. A nuclide with 49 protons and 60 neutrons has what nuclide symbol?

* * * * *

A. Atoms with 49 protons must be named silver, symbol **Ag**. The mass number of this nuclide is 49 protons + 60 neutrons = **109** . This isotope is called silver-109 and its symbol is $^{109}\mathbf{Ag}$ **.**

Nuclide symbols may also be written with the **nuclear charge** below the mass number. This is called **A-Z notation**, illustrated for tritium at the right. **A** is the symbol for mass number and **Z** is the symbol for nuclear charge.

$$^{3}_{1}\mathrm{H}$$

Any nucleus that includes protons is by definition an atom, and since the atom symbol also identifies the number of protons in the nucleus, Z values are not required to identify a nucleus. However, for many subatomic particles, the nuclear charge is not the same as the number of protons, and showing the nuclear charge will be helpful in clearly identifying these particles. In addition, for problems in which we must *balance* nuclear reactions, knowing the nuclear charge (Z) is necessary, and showing Z will be helpful.

Practice: First learn the rules above, then complete these problems.

1. The charge on the nucleus is determined by its number of ________________.
2. The mass number of a nucleus is determined by its number of ____________________.
3. Isotopes have the same number of ___________ but different numbers of __________.
4. Of the three sub-atomic particles, the two with the highest mass are ________________ and ________________.
5. Write the nuclide (isotope) symbol for a single proton using A-Z notation.
6. Consulting a table of atoms or periodic table, fill in the blanks below.

Atom Name	Atom Symbol	Protons	Neutrons	Atomic Number	Mass Number	Nuclide Symbol
Helium			2			
	Au		118			
		82			206	
						^{242}Pu

ANSWERS

1. **Protons** 2. **Protons + Neutrons** 3. Same number **Protons**, different number of **Neutrons**

4. **Protons and Neutrons.** 5. $^{1}_{1}H$ (A particle with one proton is always given the symbol H.)

6.

Atom Name	Atom Symbol	Protons	Neutrons	Atomic Number	Mass Number	Nuclide Symbol
Helium	**He**	2	**2**	**2**	**4**	**^{4}He**
Gold	Au	**79**	118	**79**	**197**	**^{197}Au**
Lead	**Pb**	82	**124**	**82**	206	**^{206}Pb**
Plutonium	**Pu**	**94**	**148**	**94**	**242**	^{242}Pu

In writing the nuclide or isotope symbols for atoms, the nuclear charge below the mass number is optional.

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Lesson 25B: Radioactive Decay Reactions

Stable Nuclei

Protons have a positive electrical charge. If there is more than one proton in a nucleus, the like charges of the protons repel and the nucleus will have a tendency to fly apart.

Neutrons are neutral: they have a zero electrical charge and they do not repel other particles or each other. Neutrons act in some way as the "glue" of the nucleus: if the *right* number of neutrons is mixed with the protons, the repelling protons remain in the nucleus, and the nucleus is stable.

For small nuclei, the neutron to proton ratio that results in a stable nucleus is about one to one. As the number of protons in nuclei increases, the number of neutrons needed to form a stable nucleus increases slightly faster: the n^0/p^+ ratio gradually increases.

For example:

- All stable fluorine nuclei have 9 protons and 10 neutrons.
- Chlorine has two stable nuclei: both have 17 protons, while one has 18 and the other 20 neutrons.
- All lead have 82 protons. The four stable isotopes of lead have 122, 124, 125, or 126 neutrons.

Radioactive Decay

An unstable nucleus does not have a stable ratio of neutrons to protons. Making a nucleus stable is more complex than just adding more neutrons as "glue" or being in the right range of ratios. Certain combinations of protons and neutrons are stable, but others are not.

A nucleus that is **radioactive** is between stable and unstable: it will have a tendency to *gradually* expel particles until a stable neutron and proton combination is achieved. The process of expelling particles from the nucleus is termed **radioactive decay**. Depending on the nucleus, radioactive decay may occur on average in seconds or gradually for up to billions of years.

Radioactive decay can be a powerful tool in the study of chemical reactions.

For example, for most atoms with less than 84 protons, stable isotopes exist, but radioactive isotopes also exist: the radioactive nuclei can either be found in nature or synthesized in nuclear reactors. A radioactive atom will undergo *decay* that we can detect, but the radioactive and non-radioactive forms of the atom have essentially the same behavior in chemical reactions.

By substituting a radioactive nucleus for a stable nucleus, we can "tag" the atoms in substances. Because we can detect the location of radioactive nuclei as they decay, we can track where they go and how they behave during chemical reactions, including reactions in biological systems. The use of *radioactive dyes* in medical imaging is one example of the importance of nuclear chemistry.

There are several types of radioactive decay, but the two types encountered most often in chemistry are **alpha (α) decay** and **beta (β) decay**.

Alpha Decay

In alpha decay, a particle with 2 protons and 2 neutrons is ejected from the nucleus. Such a particle is termed an **alpha particle**. Because it has 2 protons and 2 neutrons, an alpha particle has the same structure as a helium-4 nucleus, and it is given the same isotopic symbol as an He-4 nucleus.

Symbol for alpha particle = α particle = ${}^{4}_{2}\text{He}$

Example: The isotope U-238 undergoes radioactive decay by emitting an alpha particle. This nuclear reaction is written as

$$ {}^{238}_{92}\text{U} \xrightarrow{\alpha} {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He} $$

Alpha decay lowers the atomic number (and nuclear charge) of a nucleus by 2 and its mass number by 4.

Balancing Nuclear Reactions

Nuclear reactions balance differently than chemical reactions, but they balance relatively easily. The rule is:

> In nuclear reactions, both mass numbers and nuclear charge must be conserved.
>
> In a balanced *nuclear* reaction, on both sides of the arrow,
>
> - The *sum* of the *mass numbers* (A, on top) must be the same, and
> - The sum of the *nuclear charge*s (Z, on the bottom) must be the same.

The result is that nuclear reactions can be balanced by simple addition and subtraction.

Q. Use the nuclear balancing rule to write below the symbol for the nucleus remaining after the alpha decay of plutonium-242.

$$ {}^{242}_{94}\text{Pu} \xrightarrow{\alpha} \underline{\qquad\qquad} + {}^{4}_{2}\text{He} $$

* * * * *

The mass numbers on top add up to 242 on both sides. The nuclear charges on the bottom total 94 on both sides. The nuclear charge must be 92, and that means the atom is uranium. The mass number of the nucleus must be 238. The balanced nuclear reaction is:

$$ {}^{242}_{94}\text{Pu} \xrightarrow{\alpha} {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He} $$

Try one more.

Q. Which isotope is produced by the alpha decay of radium-226?

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A key to balancing nuclear reactions is to write the isotope symbols in A-Z notation. Start there for Ra-226.

* * * * *

Radium by definition has a nucleus with 88 protons, so this reaction begins

$^{226}_{88}\text{Ra} \xrightarrow{\alpha}$ Fill in the missing symbols.

* * * * *

In alpha decay, one product is always an alpha particle. Add its symbol on the right.

* * * * *

$$^{226}_{88}\text{Ra} \xrightarrow{\alpha} {}^{4}_{2}\text{He} +$$

Use the balancing rule to write the isotopic formula for the remaining particle: the nucleus left behind after the alpha particle is expelled.

* * * * *

$$^{226}_{88}\text{Ra} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{222}_{86}\text{Rn}$$

After the decay, the nucleus has 86 protons, so it must be radon (Rn). For the mass numbers to balance, the Rn nucleus must have a mass number of 222.

Practice A

1. Write the balanced equation for the alpha decay of radon-219.
2. How many protons and how many neutrons are in Rn-219?
3. How many protons and how many neutrons are in the nucleus left behind after the alpha decay of Rn-219? How many protons and neutrons are lost in the decay?
4. Lead-206 can be formed by the alpha decay of which radioactive isotope?

Beta Decay

Beta decay is another type of radioactive decay. In beta decay, a neutron decays into a proton and an electron, and the electron is expelled from the nucleus at high speed. An electron formed in this manner is termed a **beta particle**.

- Because an electron has no protons and neutrons, its mass number is zero.
- Because an electron has a negative charge, when it is formed in the nucleus its "nuclear charge" is negative one.

In nuclear reaction equations, a beta particle can be represented in these ways:

$$\text{Symbol for beta particle} = \beta \text{ particle} = {}^{0}_{-1}\beta \quad \textit{or} \quad {}^{0}_{-1}\text{e}$$

Normally, in the nucleus of an atom, protons are the only particles with a charge. However, on a *sub*atomic particle, the charge may be positive, zero, or negative. In the special case of an electron formed in the nucleus by beta decay, briefly, before the electron is expelled, it is a nuclear particle with a negative one charge.

In beta decay, the number of neutrons in a nucleus *decreases* by one, but the number of protons increases by one, so the mass number of the isotope stays the same.

Example: The equation for the beta decay of the radioactive isotope lead-210 can be written as

$$^{210}_{82}\text{Pb} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{210}_{83}\text{Bi}$$

Is the above equation balanced?

* * * * *

Before and after the reaction, the mass numbers total 210 and the nuclear charges total 82, so this is a balanced nuclear equation.

Using the rules for nuclear balancing, we can predict the structure and symbol for the products of beta decay. Apply the rules to this question.

Q. The isotope carbon-14 undergoes beta decay. Write the isotope symbols for the two nuclear particles formed in this reaction.

* * * * *

Begin by converting to A-Z notation: $^{14}_{6}\text{C} \xrightarrow{\beta}$

* * * * *

One product must be a beta particle. For its symbol, you may use a β or an e .

$^{14}_{6}\text{C} \xrightarrow{\beta} {}^{0}_{-1}\text{e} +$ Complete the balancing.

* * * * *

$$^{14}_{6}\text{C} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N}$$

The isotope formed by the beta decay of carbon-14 is nitrogen-14.

Practice B

1. From memory, write the symbols for an alpha particle and a beta particle.
2. Write balanced equations for these decay reactions.

   a. $^{40}\text{K} \xrightarrow{\beta}$ b. $^{239}\text{Pu} \xrightarrow{\alpha}$

3. The isotope iodine-131 is used for the treatment of hyperthyroidism: a condition in which the thyroid gland produces too much thyroid hormone. In the body, iodine is absorbed by the thyroid gland. If I-131 is administered to a patient, its beta decay kills cells in the thyroid. The result is a reduced level of thyroid hormone without surgery. Write the symbol for the nucleus produced by the beta decay of I-131.

4. Lead-206 can be produced by the beta decay of which nucleus?
5. In a part of what is termed a **radioactive decay series**, nucleus A with 88 protons and 140 neutrons can beta decay to form nucleus B. B can emit a high speed electron to form nucleus C, which can α decay to form nucleus D. Write isotopic symbols for A, B, C, and D.
6. Radioactive atom A with 82 protons and 132 neutrons emits a beta particle to become atom B. Atom B emits an alpha particle to become atom C, which can emit a high speed electron from the nucleus to form Atom D. Write the isotopic formulas for A, B, C, and D.

ANSWERS

Practice A

1. $^{219}_{86}\text{Rn} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{215}_{84}\text{Po}$

2. **86 Protons** and **133 neutrons.**

3. **84 protons** and **131 neutrons: 2 protons and 2 neutrons** are always lost in alpha decay.

4. $\boxed{^{210}_{84}\text{Po}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{206}_{82}\text{Pb}$

Practice B

2a. $^{40}_{19}\text{K} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{40}_{20}\text{Ca}$

2b. $^{239}_{94}\text{Pu} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{235}_{92}\text{U}$

3. $^{131}_{53}\text{I} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \boxed{^{131}_{54}\text{Xe}}$

4. $\boxed{^{206}_{81}\text{Tl}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{206}_{82}\text{Pb}$

5. $\underset{\textbf{A}}{\boxed{^{228}_{88}\text{Ra}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{B}}{\boxed{^{228}_{89}\text{Ac}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{C}}{\boxed{^{228}_{90}\text{Th}}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + \underset{\textbf{D}}{\boxed{^{224}_{88}\text{Ra}}}$

6. $\underset{\textbf{A}}{\boxed{^{214}_{82}\text{Pb}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{B}}{\boxed{^{214}_{83}\text{Bi}}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + \underset{\textbf{C}}{\boxed{^{210}_{81}\text{Tl}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{D}}{\boxed{^{210}_{82}\text{Pb}}}$

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Lesson 25C: Fission and Fusion

Fission

Nuclear fission is a reaction in which a large nucleus divides into two smaller nuclei, both of which contain more than two protons. If a fission reaction is accompanied by the creation of free neutrons, those neutrons can collide with other nearby fissionable nuclei and cause them to split.

If this "splitting of atoms" begins in a sample of fissionable nuclei that is large enough to have **critical mass**, the result can be a **chain reaction** which releases large amounts of energy. If the chain reaction is not controlled, the result is an **atomic bomb**.

In a **nuclear power plant** (a type of **nuclear reactor),** a chain reaction is controlled by adding materials that absorb some of the free neutrons. The large amount of energy produced by a chain reaction is then released gradually, and the resulting heat can be harnessed to drive turbines that produce electricity.

The example of nuclear fission encountered most often is the splitting of uranium-235. A typical reaction is

$$^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{236}_{92}U \rightarrow {}^{140}_{54}Xe + {}^{94}_{38}Sr + 2\ {}^{1}_{0}n + 2 \times 10^{10} \text{ kJ/mol}$$

In this reaction, a U-235 nucleus is struck by a free neutron. The neutron is at first absorbed, but this unstable nucleus then splits into two smaller nuclei plus 2 free neutrons. Those two neutrons can collide with other fissionable nuclei to create a chain reaction. These reactions produce amounts of energy per mole that are millions of times larger than that produced by chemical reactions such as the burning of fossil fuels.

A disadvantage of using fission for electricity generation is that the products include highly radioactive isotopes. Exposure to the radiation released by radioactive decay can cause cancer, and some of the waste products of fission remain significantly radioactive for thousands of years. A major issue in nuclear power generation is how to store the waste products so that they will not escape into the earth's biological environment.

Isotopic Separation

Both U-235 and U-238 can be split, but in practice only U-235 is an effective fuel for chain reactions. For use in nuclear power plants or weapons, naturally occurring uranium must **enriched**: meaning that the percentage of nuclei that are U-235 must be increased. In mined uranium ore, 99.3% of nuclei are U-238 and 0.7% are U-235. For nuclear weapons, uranium must be enriched to 20-80% U-235, and over 50 kilograms of this "weapons grade" uranium must be collected.

Since all isotopes, including U-235 and U-238, have the same tendency to react chemically, chemical reactions cannot effectively separate isotopes. However, particles with the lighter isotopes will be less dense, and they will move faster at a given temperature. Since atoms containing U-235 are lighter than atoms with U-238, in gaseous substances containing uranium the particles containing U-235 atoms diffuse slightly faster. Gaseous diffusion is therefore one method that is used to separate uranium isotopes. Because particles that are less dense tend to be moved toward the inside when spun in a circle at high speed, use of a centrifuge is another way to separate uranium isotopes.

Both gaseous diffusion and centrifugation of uranium-containing substances are slow and expensive processes. This makes it difficult (thankfully) to obtain the amount of highly enriched uranium needed to build a nuclear weapon.

Practice A

1. From memory, write the isotopic symbol for a free neutron.
2. Briefly describe the difference between fission and fusion.
3. Fill in the one missing isotopic symbol in this nuclear fission reaction.

$$^{235}_{92}U + ^{1}_{0}n \rightarrow ^{87}_{35}Br + \qquad\qquad + 6\ ^{1}_{0}n$$

4. Fill in the missing isotopic symbol that is the first reactant in this fission equation.

$$\qquad + ^{1}_{0}n \rightarrow ^{91}_{38}Sr + ^{146}_{56}Ba + 3\ ^{1}_{0}n$$

Fusion

Nuclear fusion is a reaction that combines two small nuclei to make a larger one. When a small nucleus such as helium is the product of fusion, this reaction produces large amounts of energy. An example of a fusion reaction is

$$^{2}_{1}H + ^{3}_{1}H \rightarrow ^{4}_{2}He + ^{1}_{0}n + 2 \times 10^{9}\ kJ/mol$$

In this reaction, two hydrogen nuclei are fused, and one product is a heavier helium nucleus. Fusion is a reaction that takes place in stars, including our sun, and in hydrogen bombs.

The primary reaction that causes stars to "burn" (release energy) is the conversion from hydrogen to helium. At the extremely high temperatures and pressures found in stars, lighter nuclei can fuse to form heavier nuclei, and those nuclei can undergo successive fusion reactions. After long periods of making heavier nuclei, some stars become unstable and explode. The atoms scattered into space from exploded stars can accumulate over time due to gravitational attraction, forming new stars and planets. The atoms that coalesced to form our own planet billions of years ago are nuclei, or the decay products of nuclei, that were originally formed by fusion in a star.

Fusion can use hydrogen as its fuel, and the products of hydrogen fusion are generally stable nuclei rather than long-lived radioactive isotopes. A nuclear reactor that could slow and control the fusion of hydrogen would therefore be a source of inexpensive and clean energy. To produce the energy needed for our society, such fusion reactors could replace the burning of fossil fuels and current nuclear power plants, both of which form products that can harm our environment. However, while nuclear reactors can control the rate of nuclear fission, currently no way has been discovered to engineer the gradual release of the energy of nuclear fusion.

Summary: To balance nuclear reactions, the rules you need *in memory* are

1. α particle = ${}^{4}_{2}\text{He}$; β particle = ${}^{0}_{-1}\beta$ *or* ${}^{0}_{-1}\text{e}$, neutron = ${}^{1}_{0}\text{n}$
2. Always use A-Z notation: mass number on top, nuclear charge on the bottom.
3. The mass numbers and nuclear charges must be conserved: both must *add* to give the same number on both sides of the arrows.
4. Fission splits a nucleus, fusion combines nuclei.

Practice B

1. If a single nucleus is formed as the product of this reaction, write its isotope symbol.

   $${}^{2}_{1}\text{H} + {}^{2}_{1}\text{H} \rightarrow$$

2. In stars that are *red giants*, helium-4 can fuse with beryllium-8 to form a single nucleus. Write the equation for this reaction.
3. Assuming that *one* isotope symbol is missing from these equations, fill in the missing isotope, then write the name for this *type* of reaction.

Type:

a. ${}^{235}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{92}\text{Kr} + \qquad + 3\ {}^{1}_{0}\text{n}$ ____________

b. ${}^{238}\text{U} \rightarrow {}^{4}\text{He} +$ ____________

c. ${}^{1}\text{H} + {}^{3}\text{H} \rightarrow$ ____________

d. ${}^{234}\text{Th} \rightarrow {}^{0}_{-1}\text{e} +$ ____________

ANSWERS

Practice A

1. ${}^{1}_{0}\text{n}$ 2. Fission splits a nucleus, fusion combines nuclei.

3. ${}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{87}_{35}\text{Br} + \boxed{{}^{143}_{57}\text{La}} + 6\ {}^{1}_{0}\text{n}$

4. $\boxed{{}^{239}_{94}\text{Pu}} + {}^{1}_{0}\text{n} \rightarrow {}^{91}_{38}\text{Sr} + {}^{146}_{56}\text{Ba} + 3\ {}^{1}_{0}\text{n}$

Practice B

1. $^{2}_{1}H + ^{2}_{1}H \rightarrow \boxed{^{4}_{2}He}$ 2. $^{4}_{2}He + ^{8}_{4}Be \rightarrow ^{12}_{6}C$

3a. $^{235}_{92}U + ^{1}_{0}n \rightarrow ^{92}_{36}Kr + \boxed{^{141}_{56}Ba} + 3\ ^{1}_{0}n$ **fission**

3b. $^{238}_{92}U \rightarrow ^{4}_{2}He + \boxed{^{234}_{90}Th}$ **alpha decay**

3c. $^{1}_{1}H + ^{3}_{1}H \rightarrow \boxed{^{4}_{2}He}$ **fusion**

3d. $^{234}_{90}Th \rightarrow ^{0}_{-1}e + \boxed{^{234}_{91}Pa}$ **beta decay**

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Lesson 25D: Fractions and Percentages

Pretest. If you earn a perfect score, you may skip this lesson.

1. 0.6% is what decimal equivalent?
2. 45/10,000 is what decimal equivalent and what percent?

* * * * *

Fractions and Decimal Equivalents

A **fraction** is a ratio: one quantity divided by another. In math, a fraction can be any ratio, but in science, "fraction" often (but not always) refers to a *part* of a larger total: a smaller quantity over a larger quantity. In dealing with percentages and fractions, we will call this

> **Rule 1.** **Fraction** = $\frac{\text{Quantity A}}{\text{Quantity B}}$ and often equals $\frac{\text{Part}}{\text{Total}}$ = $\frac{\text{Smaller number}}{\text{Larger number}}$

The **decimal equivalent** of a fraction is a number that results by dividing the top number of the fraction (the **numerator**) by the bottom number (the **denominator**).

An example of a fraction and its decimal equivalent is 1/2 = 0.50

> **Rule 2.** To find the decimal equivalent of a fraction, divide the top by the bottom.

Use your calculator if needed to answer: **Q.** The decimal equivalent of 5/8 = ________

* * * * *

A. 0.625

In chemistry calculations, the term "fraction" can refer to any fixed decimal number from 1.00 to 0.00 (such as 0.25) that can be obtained by dividing one quantity by another. In terms of numeric value, a fraction and its decimal equivalent are the same.

> Fraction = Decimal equivalent

In chemistry, both 1/2 and 0.50 are termed fractions. Depending on the context, when a chemistry problem asks "what is the fraction…" it may be asking for

- a fraction in an x/y format, or
- a decimal equivalent number such as 0.25 .

Usually, from the context and examples of related problems, it will be clear what type of fraction is wanted.

Calculating Percentages

A **percent** multiplies a decimal equivalent by 100%.

A familiar example is 1/2 = 0.50 x 100% = 50%.

For those who are not math-inclined, percentages provide more familiar numbers to measure change than numbers with decimals. However, when a *percent* is required for an answer, in most chemistry calculations you will need to solve for the *decimal equivalent* first, then convert to percent. To convert a decimal equivalent to a percent, multiply by 100% (moving the decimal twice to the right). Let's call this

> **Rule 3**. **Percent** = fraction x 100% = (decimal equivalent) x 100%
>
> To find a %, calculate the decimal equivalent *first*.
>
> To find a %, write the fraction, then the decimal equivalent, then the %.

Example: 1/8 is what percent?

Write the fraction, then its decimal equivalent, then multiply by 100%.

1/8 = 0.125 x 100% = **12.5%**

Apply Rule 3 to this problem: **Q.** 25 is what percentage of 400?

* * * * *

The rule: if you WANT a percentage, first write the fraction, then its decimal equivalent.

In this problem, the question is: the smaller number is what part of the larger number? Write the fraction definition and fill in the numbers.

* * * * *

$$\text{Fraction} = \frac{\text{Part}}{\text{Total}} = \frac{\text{Smaller}}{\text{Larger}} = \frac{25}{400} = \_\_\_\_\_\_\_\_$$ (fill in the decimal equivalent)

* * * * *

$$\text{Fraction} = \frac{\text{Part}}{\text{Total}} = \frac{\text{Smaller}}{\text{Larger}} = \frac{25}{400} = 0.0625$$ What will the percentage be?

* * * * *

Percent = fraction x 100% = (decimal equivalent) x 100% = 0.0625 x 100% = **6.25%**

Similarly, if you are *given* a percentage to use in a calculation, you must *change* the percentage to its decimal equivalent. Conversion calculations and mathematical equations nearly always require numeric values (the fraction or its decimal equivalent), not the percentages that are "values x 100%".

Since Percent = (decimal equivalent) x 100% , | Decimal equivalent = Percent / 100% |.
Let's call this

<u>Rule 4</u> . Change a percentage to its decimal equivalent before use in conversions.
To change a percentage to its decimal equivalent, divide by 100% (moving the decimal twice to the left).
Decimal equivalent of percent = Percent / 100%

<u>Examples</u>: In calculations, change 25% to 0.25 ; change 0.50% to 0.0050

Apply Rule 4 to this problem. **Q.** 3.5 percent of 12,000 is ?

* * * * *

If fractions for you are easy math, solve in any way you wish. If you need a systematic approach, try the following.

For percentages in calculations, the fundamental rules are,

- if you are *given* a percentage, as step one convert it to a decimal equivalent;
- if you WANT a percentage, first find the decimal equivalent WANTED.

In this problem, since we were *given* a percent, step one is to convert to the decimal equivalent.

Decimal equivalent = percentage / 100% = 3.5% / 100% = **0.035**

There are many ways to solve from here. Having converted to the fraction, it may be intutitive that

3.5% of 12,000 = 0.035 x 12,000 =

If not, a more methodical way is to solve this equation using our equation method:

Fraction = decimal equivalent = $\frac{\text{Smaller}}{\text{Larger}}$

Make a data table that matches the terms in the boxed equation.

* * * * *

DATA:

Decimal equivalent = 0.035

Smaller = ? (Since the number be asked for is less than 100% of 12,000, it is smaller)

Larger = 12,000

SOLVE: (solve the boxed equation for the WANTED variable, then substitute.)

? = Smaller = (decimal equivalent) x (larger) = 0.035 x 12,000 = **420**

Sanity check: since 10% of 12,000 is 1,200 , 3.4% should be *about* 400? Check.

There are many ways to solve fraction and percentage calculations. Use one that works for you.

Practice: First memorize the rules above, then do each of these problems.

1. 1/5 is what decimal equivalent and what percent?
2. 4.8% is what decimal equivalent?
3. 9.5/100,000 is what decimal equivalent and percent?
4. What percentage of 25 is 7?
5. What amount is 0.450% of 7,500. ?
6. Twelve is what percent of 24,000?
7. Complete the two problems on the pretest to this lesson.

ANSWERS

Pretest: 1. 0.006 2. 0.0045, 0.45%

Practice

1. Decimal equivalent of 1/5 = **0.20.** Percent = decimal equivalent x 100% = **20.%**
2. Decimal equivalent = percent / 100% = 4.8% /100% = **0.048**
3. Move the decimal 5 times to divide by 100,000. Decimal equivalent = **0.000095 = 9.5 x 10^{-5}**

   Percent = decimal equivalent x 100% = 0.000095 x 100% = **0.0095% = 9.5 x 10^{-3} %**
4. To calculate percent, calculate fraction, then decimal equivalent, then percent.

   What fraction of 25 is 7? Fraction = $\frac{\text{Part}}{\text{Total}} = \frac{7}{25} = 0.28$

   Percent = fraction x 100% = decimal equivalent x 100% = 0.28 x 100% = **28%**
5. To calculate an amount, change % to decimal equivalent by dividing by 100. 0.450% = 0.00450

   ? = 7,500 x 0.00450 = **34**
6. To calculate percent, write fraction, then decimal equivalent, then percent.

   12 is what part of 24,000? = 12 is what fraction of 24,000?

   Fraction = $\frac{\text{Part}}{\text{Total}} = \frac{12}{24{,}000} = \frac{12}{24 \times 10^3} = 0.5 \times 10^{-3} = 5.0 \times 10^{-4}$

   This number in exponential notation is a decimal equivalent. A decimal equivalent is any numeric value that has no denominator (which means 1 is the denominator) and is derived from a fraction.

   Percent = fraction x 100% = decimal equivalent **x** 100% =

   = 5.0 x 10^{-4} x 100% **= 5.0 x 10^{-2} % = 0.050 %**

7a. Decimal equivalent of percent = percent / 100% = 0.6% /100% = **0.006**

7b. To find the decimal equivalent of a fraction, divide.

45/10,000 = **0.0045**

Percent = fraction x 100% = decimal equivalent x 100% = 0.0045 x 100% = **0.45%**

* * * * *

Lesson 25E: Natural Logarithms

Prerequisites: Complete the earlier lesson on Base 10 Logarithms before beginning this lesson.

* * * * *

I. Review of Base 10 Logs

To solve half-life calculations, you will need to know the rules for **natural log (ln)** calculations. The rules for natural logs parallel base 10 logs, and because our number system is based on 10, it is easier to learn the logic of the base 10 rules first.

If you have not completed the lesson on *Base 10* Logarithms in a prior module, do so now. If you have completed that lesson, review the rules in the following summary, then complete the practice set below.

Summary: Log Rules to Commit to Memory

1. A *logarithm* is simply an exponent: the power to which a base number is raised.
2. A logarithm answers the question: if a number is written as a base to a power, what is the power?
3. [log] buttons on a calculator find the *power* of a number written as 10 to a power.
4. To check log answers: when a number is written in scientific notation, its power of 10 must agree with its base 10 logarithm within ± 1.
5. The equation defining a **log** is $\boxed{\log 10^x = x}$; the *log* of 100 is 2 .
6. Knowing the log of a number **x**, find the number by "taking the antilog" or "finding the inverse log." Perform the operation $10^{\log x}$.

   $\boxed{10^{\log x} = x}$. Recite and repeat to remember: "10 to the log x equals x."

   As an easy example, remember: $10^{\log 100} = 10^2 = 100$
7. On a calculator, to convert a log value to a number,
   - input the log value, then press [INV] [LOG] ; or [2nd] [LOG] ; *or*
   - Input the log, then press [10^x]. *or* input 10, [x^y] , input the log , [=].

Practice A: Practice with the calculator you will use on tests. If you have problems with *any* of these, review the previous lesson on base 10 logarithms.

1. Do these without a calculator.

   a. $\text{Log}(10^{14}) =$ ________________________________ (expo ± 1 ? __)

   b. Log(10) = ________________________________

   c. Log(1) = ________________________________

Do those below with a calculator.

2. $10^{3.2}$ = (in scientific notation):______________________________ (expos ± 1 ?) ____

3. $10^{-12.3}$ = ___________________________ (expos ± 1 ?) ____

4. $10^{-0.2}$ = (number): _________(scientific notation): _____________(expos ± 1 ?) ____

5. $\text{Log}(2.0 \times 10^{14}) =$ ________________________________ (expo ± 1 ? __)

6. $\text{Log}(2.0 \times 10^{-14}) =$ ____________________________ (expo ± 1 ? __)

7. Log 5.0 = _________________________________ (expo ± 1 ? __)

8. Log 0.0050 = _________________________________ (expo ± 1 ? __)

9. Log x = 4.7 , x = ___________________________ (expo ± 1 ? ___)

10. Log A = –8.2 , A= __________________________ (expo ± 1 ? ___)

11. Log D = –0.50 , D = __________________________ (expo ± 1 ? ___)

12. Log x = 6.6 , antilog = __________________________ (expo ± 1 ? ___)

13. $10^{-9.5}$ = ________________________________ (expos ± 1 ?) ____

14. Log (3.0×10^{-5}) = ______________________________ (expo ± 1 ? __)

II. Base *e* Rules

a. The Symbol *e*

In mathematical and scientific equations, the lower-case ***e*** is an abbreviation for a number: **2.7182818…** For calculations, the value ***e*** = 2.718 must be memorized.

The number ***e*** has many interesting mathematical properties. In science, ***e*** is found in many equations that predict natural phenomena. In these equations, ***e*** is the base for values expressed in the form e^x, and ***e*** is termed the **natural exponential**.

To solve calculations involving radioactive half-life, we will use both the natural exponential ***e*** and the natural log function **ln**. Let's consider calculations with ***e*** first.

b. Calculating with natural exponentials

We know that e^1 = __________(what number?)

* * * * *

2.718…. Using 1 and the [e^x] button, write the key sequence that produces that answer for e^1 on your calculator.

* * * * *

- A standard TI-type calculator might use: 1 [e^x].
- On an RPN scientific calculator, try: 1 [enter] [e^x].

Circle or write a key sequence that works on your calculator, then use your key sequence to do these, then check your answers below.

1) e^2 = 2) $e^{2.5}$ = 3) e^{-1} = 4) $e^{-2.5}$ =

(Because the statistical basis for significant figures does not apply to logarithmic calculations, we will use this general rule: during ***e*** and **ln** calculations, round numbers in answers to 3 significant figures.)

* * * * *

1) e^2 = **7.39** 2) $e^{2.5}$ = **12.2**

Recall that to enter a negative number, you usually use a [**+/-**] key.

3) e^{-1} (= 1/e = 1/2.718..) = **0.368** 4) $e^{-2.5}$ (= $1/e^{2.5}$) = **0.0821**

c. **Calculating Natural Logs**

The **ln** function (the **natural log**) answers this question: if a number is written as ***e*** to a power, what is the power?

Just as by definition, **log** $10^x \equiv x$, the natural log definition is $\boxed{\ln e^x \equiv x}$

Use the natural log definition to do these with*out* a calculator.

1) $\ln e^0 =$ ______ 2) $\ln e^1 =$ ______ 3. $\ln e^{-4} =$ ________

* * * * *

1) $\ln e^0 = \mathbf{0}$ 2.) $\ln e^1 = \mathbf{1}$ 3) $\ln e^{-4} = \mathbf{-4}$

By definition, $\mathbf{\ln e} =$ _____ .

* * * * *

$\ln e = \ln e^1 = \mathbf{1.}$

Try this one in your head: **ln(2.718)** should equal about ____________ .

* * * * *

$\ln(2.718) \approx \ln e \approx \ln e^1 \approx \mathbf{1.}$

Now use your calculator for the same calculation: **ln(2.718)** = ___________

* * * * *

Is the calculator answer close to the mental arithmetic answer?

Write down the key sequence that works for **ln(2.718)** above.

The same steps should take the natural log of any positive number. Try these.

1) ln 314 = ________________

* * * * *

1) ln 314 = **5.75**

To *check* an answer, *after* writing it down, use the $\boxed{e^x}$ key and see if you *return* to the number you were taking the **ln** of. Try that as a check on these:

2) ln 0.0050 = _________________ (after writing answer, use $\boxed{e^x}$. Check? ___

3) $\ln (6.02 \times 10^{23}) =$ _________________________ Check? ___

4) $\ln (19.29 \times 10^{-15}) =$ _________________________ Check? ___

* * * * *

2) ln 0.0050 = **– 5.30** 3) $\ln(6.02 \times 10^{23}) =$ **54.8** 4) $\ln(19.29 \times 10^{-15}) =$ **– 31.6**

Note in part 4) that a calculator does *not* require the input of *scientific* notation. However, if you use the $\boxed{e^x}$ key to check your answer, it will likely return the original number *converted* to scientific notation.

Practice B: Use the calculator you will use on quizzes and tests.

1. $e^{2.0}$ =
2. $e^{-4.7}$ =
3. e^{-11} =
4. ln 42 =
5. ln 0.020 =
6. ln(9 x 10^5) =
7. ln(5.0 x 10^{-4}) =
8. ln(10^{-4}) =

d. **Converting ln Values to Numbers**

A base **10** definition: $\mathbf{10^{\log x} = x}$

A base *e* definition: $\mathbf{e^{\ln x} = x}$ Note the similarities.

Using the bottom equation, for some calculations involving **ln** and ***e*** you will not need a calculator. Try this:

$e^{\ln(-11)}$ = ________

* * * * *

$e^{\ln(-11)}$ = **–11**

The equation $\mathbf{e^{\ln x} = x}$ also means that if you know the **ln** value, to find the corresponding fixed decimal *number*, make the ln value a power of ***e***.

If the ln value = 1, the number (do this one in your head) is ___________

* * * * *

e^1 = **2.718…**

Knowing that answer, do the same **ln** to number conversion on your calculator by taking the antilog.

If the **ln value** = 1, the **number** obtained using the calculator is ___________

* * * * *

Input the **ln**, then press [INV or 2nd] [**ln**] *or* press [e^x].

Write down or circle the key sequence that converted **ln** = 1 to the number 2.718…

Use your key sequence to convert the following **ln** values to numbers. Write first write the number in terms of *e*, then the number, then the number in scientific notation.

1) If **ln = 6** , number = *e* = (number): ________ = (sci. notation):_____________

* * * * *

1) If ln = 6 , number = e^6 = (nbr): **403** = (sci. notation): $\mathbf{4.03 \times 10^2}$

Unlike base 10, in base ***e*** calculations there is no obvious correlation between the scientific notation exponent and the base ***e*** logarithm that helps in checking your answer. However, you can check by taking the **ln** of the number answer and see if it returns to the original **ln** value.

Try these.

2) If **ln = –4.5** , number = *e* = (nbr): _________ = (sci. notation):_____________

3) **ln = 57.2** , number =________________________

4) **ln [A] = 0.0300** , [A] = *e* = (nbr. and unit): _________________

* * * * *

2) If ln = –4.5 , number = $e^{-4.5}$ = **0.0111** = $\mathbf{1.11 \times 10^{-2}}$

3) ln = 57.2 , number = $e^{57.2}$ = $\mathbf{6.94 \times 10^{24}}$

4) If ln [A] = 0.0300 , [A] = $e^{0.0300}$ = **1.03 M**

e. **Units and Logarithms**

Note that in 4) above, the unit expected for a concentration has been added. From a strict mathematical perspective, logarithms cannot be taken of values with units, and logarithm values do not have units. All precisely stated scientific relationships obey these rules.

However, some of these precise scientific equations can be very complex, or they involve quantities that are difficult to measure. In those cases, the equations we write in chemistry are often "shortcuts" which simplify the complex relationships in order to speed problem solving.

When using these shortcut equations, the rules for dimensional homogeneity and unit cancellation may not apply, and special rules may be needed to assign units to answers. To make shortcut equations work, two of our rules will be

> When taking the logarithm of a value with units, write the result as a value without units.
>
> If a WANTED quantity is based on a logarithm value, first (if needed) make all of the supplied units in the problem *consistent*, then *add* the appropriate consistent unit to the answer.

The quantity involved most often in log calculations will be concentration in moles per liter. The rule will be: if a [x] is WANTED, add moles/liter (M) to the answer.

Apply that rule to the following problems. Write the answer as a number *or* in scientific notation. When in doubt, check answers as you go.

5) **ln [Z] = –12.5** , [Z] =________________________

* * * * *

5) [Z] **=** $e^{-12.5}$ = **3.73 x 10^{-6} M** (add the *unit* of concentration: mol/L)

6) ln [R] = – 0.17 , [R] =________________________

7) [D] = $e^{-1.39}$, [D] =________________________

8) ln(0.250 M) = ________________________

9) ln [A] = – 2.63 , [A] = ______________

* * * * *

6) [R] = $e^{-0.17}$ **= 0.844 M** 7) [D] = **0.249 M (add M)**

8) ln(0.250 M) = **– 1.39 (drop the unit)** 9) **[A]** = $e^{-2.63}$ **= 0.0721 M**

f. **Notation with *e* and ln**

- Some calculators use an **E** at the right side of the answer screen to show the power of **10** for numbers in scientific notation. Note that this is *not* the same as the symbol ***e*** for the natural exponential.
- Be careful to distinguish "taking the **ln**" from "the **ln** value."

Ln(7.389) = _____________ . Try it. You should get close to 2.

But if **ln = 7.389** , the number *with* that **ln** is __________ Try it.

* * * * *

If **ln** = 7.389, the number is $e^{7.389}$ = **1,620**

If you get lost on a natural log calculation, a good strategy is to do a similar and simple base-10 mental and calculator computation, and then apply the same logic to the natural log case. Simple base 10 calculations can often be solved in your head, and the formulas and steps for base 10 and base ***e*** calculations are parallel.

g. Converting between base 10 and natural logs

A general rule for logarithms of any base is $\log_b(x) = \ln(x)/\ln(b)$ where **b** is the base. For base 10 logs, this equation becomes

$\text{Log}_{10}(x) = \ln(x)/\ln(10)$ = $\text{Log}_{10}(x) = \ln(x)/2.303$

This relationship is generally memorized as **ln(x) = 2.303 log(x)**

"The *natural* log of a number is always 2.303 times higher than the base 10 log."

Summary: Add these rules to your in-memory log-rule list.

9. The symbol ***e*** is an abbreviation for a number with special properties: **e = 2.718...**
10. The **ln** (**natural log**) function answers the question: if a number is written as ***e*** to a power, what is the power?
11. Knowing the **ln**, to find the number, take the *antilog*. On a calculator,
    - input the **ln** value, then press INV ln ; or
    - Input the **ln** value, then press e^x. An ln is simply an exponent of ***e***.
12. When you encounter **log** or **ln** calculations, it helps to write:

    $\log 10^x = x$ and $10^{\log x} = x$. "The log of 10 to the x is x; 10 to the log x is x."

    $\ln e^x = x$ and $e^{\ln x} = x$. Write the base 10 rules, then substitute ***e*** and **ln**.

    Note the patterns. Note the logic: A log is an exponent.
13. **ln(x) = 2.303 log(x)**

Practice C: Try the odd-numbered problems first. Complete the even numbered problems for additional practice or pre-test review.

1. $e^{5.2}$ =
2. $e^{-1.7}$ =
3. $e^{-20.75}$ =
4. ln 1066 =
5. ln 0.0050 =
6. ln(3×10^8) =
7. ln (14.92×10^{-6}) =
8. ln $e^{6.2}$ =
9. $e^{\ln(-42)}$ =
10. If ln = –6.8 , number = e = (number in sci. notation):____________________
11. If ln D = 7.4822 , D =
12. If ln = –12.5 , antilog =
13. If log [A] = –9 , [A] =
14. If log x = 13.7 , x =
15. Log A = –13.7 , A =
16. $10^{-11.7}$ =
17. ln [B] = –13.7 , [B] =
18. $e^{-11.7}$ =
19. ln(0.050 M) =
20. $e^{-0.693}$ =
21. If log(x) = 5.0 , ln(x) =
22. if ln(x) = 34.5 , log(x) =
23. If **ln(x) =** – (0.075 day^{-1})(4.0 days) **; x = ?**

24. Given that $\boxed{\ln[A]_t = (-\ 0.0173\ s^{-1})\ (\ t\ )}$

a. If t = 20. s, [A] = ?   b. If [A] = 0.500 M, t = ?

25. Given that $\boxed{\ln[A]_t = (-\ 0.0241\ yrs.^{-1})\ (\ t\ )}$

a. If [A] = 0.0025 M, t = ?   b. If t = 28.8 years, [A] = ?

ANSWERS

Practice A

1a. $Log(10^{14}) =$ **14**4. 1b. $Log(10) = Log(10^{1}) =$ **1** 1c. $Log(1) = Log(10^{0}) =$ **0**

2. $10^{3.2} =$ **1.6×10^{3}** 3. $10^{-12.3} =$ **5.0×10^{-13}** 4. $10^{-0.2} =$ **0.63** = **6.3×10^{-1}**

5. $Log(2.0 \times 10^{14}) =$ **14.3** 6. $Log(2.0 \times 10^{-14}) =$ **-13.7** 7. Log 5.0 = **0.70**

8. Log 0.0050 = **-2.3** 9. Log x = 4.7 , x = **5.0×10^{4}**

10. Log A = -8.2 , A= **6.3×10^{-9}** 11. Log D = -0.50 , D = **0.32**

12. Log x = 6.6 , antilog = **4.0×10^{6}** 13. $10^{-9.5} =$ **3.2×10^{-10}**

14. $Log(3.0 \times 10^{-5}) =$ **-4.5**

Practice B

1. $e^{2.0} =$ **7.39** 2. $e^{-4.7} =$ **9.10×10^{-3}** 3. $e^{-11} =$ **1.67×10^{-5}** 4. ln 42 = **3.74**

5. ln 0.020 = **-3.91** 6. $\ln(9 \times 10^{5}) =$ **13.7** 7. $\ln(5.0 \times 10^{-4}) =$ **-7.60**

8. $\ln(10^{-4}) = \ln(1 \times 10^{-4}) =$ **-9.21**

Practice C

1. $e^{5.2} =$ **181** 2. $e^{-1.7} =$ **0.183** 3. $e^{-20.75} =$ **9.74×10^{-10}**

4. ln 1066 = **6.97** 5. ln 0.0050 = **-5.30** 6. $\ln(3 \times 10^{8}) =$ **19.5**

7. $\ln(14.92 \times 10^{-6}) =$ **-11.1** 8. $\ln e^{6.2} =$ **6.2** 9. $e^{\ln(-42)} =$ **-42**

10. If ln = -6.8 , number = $e^{-6.8}$ = (number in std. notation): **1.11×10^{-3}**

11. If ln D = 7.4822 , D = **1780** 12. If **ln = -12.5** , antilog = **3.73×10^{-6}**

13. If log [A] = -9 , [A] = **10^{-9} M** 14. If log x = 13.7 , x = **5.01×10^{13}**

15. Log A = -13.7 , A= **2.00×10^{-14}** 16. $10^{-11.7} =$ **2.00×10^{-12}**

17. ln [B] = -13.7 , [B] = **1.12×10^{-6} M** 18. $e^{-11.7} =$ **8.29×10^{-6}**

19. ln(0.050 M) = **-3.00** (drop the unit) 20. $e^{-0.693} =$ **0.500**

21. If log(x) = 5.00 , ln(x) = ? $\boxed{\ln(x) = 2.303 \log(x)}$; ln(x) = (2.303)(5.00) = **11.5**

22. If $\ln(x) = 34.5$, $\log(x)$ = ? | $\ln(x) = 2.303 \log(x)$ | ; $\log(x) = 34.5/2.303 =$ **15.0**

23. If $\ln(x) = -(0.075 \text{ day}^{-1})(4.0 \text{ days})$ **; x = ?**

$\ln(x) = -0.300 \ (\text{day}^{-1})(\text{days}) = -0.300 \text{ day}^0 = -0.300\ (1) = -0.300$

$x = e^{\ln(x)} = e^{-0.300} =$ **0.741**

24. Given that $\ln[A]_t = (-0.0173 \text{ s}^{-1})(t)$

a. Strategy: to find $[A]_t$, first solve for **ln**$[A]_t$

$$? = \ln[A]_t = (-0.0173 \text{ s}^{-1})(\mathbf{20.0\ s}) = \mathbf{-0.346}$$

WANTED is **[A]** at 20 s. Known is: $\ln[A]_{20\ s} = -0.346$ Solve for $[A]_{20\ s}$.

* * * * *

$$[A] = e^{\ln[A]} = e^{-0.346} = \mathbf{0.708\ M}$$ (If a [ ] is wanted, add M as unit)

b. Strategy: Solve for t in symbols first.

* * * * *

$$t = \frac{\ln[A]}{-0.0173 \text{ s}^{-1}} = \frac{\ln[0.500\text{ M}]}{-0.0173 \text{ s}^{-1}} = \frac{-0.693}{-0.0173 \text{ s}^{-1}} = \boxed{\mathbf{40.0\ s = t}}$$ $(1/\text{s}^{-1} = (\text{s}^{-1})^{-1} = \text{s})$

25. Given that $\ln[A]_t = (-0.0241/\text{yr.})(t)$

a. Strategy: Solve for t in symbols first.

* * * * *

$$t = \frac{\ln[A]}{-0.0241\text{ /yr.}} = \frac{\ln[0.0025\text{ M}]}{-0.0241\text{ /yr.}} = \frac{-5.99}{-0.0241\text{ /yr.}} = \boxed{\mathbf{249\ years\ = t}}$$

b. Strategy: to find $[A]_t$, first solve for **ln**$[A]_t$

$$? = \ln[A]_t = (-0.0241\text{ /yr.})(\mathbf{28.8\ yr.}) = \mathbf{-0.694}$$

WANTED is **[A]**. Known is: $\ln[A] = -0.694$ Solve for [A].

* * * * *

$$[A] = e^{\ln[A]} = e^{-0.694} = \boxed{\mathbf{0.500\ M}}$$ If a [ ] is wanted, add M.

* * * * *

Lesson 25F: Radioactive Half-Life Calculations

Half-Lives: Simple Multiples

The half-life of a reactant (symbol $t_{1/2}$) is the time required for half of the particles of the reactant to be used up in a reaction.

A radioactive nucleus has a characteristic half-life: in any sample, the time in which half of the nuclei decay is *constant*.

The rate of *chemical* reactions changes with changes in temperature, but in radioactive decay, a *nuclear* reaction, the time of a half-life does not change significantly at temperatures up to several thousand kelvins.

This means that each radioactive nucleus has a *characteristic* half-life. Some radioactive nuclides have a half-life of a few seconds; others have a half-life of billions of years. It is not possible to predict when any one nucleus will decay. However, in any sample of more than a few hundred of a given nucleus, the time in which *half* of the nuclei decay is always the same.

If we can calculate (or look up) the half-life, we know how long it will take for half of the nuclei to decay. By solving half-life equations, we can also calculate how long it will take for any percentage of the nuclei in a sample to decay.

Half-Life Calculations For Simple Multiples

In calculations involve half-life, the two variables will generally be the *time* over which a given nuclide in a sample decays and the *percentage* of those nuclei that remain. If the time period for the decay is equal to either the half-life or a simple multiple of the half-life, answers can be calculated by mental arithmetic.

- If a sample of a given nucleus has decayed for a time equal to one half-life, 1/2 of the original nuclei have decayed and half remain.

  If the nuclei are in a sample that has a constant volume (which should be assumed unless other conditions are stated), 1/2 of the original *concentration* of the reactant also remains after one half-life.

- After two half-lives (double the time of the half-life), the number of nuclei remaining is half *of* the half that remained after the first half-life: half of 1/2 = 1/4 (25%) of the original nuclei remain and 75% have decayed.
- At triple the half-life, 1/2 of 1/4 = 1/8 (12.5%) of the original nuclei remain.

Apply the rules above to the following problem.

Q. Fluorine-18, a radioactive isotope used in nuclear medicine, has a 1.8 hour half-life. How long will it take for 87.5% of the F-18 nuclei in a sample to decay?

* * * * *

A. If 87.5% has decayed, 12.5% remains. How many half-lives are required? How much time would this be?

* * * * *

12.5% remains at 3 half-lives; 3 x 1.8 hours = **5.4 hours**

To solve radioactive decay calculations for these "simple multiple" cases, given any one of headings in the table below, you will need to be able to fill in the rest of the table from memory. This should not be difficult: note that in the two middle columns, each number is simply half of the one above.

For Radioactive Nuclei, at time =	Fraction Remaining	Percent Remaining	Percent Decayed
0	1	100%	0%
One half-life	1/2	50%	50%
Two half-lives	1/4	25%	75%
Three half-lives	1/8	12.5%	87.5%

For calculations that are not easy multiples, we will use these rules to make estimates of answers.

Practice A: Write the table above until, given the top row, you can fill in four rows below from memory. Then complete these problems.

1. The nucleus of Pu-239 undergoes radioactive decay with a half-life of 24,400 years. In a sample of constant volume containing Pu-239,
   a. After how many years will 25% of the original Pu-239 nuclei remain?
   b. After how many half-lives will the [Pu-239] be 1/16th of its original concentration?
   c. What percentage of the Pu-239 has decayed after exactly 4 half-lives?

Rate Constants for Radioactive Decay

In half-life calculations that do not involve simple multiples, we can solve using rate equations and the math of natural logs.

Each radioactive nucleus has a rate constant for decay (**k**) that is characteristic: a value that is constant. Different radioactive nuclei will have different values for **k** .

One way to write the equation that predicts the decay rate for radioactive nuclei is

$$\ln \left[\frac{[A]_t}{[A]_0} \right] = -kt \quad \text{which can be written} \quad \boxed{\ln(\textit{fraction remaining}) = -kt}$$

In the first equation, $[A]_t / [A]_0$ is the *fraction* of nuclei *remaining* at time = t .

For example: after one half-life, half (50%) of the original sample remains and the *fraction remaining* is **0.50** .

After two radioactive half lives, the fraction remaining would be?

* * * * *

After two half-lives, the percentage remaining is 25% (1/4), and the fraction remaining is therefore **0.25** .

Since the reactant is being used up over time, the value of [A] after time = **t** will be less than it was at time = **0**, and the value of the fraction will be less than one. The decimal equivalent of the fraction will always have the form **0.**xxx .

The units used to calculate the fraction can be concentration *or* any consistent units that are proportional to concentration, including the mass or number of particles in a sample that has a fixed volume.

Radioactive half-life calculations often involve fractions or percentages, and in those cases the form of the equation above that includes (*fraction remaining*) will be the most convenient to use. However, to use the equation with the term *(fraction remaining),* you must calculate using fractions or the decimal equivalent of fractions, rather than percentages.

If a percentage is WANTED, you will need to solve for the fraction first. If a percentage is *given*, you will need to convert to its decimal equivalent to use in the equation.

When using fractions and percentages, recall that

- A fraction can be expressed as x/y or as a decimal equivalent value that is x divided by y.
- Percentage = fraction x 100% *and* decimal equivalent = percent/100%
- Percentage remaining = 100% — percentage decayed
- Fraction remaining = 1.000 — fraction decayed
- In decay calculations, the fractions will have a value between 1.00 and 0 (such as 0.25).

(For practice in converting between percentages and fractions, see Lesson 25C.)

Practice B: Commit to memory the equation above that includes *(fraction remaining)*, the complete each of these.

1. If 90% of a sample has decayed, what is the fraction remaining?
2. If the fraction of a sample that has decayed is 0.40, what percent remains?
3. If the value for **ln**(fraction of sample remaining) is -1.386 ,
   a. What is the fraction of the sample remaining?
   b. What percentage has decayed?
4. For the decay of a radioactive nucleus, if the rate constant of the reaction is **k** = 0.04606 hours^{-1} , what percentage remains after 50.0 hours?

Half-Life Calculations For Non-Simple-Multiples

For a radioactive nucleus, after a time equal to one half-life ($t_{1/2}$), half of a sample has decayed and *half* remains. Substituting into the equation

ln(*fraction remaining*) = −kt at a time equal to one half-life,

we can write ln(**1/2**) = −k $t_{1/2}$ Solve this equation in symbols for half-life.

* * * * *

One way of several to write the equation is

$t_{1/2} \equiv -\ln(1/2) / k$ This equation is one way to *define* radioactive half-life.

In the above equations, the rate constant (**k**) and half-life ($t_{1/2}$) are variables: their numeric values will differ for different radioactive nuclei. The term **ln(1/2)** is a constant: it can be converted to a number. Use your calculator to find its fixed decimal value.

ln(1/2) = ________________

* * * * *

ln(1/2) = ln(0.500) = **− 0.693**

Substitute this numeric value into the equation above that defines half-life and simplify.

* * * * *

$t_{1/2} = -(-0.693) / k$ which simplifies to $t_{1/2} = 0.693 / k$

This last equation above is often listed in textbooks as a definition of radioactive half-life. From this form, it is clear that if you know the half-life, you can find the rate constant **k**, and if you know the rate constant you can find the half-life.

To solve decay calculations, we need equations that relate the fraction remaining, half-life, time, and **k** . Several combinations of the equations above can be used, but the best equations may be those that are easy to remember. In these lessons, we will solve using what we will call the

Radioactive Decay Prompt

If *radioactive decay* calculation includes or *half-life* and a *fraction* or *percentage* of a sample, and the answer cannot be calculated using simple multiples, write in the DATA:

ln(fraction *remaining*) = −kt and ln(**1/2**) = −k $t_{1/2}$

Note that the second equation is simply a special case of the first: when the fraction remaining is 1/2, the time is equal to the half-life.

Commit the radioactive decay prompt to memory, then apply it to solve this problem.

Q. Iodine-131, a radioactive isotope used to treat thyroid disorders, has a half-life of 8.1 days. What percentage of an initial [I-131] remains after 48 hours?

* * * * *

WANT: Percent [I-131]$_{48\ \text{hrs.}}$ = % remaining

DATA: $t_{1/2}$ = 8.1 **days** = radioactive half-life

48 hours = 2.0 **days** = **t** (choose any *consistent* time unit)

Strategy: Write the equations that relate the symbols in the problem.

For radioactive decay calculations that include half-life and fraction or *percentage*, write and use

ln(fraction *remaining*) = −kt and ln(**1/2**) = **−k $t_{1/2}$**

Percentage remaining = **fraction remaining** x 100%

If needed, adjust your work and solve from here.

* * * * *

The variable that links both of the prompt equations is **k** .

Since we know the half-life, the second prompt equation will find **k** .

Knowing **k** and **t**, the first prompt equation will find **ln(**fraction remaining**)**.

Knowing ln(fraction remaining), the fraction remaining can be found using

Fraction = $e^{\ln(\text{fraction})}$ and Percentage [I-131] = Fraction x 100%

Apply those steps and solve.

* * * * *

ln(**1/2**) = **−k $t_{1/2}$**, has two variables **k** and $t_{1/2}$, and we know $t_{1/2}$.

k = − ln(1/2) / $t_{1/2}$ = − (− 0.693) / $t_{1/2}$ = + 0.693 / 8.1 days = **0.0856 day^{-1} = k**

ln(fraction remaining) = −kt = − (0.0856 day^{-1})(2.0 days) = **− 0.171**

Fraction = $e^{\mathbf{ln}(\text{fraction})}$ = $e^{-0.171}$ = 0.843 = **84 %** I-131 remains after 2 days

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Practice C: If you are unsure of the answer to a part, check it before doing the next part.

1. The rate constant for the decay of the tritium isotope of hydrogen is 0.0562 years^{-1} . Calculate the half-life of tritium.

2. Strontium-90 is a radioactive nuclide found in **fallout**: dust particles in the cloud produced by the atmospheric testing of nuclear weapons. In chemical and biological systems, strontium behaves much like calcium. If dairy cattle consume crops exposed to dust or rain containing fallout, dairy products containing calcium will also contain Sr-90. Similar to calcium, Sr-90 will be deposited in the bones of dairy product consumers, including children. In part for this reason, most (but not all) nations conducting nuclear tests signed a 1963 treaty which banned atmospheric testing.

Strontium-90 undergoes beta decay with a half-life of 28.8 years. What percentage of an original [Sr-90] in bones will remain after 40.0 years?

a. Estimate the answer. b. Calculate the answer.

3. The element Polonium was first isolated by Dr. Marie Sklodowska Curie and named for her native Poland. Radioactive Po-210 is found in significant concentrations in tobacco. If 20.0% of Po-210 remains in a sample after 321 days of alpha decay,
   a. Estimate the half-life of Po-210.
   b. Calculate a precise half-life of Po-210. Compare it to your Part A estimate.
4. If 10.0% of a sample of Rn-222 remains after 12.6 days,
   a. estimate the half-life for Rn-222.
   b. Calculate a precise half-life of Rn-222. Compare it to your Part A estimate.
5. If the half-life of carbon-14 is 5,730 years, what fraction of the original carbon-14 in a sample has decayed after 1650 years? Estimate, then calculate.

ANSWERS

Practice A

1a. First-order half-life is constant. Half remains after one half-life, half of that half (25%) remains after two half-lives. Two half-lives = 2 x 24,400 years = **48,800 years.**

1b. Half remains after one half-life, 1/4th after two, 1/8th after three, 1/16th after **four** half-lives.

1c. Half remains after one half-life, 1/4th after two, 1/8th after three; 1/16th after **four** half-lives. 1/16 = 0.0625 = 6.25% remains, so **93.75% has decayed.**

Practice B

1. If 90% has decayed, 10% remains, and fraction remaining = 10% / 100% = **0.100**
2. If fraction decayed = 0.40, percentage decayed = 0.40 x 100% = 40%, and percentage remaining = 100% — 40% = **60%**

3a. WANT: fraction of sample remaining

DATA: ln(fraction remaining) = — 1.386

Knowing a value for ln(fraction remaining), to find fraction remaining, use

Fraction remaining = $e^{\ln(\text{fraction remaining})}$

* * * * *

$= e^{-1.386} =$ **0.250**

3b. If the fraction remaining is 0.250, the percentage remaining is 25.0%, and the percentage decayed is **75.0%.**

4. WANT: % of sample remaining . To find percentage, find fraction first.

DATA: 0.04606 hours^{-1} = **k**

50.0 hours = **t**

Strategy: The equation that relates the three equation terms is

ln(fraction *remaining*) = —kt

Knowing **k** and **t**, **ln**(fraction) and then fraction can be found.

* * * * *

ln(fraction remaining) = —kt = — (0.04606 hours^{-1})(50.0 hours) = **— 2.303**

Fraction remaining = $e^{\ln(\text{fraction remaining})}$ = $e^{-2.303}$ = **0.100**

The percentage remaining is fraction x 100% = **10.0%**

Practice C

1. WANT: $\mathbf{t_{1/2}}$ for tritium

DATA: 0.0562 years^{-1} = **k**

Strategy: In radioactive decay calculations that include half life and fraction or percentage, write

ln(fraction *remaining*) = —kt and ln(1/2) = —k $t_{1/2}$

In this problem, only the second equation is needed to relate the symbols in the WANTED and DATA.

SOLVE: $\mathbf{t_{1/2}}$ **=** ln(**1/2**) / **—k** **=** (— 0.693) / — 0.0562 years^{-1} = **12.3 years**

(1/ years^{-1} = (years^{-1})$^{-1}$ = years)

2a. WANT: Estimate of % [Sr-90] remaining after 40 years. If one half life is about 30 years and 50% remains, and two half-lives is about 60 years and 25% remains, then at 40 years, about…. 40% remains?

2b. WANT: % [Sr-90]$_{40.0\text{ yrs.}}$ remaining

DATA: 28.8 yrs. = $\mathbf{t_{1/2}}$

40.0 yrs = $\mathbf{t_{1/2}}$.

In radioactive decay calculations that include half life and fraction or percentage, write

ln(fraction *remaining*) = —kt and ln(1/2) = —k $t_{1/2}$

Percentage = Fraction x 100%

From half-life, **k** can be found. From **k** and **t**, **ln**(fraction) and then fraction can be found.

SOLVE: Since ln(**1/2**) **= —k** $\mathbf{t_{1/2}}$,

k = — ln(1/2) / $t_{1/2}$ = — (— 0.693) / $t_{1/2}$ = 0.693 / 28.8 yrs. = **0.02406 yrs.$^{-1}$ = k**

ln(fraction *remaining*) = —kt = — (0.02406 yrs.$^{—1}$)(40.0 yrs.) = **— 0.9624 = ln**(fraction)

Fraction = $e^{\ln(\text{fraction})}$ = $e^{—0.9624}$ = 0.382 = **38.2 %** Sr-90 remains after 40 years

Compare this to the estimate.

3a. Estimate 20% remains after 321 days. 50% remains after one half-life, and 25% after 2 half -lives.

At about 300 days, 25% would remain, which is 2 half-lives, so one half-life is about …..

* * * * *

About 150 days.

3b. WANT: $t_{1/2}$

DATA: 20.0% Po-210 remains

fraction remaining = 20% / 100% = 0.200

t = 321 days.

See radioactive decay, half life, and fraction or percentage? Write

ln(fraction *remaining*) = —kt and ln(**1/2**) **= —k $t_{1/2}$**

Percentage = **Fraction** x 100%

Knowing the fraction and t, **k** can be found from the first prompt equation. Half-life can then be found from the second equation. Solving for **k** in symbols first:

k = — ln(fraction) }/ t = — {ln(0.200) }/ (321 days) = — (—1.61) / (321 days) **= 5.01 x 10$^{—3}$ days$^{—1}$**

$t_{1/2}$ = —ln(1/2) / k = + 0.693 / 5.01 x 10$^{—3}$ days$^{—1}$ = **138 days =** half-life of Po-210

Is this answer close to the estimate in Part A?

4a. Estimate 10% remains after 12.6 days. 12.5% remains after three half –lives, which is close to 10%.

If three half-lives is about 12 days, then one half life would be….

* * * * *

About 4 days.

4b. WANT: $t_{1/2}$

DATA: 90.0% Rn-222 has decayed, so 10.0% remains, and fraction remaining = 0.100.

t = 12.6 days.

ln(fraction *remaining*) = —kt and ln(1/2) = —k $t_{1/2}$

Fraction remaining = 10% / 100% = 0.100

Knowing the fraction and t, **k** can be found from the first equation. Half-life can then be found from the second equation.

k = — ln(fraction) / t = — ln(0.100) **/** (12.6 days) = — (—2.30) **/** (12.6 days) **= 0.183 days^{-1}**

$t_{1/2}$ **= — ln(1/2) / k = +**0.693 **/** 0.183 days^{-1} **=** | **3.79 days =** half-life of Rn-222 |

Close to the estimate in Part A?

5. Estimate: 0.50 is the fraction gone after about 6,000 years, so maybe 0.15 is the fraction gone in about a third of that time?

Calculate:

WANT: fraction [C-14] *decayed* = 1.000 — fraction remaining

DATA: $t_{1/2}$ = 5,730 yrs.

t = 1,650 yrs.

In radioactive decay calculations that include half life and fraction or percentage, write

| **ln(fraction remaining) = —kt** | and | ln(**1/2**) **= —k** $t_{1/2}$ |

From half-life, **k** can be found. From **k** and **t**, **ln**(fraction) and then fraction can be found.

SOLVE: **k = —** ln(1/2) / $t_{1/2}$ **= —** (— 0.693) / $t_{1/2}$ = + 0.693 / 5730 yrs. **= 1.21 x 10^{-4} years^{-1}**

ln(fraction remaining) = —kt = — (1.21 x 10^{-4} yr^{-1})(1,650 yrs.) **=** | **— 0.1996** |

Fraction remaining = $e^{\ln(\text{fraction remaining})}$ = $e^{-0.1996}$ = **0.819** fraction C-14 remaining

If 0.819 = fraction remaining, 1.000 — 0.819 = **0.181 =** fraction C-14 *decayed*

* * * * *

Summary: Nuclear Chemistry

1. α particle = ${}^{4}_{2}\text{He}$; β particle = ${}^{0}_{-1}\beta$ *or* ${}^{0}_{-1}\text{e}$, neutron = ${}^{1}_{0}\text{n}$
2. To balance nuclear reactions,
   a. Always use A-Z notation: mass number on top, nuclear charge on the bottom.
   b. Both mass numbers and nuclear charge must be conserved; each must *add* to give the same number before and after the reaction.
3. Fission splits a nucleus, fusion combines nuclei.
4. In the decay of radioactive isotopes, the half-life is constant.
   - At the half-life, 1/2 of the original number of nuclei remain;
   - At double the time of the half-life, 1/4 of the original nuclei remain;
   - At triple the half-life, 1/8 of the original number of nuclei remain.
5. **Radioactive Decay Prompt**

   If *radioactive decay* calculation includes *half-life* and a *fraction* or *percentage* of a sample, and the answer cannot be calculated using simple multiples,

   write in the DATA: **ln(fraction *remaining*) = −kt** and ln(**1/2**) = **−k $t_{1/2}$**

   and use the math of natural logs (Lesson 25D) to solve.
6. **Fraction** = $\frac{\text{Quantity A}}{\text{Quantity B}}$ and often equals $\frac{\text{Part}}{\text{Total}}$ = $\frac{\text{Smaller number}}{\text{Larger number}}$

   A numeric fraction may be expressed in an x/y format *or* as a decimal equivalent.

   In chemistry, both 1/2 and its decimal equivalent 0.50 are termed fractions.
7. In calculations,
   - If you are *given* a percentage, as step one convert it to a decimal equivalent;
   - If you WANT a percentage, first find the decimal equivalent WANTED.
   - Fraction = percent/100% *and* Percentage = fraction x 100%
8. When using the radioactive decay prompt,
   - To use the equation with *(fraction* remaining*)*, you must work in fractions, not percentages.
   - Fraction remaining = 1.000 − fraction decayed
   - Percentage remaining = 100% − percentage decayed
   - The fractions in decay calculations will have a value between 1.00 and 0 (such as 0.25).

# # # # #