

These lessons in this download are *beta* versions. They are being posted in draft form to provide help on as many topics as possible during this semester. If you have need for help in this topic now, they will be useful, but not all topics are covered, and they will have a higher than usual rate of typos and errors.

If you return to this topic, download a newer version of this packet at www.ChemReview.Net.

Modules 19 and above have been re-numbered.

Module 19 – Graphing is now Module 20.

If you are looking for Graphing topics, check Module 20

Calculations In Chemistry

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Module 19 – Kinetic Molecular Theory and Graham's Law

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For additional modules, visit www.ChemReview.Net

Module 19 — Kinetic Molecular Theory

Lesson 19A: Squares and Square Roots

Timing: Complete this lesson before calculations involving *root mean square velocity* or Graham's law.

Prerequisites: Review Module 1 on Scientific Notation if needed.

Pretest: If you solve all of these problems correctly, skip to Lesson 19B. Express your final answers in scientific notation. Answers are at the end of this lesson.

- Solve without a calculator: $(7.0 \times 10^6)^2 =$
- Use a calculator. $(9.5 \times 10^{-4})^2 =$
- Do not use a calculator. The square root of $1.6 \times 10^{-11} =$
- You may use a calculator for all or part of this problem. $(4.9 \times 10^5)^{1/2} =$
* * * * *

Squaring Numbers

Most calculators have an $\boxed{x^2}$ or $\boxed{x^{\wedge}2}$ key, but different calculators use keys in different sequences. To test using the square key, cover the answer below and do this calculation in your head.

$$5^2 = \underline{\hspace{2cm}}$$

* * * * *

$5^2 = 25$. Now, using the calculator you will use on tests,

write a key sequence that is used to square 5: _____.

To test your key sequence, first solve in your head, then use your sequence to solve the following problems. Make sure that you get the same answers both ways.

- a. $6^2 =$ _____ b. $9^2 =$ _____ c. Do *two* ways on keys: $12^2 =$ _____ .

* * * * *

Answers: a. 36 b. 81 c. $12 \times 12 = 144$

Note that all of the calculations above can be done at least two ways. Always try to do calculator calculations in two different ways to check your calculator use.

Taking Exponential Notation to a Power Without a Calculator

- Memorize this rule: To take *exponentials* to a power, *multiply* the exponents. (As used here, *power* and *exponent* have the same meaning.)

Examples: $(10^3)^2 = 10^6$ $(10^{-5})^2 = 10^{-10}$

Without a calculator, write answers to these, then check below.

- a. $(10^6)^2 =$ b. $(10^{-5})^2 =$ c. $(10^{-12})^2 =$

* * * * *

For both: $(10^{-5})^2 = 10^{-10}$; (b) $7 \times 7 = 49 \times 10^{-10}$ (c) $8 \times 8 = 64 \times 10^{-10}$

d. Based on (b) and (c), write an *estimate* of $(7.5 \times 10^{-5})^2 =$ _____.

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7.5 is half way between 7 and 8, so you might estimate that half way between 49 and 64 is *about* 55×10^{-10} .

e. Using a calculator, $(7.5)^2 =$ _____

f. Multiply: $7.5 \times 7.5 =$ _____. Does this match your part (e) answer ?

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g. Based on (e) and (f), write a precise answer to $(7.5 \times 10^{-5})^2$. Compare your answer to your estimate in (d) above. Then write your answer in scientific notation

with correct *sf*: _____

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$$56.2 \times 10^{-10} = \boxed{5.6 \times 10^{-9}} .$$

h. Now try $(7.5 \times 10^{-5})^2$ by plugging everything into the calculator. .

- A “standard TI-type” calculator *may* use 7.5 5
- A graphing calculator *might* use 7.5 5
- On an RPN calculator, *try* 7.5 5

An online search with your calculator name and model number and “exponential notation” may offer a better approach.

Write the calculator answer, rounding to two digits: _____ .

h. Compare answers in steps (g) and (h). They should agree. They should also be close to the value of your estimate in step (d).

You may use the calculator in any way you choose, but calculations should always be done two ways to check your work. Estimating an answer by mental arithmetic is often the fastest and most reliable way to check answers at the end of a problem or test.

Practice B: On these

- first write down an *estimated* answer, then re-write it in scientific notation;
- then use the calculator for the parts you wish and get a *final* answer in scientific notation. Finally, compare the last two columns: they should be close.

Try any two. Need more practice? Do more.

1. $(2.1 \times 10^6)^2$ Est = _____ Sci Note = _____ Calc = _____

2. $(3.9 \times 10^{-2})^2$ Est = _____ Sci Note = _____ Calc = _____

3. $(7.7 \times 10^4)^2$ Est = _____ Sci Note = _____ Calc = _____

4. $(5.50 \times 10^{-2})^2$ Est = _____ Sci Note = _____ Calc = _____

Taking Square Roots

“Taking the square root” of a quantity is the same as taking a quantity to the $1/2$ power.

a. $\sqrt{x} = x^{1/2} = x^{0.5}$

b. The *square* root of 4.7 can be written as $4.7^{1/2}$. $10^{1/2}$ means the *square* root of 10.

To calculate a square root, some calculators have a button labeled \sqrt{x} or $x^{1/2}$.

Other calculators use this two-key sequence: 2^{nd} or INV x^2 or $x^{^2}$.

To test calculator sequences, use sample calculations that are easy to check. Do the following calculation in your head.

The square root of 81 = _____

* * * * *

The square root of 81 = 9. Now, by entering 81 and one of the key sequences above, find two key sequences that give the same *answer* for the square root on your calculator.

* * * * *

One or more of these sequences *may* work. Others may work as well.

- On a standard TI-type, try 81 2^{nd} or INV x^2 =
and/or try 81 \sqrt{x} or $x^{1/2}$ =
and/or try 81 y^x 0.5 =
- On a graphing calculator (*if* allowed), try 2^{nd} x^2 81) enter
and/or try 81 \wedge (1 \div 2) enter
and/or try 81 \wedge 2 $1/x$ or x^{-1} enter
- On an RPN calculator, try 81 enter \sqrt{x} or $x^{1/2}$
and/or try 81 enter 0.5 y^x

Write or circle one or if possible *two* key sequences that work and make sense to you. Then, on the problems below, *check* your key sequence: try the calculation two ways using the calculator.

a. $36^{1/2} =$

b. $\sqrt{625} =$

c. $\sqrt{7225} =$

d. $(0.3025)^{1/2} =$

* * * * *

Answers: a. 6.0 b. 25.0 c. 85.00 d. 0.5500

Roots of Divisible Powers of 10

To take the root of the exponential term is easy if the exponent is an *even* number.

1. To find the square root of an even exponential term, write the term to the $1/2$ power.

Example: Write the square root of 10^{-8} as $(10^{-8})^{1/2}$.

2. Apply the rule: to take an exponential term to a power, multiply the exponents.

Examples: The square root of $10^{-8} = (10^{-8})^{1/2} = 10^{-4}$

 The square root of $10^{30} = (10^{30})^{1/2} = 10^{15}$

This rule can be stated as:

To take the square root of an *even* exponential, cut the exponent in half.

Practice C:

1. Take these roots by entering the numbers into a calculator. Try each two ways.

a. $\sqrt{9025} =$

b. $(0.00160)^{1/2} =$

2. Do *not* use a calculator. Write answers to these as powers of 10.

a. The square root of $10^{18} =$

b. $(10^{10})^{1/2} =$

c. The square root of $10^{-16} =$

d. $(10^{-12})^{1/2} =$

Roots of Exponential Notation

On this problem, apply the fundamental rule: Treat numbers as numbers, and exponents as exponents.

Without a calculator: $(64 \times 10^{12})^{1/2} =$ _____

* * * * *

Answer: The square root of 64 is 8. $(10^{12})^{1/2} = 10^6$

$$(64 \times 10^{12})^{1/2} = 8.0 \times 10^6$$

Not all roots can be solved by inspection, but by using a method similar to the above, we can estimate square roots without the calculator, and calculate a precise square root without entering the exponential term into the calculator.

For roots that cannot be solved by inspection, use these steps.

1. If the exponential term is *not even*, make the exponent one number *lower* (which makes its value 10 times smaller), and move the decimal to make the significant 10 times larger. Making the significant larger makes it easier to estimate its square root.

Try that step on $(1.764 \times 10^{-7})^{1/2} =$

* * * * *

$$(1.764 \times 10^{-7})^{1/2} = (17.64 \times 10^{-8})^{1/2}$$

To make the exponent an *even* number, it is *lowered* from 10^{-7} to 10^{-8} . When you make the exponential smaller, make the significant larger.

2. Without a calculator, write a rough *estimate* of the $\sqrt{\text{significant}}$ that is in front of the even exponential. Then find the exact root of the exponential. Combine these two parts and write the estimate for the root.

Apply step 2 to the step 1 answer: $(17.64 \times 10^{-8})^{1/2} \approx$ _____

* * * * *

$$(17.64 \times 10^{-8})^{1/2} = (17.64)^{1/2} \times (10^{-8})^{1/2} =$$

To estimate a square root of 17.64, since $4 \times 4 = 16$, and 17 is a little higher than 16, guess ≈ 4.1 . (\approx means *approximately equals*.)

Handle exponents as exponents. $(10^{-8})^{1/2} = 10^{-4}$

Combine the two parts. Estimate $\approx 4.1 \times 10^{-4}$

3. To get a precise answer,
- Start from the exponential notation that has the *even* exponent.
 - Find the *precise* root of the significant on the calculator.
 - Take the root of the exponential term *without* the calculator.

Try those steps on the above problem.

* * * * *

For $(17.64 \times 10^{-8})^{1/2}$. The calculator $\sqrt{17.64} = 4.200$

$$(10^{-8})^{1/2} = 10^{-4}. \text{ Answer} = \text{span style="border: 1px solid black; padding: 2px;">}4.200 \times 10^{-4}\text{}$$

4. Compare the step 3 calculator answer to the step 2 estimate. They should be close.
5. Now take the root by entering the *original* number in the problem into the calculator.

* * * * *

One or more of these key sequences (and others) *may* work.

• On a standard TI-type, try **1.764** EE **7** +/- 2nd or INV x² =

and/or try **1.764** E or EE or EXP **7** +/- \sqrt{x} or x^{1/2} =

• On a graphing calculator, try **1.764** EE (-) **7** enter ^ **0.5** enter

• On an RPN calculator, try **1.764** E or EE or EXP **7** +/- enter \sqrt{x} or x^{1/2}

The calculator answer should match the step 3 final answer.

Circle or write one or two key sequences that work: _____

Whatever sequences you use, work through the logic of *why* they works. Without the *why*, it will be difficult to remember the correct sequence.

Once you have debugged and *practiced* a key sequence to calculate square roots, entering both parts of the exponential notation into the calculator may be faster than converting to an even exponential. However, converting to the even exponent to estimate the root is one way to *check* the calculator result at the end of a problem or test.

Roots of Non-Divisible Powers of 10

Changing an exponent to make it *even* can put a *number* in front of the exponential term that was not there before. Try this example.

Q. Estimate this answer *without* a calculator: The square root of $10^5 =$ _____

* * * * *

To find the root without a calculator, the power of 10 must be divisible by 2. Try adjusting the exponent to make it smaller and divisible, and adjust the significand to keep the same value for the number.

* * * * *

Answer: $(10^5)^{1/2} = (1 \times 10^5)^{1/2} = (10 \times 10^4)^{1/2} = (10^{1/2} \times 10^2) \approx 3.2 \times 10^2$

Making the exponent 4 puts a 10 in front. Now try taking the square root of 10^5 on the calculator.

* * * * *

You *may* need to enter $\underline{1} \times 10^5$.

* * * * *

3.16×10^2

Summary: Squares and Square Roots

1. If you are not certain that you are using calculator keys correctly, do a *simple* similar calculation, first on paper and then on the calculator.
2. For complex operations, do each calculation a second time using different keys or estimates with rounded numbers.
3. On complex calculations, it is often easier to do the numbers on the calculator but the exponents on paper.
4. In calculations using exponential notation, handle numbers and exponential terms separately.
 - Handle numbers by number rules and exponents by exponential rules.
 - When you multiply exponentials, you add the exponents.
 - When you divide exponentials, you subtract the exponents.
 - To take an exponential to a power, multiply the exponentials.
 - To take the square root of an even exponential, cut the exponent in half.
5. To take a square root of exponential notation without entering the exponential,
 - adjust the significand and exponential to make the exponent *even*; then
 - use the calculator to take the $\sqrt{\text{significand}}$, take the $\sqrt{\text{exponential}}$ in your head, and combine the two answers.

Practice D

1. Complete the problems below using the following steps. Save part b for your next practice session. Check your answer after each part.
 - A. First convert to and write the value with an *even* exponential term.
 - B. Write an *estimated* answer for the root.
 - C. Re-write the estimate in scientific notation.
 - D. Starting from the notation with the *even* exponent, use the calculator for the root of the significant, take the root of the exponential in your head, write the answer, then convert the answer to scientific notation.
 - E. Take the square root of the original exponential notation on the calculator.
 - F. Compare your answers in steps C, D and E.
 - a. $(6.0 \times 10^{23})^{1/2}$ A: Change To Even Expo = _____
 B. Est. Root Of Even = _____ C. Est In SciNote = _____
 D. $\sqrt{\text{SignifOfEven}}$ Calc, Expo Head = _____ E. Calc = _____
 - b. $(10^{11})^{1/2}$ A: Change To Even Expo = _____
 B. Est. Root Of Even = _____ C. Est In SciNote = _____
 D. $\sqrt{\text{SignifOfEven}}$ Calc, Expo Head = _____ E. Calc = _____
 - c. $\sqrt{1.25 \times 10^{-7}}$ A: Change To Even Expo = _____
 B. Est. Root Of Even = _____ C. Est In SciNote = _____
 D. $\sqrt{\text{SignifOfEven}}$ Calc, Expo Head = _____ E. Calc = _____

* * * * *

Practice E: Work in your notebook. Use a calculator. Save one column for review.

1. $47^2 =$
 2. $(0.25)^2 =$
 3. $(6.5 \times 10^3)^2 =$
 4. $(2.0 \times 10^{-5})^2 =$
 5. $(9.3 \times 10^{-3})^{1/2} =$
 6. $(5.7 \times 10^{-4})^{0.5} =$
 7. $\sqrt{20.25} =$
 8. $\sqrt{0.01024} =$
 9. $(2.20 \times 10^4)^{0.50} =$
 10. The square root of $9.5 \times 10^{15} =$
 11. $(7.4 \times 10^{-8})^{1/2} =$
 12. $(6.5 \times 10^{-11})^2 =$
-
-

ANSWERS**Pretest:** 1. 4.9×10^{13} 2. 9.0×10^{-7} 3. 4.0×10^{-6} 4. 7.0×10^2 **Practice A**

- | | |
|--|---|
| 1. $(10^{-3})^2 = 10^{-6}$ | 2. $(10^5)^2 = 10^{+10}$ |
| 3. $(2.0 \times 10^4)^2 = 4.0 \times 10^8$ | 4. $(3.0 \times 10^{-1})^2 = 9.0 \times 10^{-2}$ |
| 5. $(8.0 \times 10^{10})^2 = 64 \times 10^{20} = 6.4 \times 10^{21}$ | 6. $(6.0 \times 10^{-11})^2 = 36 \times 10^{-22} = 3.6 \times 10^{-21}$ |

Practice B

- | | |
|---|--|
| 1. $(2.1 \times 10^6)^2 = 4.4 \times 10^{12}$ | 2. $(3.9 \times 10^{-2})^2 = 1.5 \times 10^{-3}$ |
| 3. $(7.7 \times 10^4)^2 = 5.9 \times 10^9$ | 4. $(5.50 \times 10^{-2})^2 = 3.02 \times 10^{-3}$ |

Practice C

- | | |
|--|---------------------------------|
| 1a. $\sqrt{9025} = 95.00$ | b. $(0.00160)^{1/2} = 0.0400$ |
| 2. a. The square root of $10^{18} = 10^9$ | b. $(10^{10})^{1/2} = 10^5$ |
| c. The square root of $10^{-16} = 10^{-8}$ | d. $(10^{-12})^{1/2} = 10^{-6}$ |

Practice D

- 1a. $(6.0 \times 10^{23})^{1/2} = (60. \times 10^{22})^{1/2} = (60^{1/2} \times 10^{11}) = 7.7 \times 10^{11}$
- 1b. $(10^{11})^{1/2} = (10 \times 10^{10})^{1/2} = (10^{1/2} \times 10^5) = 3.2 \times 10^5$
- 1c. $\sqrt{1.25 \times 10^{-7}} = (12.5 \times 10^{-8})^{1/2} = (12.5^{1/2} \times 10^{-4}) = 3.54 \times 10^{-4}$

Practice E

1. 2.209×10^3 2. 6.2×10^{-2} 3. 4.2×10^7 4. 4.0×10^{-10} 5. 9.6×10^{-2} 6. 2.4×10^{-2}
7. 4.500 8. 0.1012 9. 1.48×10^{-2} 10. 9.7×10^7 11. 2.7×10^{-4} 12. 4.2×10^{-21}
- * * * * *

Lesson 19B: Kinetic Molecular Theory

Timing: Complete this lesson if you are assigned problems that include *root mean square velocity*.

* * * * *

Kinetic Molecular Theory

The ideal gas laws can be derived from fundamental equations of physics based on a model for gas behavior termed the **kinetic molecular theory (KMT)**. This model assumes that

1. Gas molecules are in constant motion. (A gas will expand to fill its container.)
2. The diameter of gas molecules is very small compared to the distance between the molecules. (Most of a gas is empty space.)

- The measured *absolute* temperature of a gas is directly proportional to the average kinetic energy of the gas molecules.
- Gas molecules have perfectly elastic collisions: when two molecules collide, the total kinetic energy of the molecules is the same before and after the collision.
- The pressure exerted by a gas arises from the collisions of the molecules with the walls of the container. The rate at which particles strike the wall per unit of wall area, and the temperature of the particles (their average kinetic energy) determine the pressure.

The assumptions of the KMT are not exactly true for any real gases, but they are *close* to true for many gases if temperature is relatively high and pressure is relatively low (standard pressure or below).

Practice A: On these, a) predict the answer using the ideal gas law, then b) explain the answer in terms of KMT.

- For a sample of gas with constant moles and volume, if the absolute temperature is doubled, what happens to the pressure?
- For a sealed sample of gas at constant temperature in a cylinder with a piston (such as a syringe), if the piston is withdrawn until the volume of the gas is doubled, what happens to the pressure?

Root Mean Square Velocity

For a gas at any point in time, because of collisions between molecules, some molecules are stopped, many are going at close to the average speed, but some are going fast compared to the average speed. This results in a distribution of velocities where the median (half above and half below) and the mean (average) velocities are close, but are not quite the same.

For the KMT equations to be accurately mathematically, we must use a special type of average velocity called the **root mean square velocity** (v_{rms}).

v_{rms} = the square root of the average of the squares of the molecular velocities

$v_{\text{rms}} = (\overline{v^2})^{1/2}$ where the line over the v^2 means *average* (arithmetic mean) and the 1/2 power means square root.

Averaging the squares of the velocities, and then taking the square root, results in a value for v_{rms} that is close to, but *slightly higher* than, the arithmetic *average* of the velocities.

$$v_{\text{rms}} = (\overline{v^2})^{1/2} \approx \bar{v} \equiv \text{the average velocity of the molecules}$$

Velocities in Different Gases at the Same Temperature

Among the implications of the kinetic molecular theory (KMT) are:

- If two samples of the same gaseous substance have the same temperature, their particles must have the same average kinetic energy and the same average velocity.

Why? If the temperature is the same, the average kinetic energy is by definition must be same.

In terms of velocity, the kinetic energy of a particle in motion is by definition one half of its mass times its velocity squared: $\boxed{KE = 1/2 mv^2}$. In the two samples, since the average kinetic energy is the same, and the mass must be the same for molecules with the same molecular formula, then the root mean square (~average) velocity of molecules in the two samples must be the same.

- If the root mean square (~average) velocity of gas molecules is *doubled*, their kinetic energy and their temperature in kelvins must *quadruple*.

Why? The equation for kinetic energy is: $\boxed{KE = 1/2 mv^2}$.

The mass of molecules is constant, but doubling the average speed increases the kinetic energy of the particles by v^2 : the KE quadruples and the absolute temperature therefore quadruples.

- If two *different* gaseous substances are at the same temperature, either in separate samples or mixed together, the molecules of the gas with the *lighter molar mass* must be traveling *faster*.

Why? The average kinetic in the two samples must be the same, so we can write:

$$\boxed{KE_{ave} = 1/2 m_a v_a^2 = 1/2 m_b v_b^2}$$

If the mass of the particles of *gas a* is smaller than for *gas b*, the velocity of the *gas a* particles must be larger to equal the average kinetic energy of the particles of *gas b*.

Practice B

- In a sample of gas with constant moles and volume, if the root mean squared velocity of molecules is doubled, what happens to the pressure? Explain why in terms of KMT.

ANSWERS

Practice A

- If the absolute temperature of a sample of gas with constant moles and volume is doubled,
 - $P(\text{constant } V) = (\text{constant } n \text{ and } R)T$. When T is doubled, P must double.
 - If T is doubled, the kinetic energy of the average particle is doubled, and the molecules strike the wall twice as hard, creating twice as much pressure.
- For a sealed sample of gas at constant temperature, if the volume of the gas is doubled,
 - $PV = (\text{constant } n \text{ and } R \text{ and } T)$. If volume is doubled, P must be cut in half.
 - If volume is doubled, there are half as many particles per unit of volume, and the number of particles hitting the each section of wall from any adjacent volume is cut in half, halving the pressure.

Practice B

1. For a sample of gas at constant moles and volume, if the v_{rms} (\sim average velocity) of the gas is doubled, the energy of motion of the gas molecules quadruples. Each molecule strikes the wall with 4 times as much kinetic energy. The pressure quadruples.

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Lesson 19C: Converting to SI Base Units

Timing: Complete this lesson if you are assigned calculations that include *average kinetic energy* or *root mean square velocity*.

* * * * *

Finding the SI Base Units For Derived Quantities

In calculations based on the fundamental equations of physics, units must be consistent, and SI *base* units are the best choice to manage units and unit cancellation. However, some SI base units are different than the units commonly used in chemistry. For example, in chemistry we usually measure

- molar mass in grams per mole, but in SI base units, molar mass is measured in *kilograms* per mole (kg/mol).
- Volume in liters or milliliters, but in SI base units, volume is measured in *cubic meters* (m^3).

Converting between the usual chemistry units and SI base units is simplified if we can recall some fundamental relationships among physical quantities. Let's review.

SI units can be divided into three types: base units measuring fundamental quantities (see Lesson 11E), combinations of base units that measure derived quantities, and special unit names that are abbreviations for the combinations measuring derived quantities.

In the SI system, each *fundamental* quantity is measured by a single defined unit based on definitions from what was known historically as the **mks system**. Among these are:

- The quantity distance (symbol = d) is measured in meters (unit = **m**).
- Mass (m) is measured in kilograms (**kg**). Mass is the only fundamental quantity that has an SI *base* unit that includes a prefix.
- Time (t) is measured in seconds (**s**).
- Particle counts (n) are measured in moles (mol).
- Absolute temperature (T) is measured in kelvins (**K**).

SI base units for *derived* quantities always result from multiplying or dividing base units, and/or from taking base units to a power. The base units that measure a derived quantity can be determined from the definition of the derived quantity. For example,

- Area formulas always involve a distance times a distance (d^2), so the SI base unit of area is the distance *base* unit squared = meters squared = **m^2** .
- Velocity (v) is defined as $\delta d / \delta t$: (change in distance)/(change in time), so the SI *base* unit of velocity must be meters per second (**m/s**).

- Acceleration (a) is defined as change in velocity per unit of time ($\delta v/\delta t$), so the SI base unit for measuring acceleration is

$$\delta v/\delta t = (\text{m/s})/\text{s} = \frac{\text{m}}{\text{s}} \cdot \frac{1}{\text{s}} = \frac{\text{m}}{\text{s}^2} = \boxed{\text{m/s}^2}, \text{ which can also be written as } \text{m} \cdot \text{s}^{-2}.$$

- Force (F) is defined as mass times acceleration ($F = ma$), so the SI base unit for measuring force is

$$F = ma = (\text{kg})(\text{m/s}^2) = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} = \boxed{\text{kg} \cdot \text{m/s}^2}, \text{ also written as } \text{kg} \cdot \text{m} \cdot \text{s}^{-2}.$$

When writing units, forms such as $\text{kg} \cdot \text{m/s}^2$ and $\text{kg} \cdot \text{m} \cdot \text{s}^{-2}$ are equivalent, but the “numerator over denominator” form is more helpful during calculations to convert between units.

To determine the SI *base* units for a derived quantity, the steps are:

- Write an *equation* that calculates the quantity using other quantities for which SI base units are known.
- For each quantity in the equation, substitute the base units that measure that quantity into the equation, then simplify the units.

Apply those two steps to this problem.

Q1. Pressure is defined as *force per unit of area*. What are the SI base units for pressure?

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The definition is: Pressure = Force/area

Force is defined as mass times acceleration ($F = ma$) so in symbols: $P = m \cdot a/\text{area}$

We know the base units for mass (kg) and acceleration (m/s^2). We also know that area is distance squared, measured in base units of meters squared (m^2).

$$\text{In symbols: } P = \frac{F}{\text{Area}} = \frac{m \cdot a}{\text{area}} = \frac{m \cdot a}{d^2} = \frac{m \cdot a \cdot 1}{d^2} \quad \text{Substituting the}$$

$$\text{SI base units for } m, a, \text{ and } d: \quad P \text{ units} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{1}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2} \text{ or } \text{kg} \cdot \text{m}^{-1} \cdot \text{s}^{-2}$$

Try another.

Q2. “Mechanical work” is the form of *energy* used to define the SI energy unit. By definition: Energy = work = force x distance. What are the SI base units for energy?

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The definition is: Energy = work = *force* times distance, and force is defined as mass times acceleration ($F = ma$) so,

$$\text{In symbols: } \text{Energy (work)} = m \cdot a \cdot d \quad (\text{remembered as “work is mad!”})$$

We know SI base units for m , a , and d , so we substitute those units into the equation:

$$\text{SI base units for Energy (E)} = m \cdot a \cdot d = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2} \text{ or } \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$$

Practice A

1. The wavelength of an electromagnetic wave is the distance between the crests of the wave. What must be the SI base unit for wavelength?
 2. What are the SI base units for volume measurements?
 3. What must be the SI base unit for molar mass?
 4. Electrical power is defined as energy flow per unit of time. What are the SI base units for power?
-
-

Abbreviations for SI Base Units

Base unit combinations that are complex are often given a special name that is equivalent to the base unit combination and is used to *abbreviate* the base unit combination.

For example: For force, the SI abbreviated unit is termed the **newton** (N), a unit which is an abbreviation for the SI *base* unit of force: $\text{kg} \cdot \text{m}/\text{s}^2$.

For derived quantities that have abbreviated unit names, the abbreviated unit for the derived quantity must be memorized. For each abbreviated unit name, the two-step process above will *derive* the SI *base* unit that is equivalent to the abbreviated unit. In addition to newtons, abbreviation units that are used frequently in chemistry include

- Pressure (F/area) measured in the SI unit **pascals** (Pa) = $\text{kg}/\text{m} \cdot \text{s}^2$
- Energy (F • d or m • a • d) measured in the SI unit **joules** (J) = $\text{kg} \cdot \text{m}^2/\text{s}^2$

This table summarizes some quantities and units that we will use in upcoming lessons.

Quantity	Symbol	Definition	SI Unit	SI Abbreviated Unit
distance	d		meter (m)	
mass	m		kilogram (kg)	
time	t		second (s)	
particle count	n		mole (mol)	
temperature	T		kelvins (K)	
area	Area	d • d	m^2	
volume	V	d • d • d	m^3	
velocity	v	d/t	m/s	
acceleration	a	v/t	m/s^2	
force	F	m • a	$\text{kg} \cdot \text{m}/\text{s}^2$	newton (N)
pressure	P	F/Area	$\text{kg}/\text{m} \cdot \text{s}^2$	pascal (Pa)
energy	E	F • d = m • a • d	$\text{kg} \cdot \text{m}^2/\text{s}^2$	joule (J)
molar mass	MM	mass/count	kg/mol	

In these lessons, we will use MM as the symbol for molar mass. Many textbooks use an italicized capital M to abbreviate molar mass, but writing MM may better help you to distinguish molarity from molar mass when writing out equations by hand.

For each row in the table above, given any one item (except for the units in **bold**), you should memorize what is needed to fill in the rest of row. The units in **bold** can be derived from the third column.

Practice B: Save a few for your next practice session.

1. Memorize any rows of the table above that are unfamiliar, then cement your knowledge by filling in the blanks below.

Quantity	Symbol	Definition	SI Base Unit	SI Abbreviated Unit
distance				
			kilogram (kg)	
	t			
			mole (mol)	
temperature				
			m²	
		$d \cdot d \cdot d$		
			m/s	
		v/t		
		$m \cdot a$		
				pascal (Pa)
	E			
			kg/mol	

Converting Between SI Units and Non-SI Units Used in Chemistry

In calculations based on the fundamental equations of physics, SI base units must be used so that units are consistent and cancel. It is often necessary to convert between an abbreviated unit such as joules and the SI base units that the abbreviated unit stands for. In addition, when using the equations of physics, converting from the non-SI units used in chemistry to SI units is often necessary.

The good news is that by using our systematic WANTED, DATA, SOLVE steps, these conversions can be done with confidence and accuracy.

Try this example.

Q1. A pressure of 0.250 atm is what pressure in SI *base* units?

If you need help, peek at part of the answer below, then try again.

★ ★ ★ ★ ★

WANT: SI base units for pressure. Since $P = F/\text{Area} = m \cdot a / \text{Area}$

$$P = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \frac{1}{\text{m}^2} = \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = \text{the SI base unit for pressure, abbreviated as } \textit{pascals}.$$

So WANTED = P in the SI base unit $\text{kg /m} \cdot \text{s}^2$ (= pascals)

DATA: 0.250 atm

(What relationship is known between atm and the WANTED unit?)

1 atm = standard pressure = 101 kPa

(so above we know equalities relating *atm* to *Pascals* to *base units*.)

$$\begin{aligned} \text{SOLVE: } ? &= \text{kg /m} \cdot \text{s}^2 = \text{pascals} = 0.250 \text{ atm} \cdot \frac{101 \text{ kPa}}{1 \text{ atm}} \cdot \frac{10^3 \text{ Pa}}{1 \text{ kPa}} = 2.52 \times 10^4 \text{ Pa} = \\ &= 2.52 \times 10^4 \text{ kg /m} \cdot \text{s}^2 \end{aligned}$$

Summary: Converting to SI Base Units

To write the SI *base* units for a derived quantity, or for a unit such as Newtons, Pascals, or Joules that is an abbreviation for a combination of SI base units ,

1. Write the equation that calculates either the derived quantity, or the quantity that the abbreviated unit measures. In the equation, use quantities for which the SI base units are known.
2. Into the equation for the derived quantity, substitute the SI base units for the quantities with known base units.

Practice C

1. Using a periodic table, find the molar mass of the noble gas argon in SI base units.
2. What is the volume of one mole of an ideal gas at STP in SI base units?
3. If a gas presses on a rectangular surface that is 4.0 meters x 1.0 meter with a force of 24.0 newtons, what is the pressure in pascals?

ANSWERS

Practice A

1. Since a wavelength is a distance, in SI base units it is measured in **meters**.
2. All equations for volume can be related to multiplying three distance units. In SI base units, multiplying 3 distances results in **meters cubed: m³**.
3. In chemistry, molar mass is usually measured in grams/mole. In SI base units, moles are used to count particles, but mass must be measured in kilograms. In SI *base* units, molar mass is measured in **kg/mol**.

4. In symbols: Power = E/t = m • a • d/t Substituting the SI base units that measure those symbols:

$$E/t = m \cdot a \cdot d \cdot \frac{1}{t} = \text{kg} \cdot \frac{\text{m}}{\text{s}^2} \cdot \text{m} \cdot \frac{1}{\text{s}} = \frac{\text{kg} \cdot \text{m}^2}{\text{s}^3} \quad \text{or} \quad \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-3}$$

Practice B: See table in lesson.

Practice C

1. WANT: MM Ar in **kg/mol** In SI base units, mass must be in kilograms.

DATA: MM Ar = 40.0 g/mol

SOLVE: $?$ $\frac{\text{kg Ar}}{\text{mol}} = 40.0 \frac{\text{g Ar}}{\text{mol}} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = \mathbf{0.0400 \frac{\text{kg Ar}}{\text{mol}}}$

2. WANT: Volume in SI base unit **meters cubed: m³**

DATA: One mole of an ideal gas at STP has a volume of 22.4 L .

$$\begin{aligned} V \text{ in m}^3 &= 22.4 \text{ L} \cdot \frac{1000 \text{ cm}^3}{1 \text{ L}} \cdot \left(\frac{10^{-2} \text{ meter}}{1 \text{ cm}} \right)^3 = 22.4 \text{ L} \cdot \frac{1000 \text{ cm}^3}{1 \text{ L}} \cdot \left(\frac{10^{-6} \text{ m}^3}{1 \text{ cm}^3} \right) = \\ &= 22.4 \times 10^{-3} \text{ m}^3 = \mathbf{2.24 \times 10^{-2} \text{ m}^3} \quad \text{or} \quad \mathbf{0.0224 \text{ m}^3} = V \end{aligned}$$

(To review distance to volume conversions, see Lesson 5F.)

3. WANT: Pressure in pascals

DATA: The equation calculating pressure is $P = F/\text{Area}$. List the symbols in that equation.

Force = 24.0 newtons = m • a = 24.0 **kg • m /s²**

Area of rectangle = length x width = 4.0 meters x 1.0 meter = 4.0 **m²**

SOLVE: $P = \frac{F}{\text{Area}} = F \cdot \frac{1}{\text{Area}} = 24.0 \frac{\text{kg} \cdot \text{m}}{\text{s}^2} \cdot \frac{1}{4.0 \text{ m}^2} = 6.0 \frac{\text{kg}}{\text{m} \cdot \text{s}^2} = \mathbf{6.0 \text{ pascals}}$

Solving for pressure in SI base units, the answer unit must be equivalent to pascals, and it is.

* * * * *

Lesson 19C: KMT Calculations

Timing: Complete this lesson if you are assigned *calculations* that include *average kinetic energy* or *root mean square velocity*.

* * * * *

Relating Kinetic Energy To the Ideal Gas Variables

By treating gas molecules as particles moving in a three-dimensional container, and by applying the definitions of physics for quantities including momentum, force, and pressure, we can calculate relationships between the terms in the ideal gas law and average kinetic energy equation. The math is somewhat lengthy, but the result is the equation:

$$\frac{PV}{n} = RT = \frac{2}{3} KE_{\text{average, per mole}} \quad \text{Equation (1)}$$

The first two terms can be rearranged to $PV = nRT$. This means that the ideal gas law is one outcome of applying the fundamental equations of physics to moving particles.

In the last two terms of Equation (1), since R is a constant (the gas constant), $T \propto KE_{\text{ave}}$, which means that the definition of temperature is also a result of the equations for particles in motion.

When solving equations, units must be consistent. In calculations using physics-based equations such as Equation (1), to simplify unit cancellation, SI *base* units must be used. In KMT equations, those base units are

- distance in meters (m), mass in kilograms (kg), time in seconds (s), particle counts in moles (mol), and absolute temperature in kelvins (K).
- To do KMT calculations, the units must be converted to the base units **m, kg, s, mol, and K**, but those units may be multiplied, divided, or raised to a power.

Specifically, for the variables used in Equation (1),

- Pressures must be converted to $\text{kg/m} \cdot \text{s}^2$, the base unit abbreviated *pascals* (Pa).
- Volumes must be converted to cubic meters (m^3), and T must be in kelvins.
- Kinetic energy must be measured in $\text{kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2$, the energy unit abbreviated as **joules per mole (J/mol)**.
 - $\text{Joules} = \text{kg} \cdot \text{m}^2/\text{s}^2$ and $\text{Joules/mole} = \text{kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2$
- The R (gas constant) value that must be used is $8.31 \text{ J/mol} \cdot \text{K}$, which is one of the many value and unit combinations for R , but for unit cancellation to work in KMT calculations, R must be written using the *base* units equivalent to joules:
 - $R = 8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$

Relating Kinetic Energy To Temperature

The last two terms of Equation (1), $RT = 2/3 KE_{\text{ave}}$, mean that for any sample of a gas, if any one of the two variables of temperature and average KE per mole is known, the other variable can be calculated. Try this example.

Q1. Calculate the average kinetic energy, in J/mol, for gas molecules at room temperature (25°C). Consult the unit rules in the bullets above as needed.

* * * * *

WANT: KE_{ave} in joules/mole = KE_{ave} in $\text{kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2$

DATA: 25°C

The relationship between temperature and KE is the last two terms of Equation (1).

$$RT = \frac{2}{3} KE_{\text{average, per mole}} \quad (\text{"RT is 2/3 of average kinetic energy per mole"})$$

To provide for unit cancellation, KMT equations are solved in SI base units. To use the above equation, list each symbol, add the SI base units that measure that symbol, then an = sign and the DATA. In the table, convert DATA to SI base units if needed.

$R = 8.31 \text{ J/K} \cdot \text{mol}$ but must be in SI *base* units = $8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$, and

T must be in K = $^{\circ}\text{C} + 273 = 25^{\circ}\text{C} + 273 = 298 \text{ K}$

SOLVING for the WANTED symbol:

$$KE_{\text{ave}} = \frac{3RT}{2} = \frac{(3)}{(2)} \cdot \frac{8.31 \text{ kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K} \cdot \text{mol}} \cdot (298 \text{ K}) = 3,710 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{mol}} = 3,710 \frac{\text{J}}{\text{mol}}$$

(For help with the arrangement and cancellation of the units, see Lesson 17C.)

By using unit cancellation, after conversion from base units to the energy unit joules, the unit of the answer is the unit WANTED. The answer units must match those for the unit WANTED. If they do not, the units have indicated a correction is needed in your work.

Relating Kinetic Energy To P, V, and Moles

Looking at the *first* and *last* terms only:

$$\frac{PV}{n} = \frac{2}{3} KE_{\text{average, per mole}}$$

Equation (1) means that for any sample of a gas, if we know any 3 for the 4 variables P, V, n, and average KE per mole, the fourth variable can be calculated. Using the special rules for SI units in the bullets above, try this problem.

Q2. In a volume of $5.60 \times 10^{-3} \text{ m}^3$ of gas at a pressure of 50.5 kPa, the average kinetic energy of the molecules is 3,400 J/mol. How many moles of gas are in the sample?

* * * * *

List WANTED and DATA, assign symbols, and choose the equation that relates the symbols.

WANT: ? = moles of gas **n**

DATA: $5.60 \times 10^{-3} \text{ m}^3$ **V**

50.5 kPa **P**

3,400 J/mol **KE_{ave}**

Those variables are related by the first and last terms in Equation 1.

$$\frac{PV}{n} = \frac{2}{3} KE_{\text{average, per mole}}$$

To solve KMT equations, convert the units supplied to SI *base* units.

$$P \text{ in } \text{kg/m} \cdot \text{s}^2 = P \text{ in pascals} = 50.5 \text{ kPa} \cdot \frac{10^3 \text{ Pa}}{1 \text{ kPa}} = 5.05 \times 10^4 \text{ kg/m} \cdot \text{s}^2$$

$$KE_{\text{ave}} \text{ in } \text{kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2 = KE_{\text{ave}} \text{ in J/mol} = 3.4 \times 10^3 \text{ kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2$$

SOLVE: First solve the fundamental equation in symbols, then substitute.

$$\begin{aligned} ? = n &= \frac{3 P V}{2 KE_{\text{ave}}} = (3) \cdot \frac{5.05 \times 10^4 \text{ kg} \cdot \text{m} \cdot \text{s}^{-2}}{\text{m} \cdot \text{s}^2} \cdot \frac{5.60 \times 10^{-3} \text{ m}^3 \cdot \frac{\text{mol} \cdot \text{s}^2}{(2) 3.4 \times 10^3 \text{ kg} \cdot \text{m}^2}}{\text{m} \cdot \text{s}^2} = \\ &= 1.2 \times 10^{-1} \text{ mol} \quad \text{or} \quad 0.12 \text{ mol} \end{aligned}$$

Check that the units cancel to result in the unit WANTED.

Relating Kinetic Energy and Velocity

For a single particle in motion, its kinetic energy can be determined from its mass and velocity: $\boxed{KE = 1/2 mv^2}$.

To use that fundamental equation to calculate the average kinetic energy of the large number of moving particles in a gas, the velocity used must be the root mean square velocity: v_{rms} (a value slightly higher than the average velocity).

In gas calculations, it is preferred to calculate average kinetic energy **per mole**. If we calculate the average kinetic energy per mole of particles, the mass in $KE = 1/2 mv^2$ is the mass of a mole: the molar mass. Because we are calculating kinetic energy based on the fundamental equations of physics, we must convert to the SI base units for molar mass: *kilograms per mole* (**MM_{kg}**).

Using kilograms per mole and root mean square velocity in the kinetic energy definition equation, kinetic energy will be calculated in the SI units for gas kinetic energy: *joules per mole*. During the calculation, joules per mole will need to be written in the base units that joules is an abbreviation for:

$$\circ \text{ Joules} = \text{kg} \cdot \text{m}^2/\text{s}^2 \quad \text{and} \quad \text{Joules/mole} = \text{kg} \cdot \text{m}^2/\text{mol} \cdot \text{s}^2$$

To summarize: to relate kinetic energy, molar mass, and average velocity for a gas, the fundamental equation is:

$$\boxed{KE_{\text{average, per mole}} = 1/2 (\text{MM}_{\text{kg}}) (v_{\text{rms}})^2} \quad \text{(Equation 2)}$$

where the units must be SI units.

- KE is measured in **joules/mole**, which during KMT calculations must be written as **kg · m²/mol · s²**.
- The mass is the mass of a mole of molecules in **kilograms per mole (MM_{kg})**.
- The velocity is the root mean square velocity (v_{rms}) of the molecules, in **m/s**.

Using the rules and unit definitions above, try this example.

Q3. If the average kinetic energy of a sample of He molecules is 3,420 J/mol, find their root mean square velocity.

* * * * *

List the symbols and their base units, then = and the DATA.

WANT: v_{rms} in meters/second = ?

DATA: KE_{ave} in J/mol = KE_{ave} in $\text{kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2 = 3.4 \times 10^3 \text{ kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2$

The equation that relates the average kinetic energy to v_{rms} is Equation (2):

$$KE_{\text{average, per mole}} = 1/2 (MM_{\text{kg}}) (v_{\text{rms}})^2$$

This equation also needs the molar mass of the gas, in kg/mol.

$$MM_{\text{kg}} \text{ of He} = \frac{? \text{ kg He}}{1 \text{ mol He}} = \frac{4.00 \text{ g He}}{1 \text{ mol He}} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = \frac{4.00 \times 10^{-3} \text{ kg He}}{1 \text{ mol He}}$$

(The molar mass in **kg**/mol is always the value of the molar mass in **g**/mol *times* 10^{-3}).

SOLVE for the WANTED variable in symbols first, but the math is easier if you solve in steps: first solve for v_{rms}^2 in symbols, then get a value, then take the square root.

* * * * *

$$v_{\text{rms}}^2 = \frac{2 KE_{\text{ave}}}{MM_{\text{kg}}} = 2 KE_{\text{ave}} \cdot \frac{1}{MM_{\text{kg}}} = (2) \frac{3.42 \times 10^3 \text{ kg} \cdot \text{m}^2}{\text{mol} \cdot \text{s}^2} \cdot \frac{1 \text{ mol He}}{4.00 \times 10^{-3} \text{ kg He}} =$$

$$v_{\text{rms}}^2 = 1.71 \times 10^6 \text{ m}^2/\text{s}^2 \quad \text{Taking the square root of both sides:}$$

$$v_{\text{rms}} = (v_{\text{rms}}^2)^{1/2} = (1.71 \times 10^6 \text{ m}^2/\text{s}^2)^{1/2} = 1.31 \times 10^3 \text{ m/s}$$

Relating Temperature, Molar Mass, and Velocity

Starting from Equation 2:

$$KE_{\text{average, per mole}} = 1/2 (MM_{\text{kg}}) (v_{\text{rms}})^2$$

if we substitute the right-side symbols that are equivalent to KE_{ave} into Equation (1), the root mean square velocity can be defined as

$$v_{\text{rms}} = \sqrt{\frac{3RT}{MM_{\text{kg}}}} \quad \text{(Equation 3)}$$

Equation (3) means that for any amount of a single gas, knowing any two of the 3 variables of molar mass, root mean square velocity, or temperature, the third variable can be calculated. Using the KMT rules and SI units that apply to Equations (1), (2), and (3), try this example.

- Q4.** Using Equation 3, calculate the root mean square velocity at which the molecules of O_2 gas that you are breathing are traveling at room temperature (25°C).
($R = 8.31 \text{ J/K} \cdot \text{mol} = 8.31 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \cdot \text{K} \cdot \text{mol}$)

If you get stuck, read a part of the answer, and then try again.

* * * * *

WANT: v_{rms}

Use

$$v_{\text{rms}} = \sqrt{\frac{3RT}{MM_{\text{kg}}}}$$

DATA: List each equation symbol and SI base units that measure the symbol.

$$v_{\text{rms}} \text{ in meters/second} = ?$$

$$R \text{ in SI base units} = 8.31 \text{ J/mol} \cdot \text{K} = 8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$$

$$T \text{ in K} = ^\circ\text{C} + 273 = 25^\circ\text{C} + 273 = \mathbf{298 \text{ K}}$$

$$\text{MM}_{\text{kg}} \text{ of O}_2 = \frac{? \text{ kg O}_2}{1 \text{ mol O}_2} = \frac{32.0 \text{ g O}_2}{1 \text{ mol O}_2} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = \frac{\mathbf{0.0320 \text{ kg O}_2}}{1 \text{ mol O}_2}$$

If needed, adjust your work, SOLVE for v_{rms}^2 , then take the square root.

$$v_{\text{rms}}^2 = \frac{3RT}{\text{MM}_{\text{kg}}} = 3RT \cdot \frac{1}{\text{MM}_{\text{kg}}} = (3) 8.31 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K} \cdot \text{mol}} \cdot 298 \text{ K} \cdot \frac{1 \text{ mol O}_2}{0.032 \text{ kg O}_2} =$$

$$v_{\text{rms}}^2 = 2.32 \times 10^5 \text{ m}^2/\text{s}^2 \quad \text{Taking the square root of both sides:}$$

$$v_{\text{rms}} = (v_{\text{rms}}^2)^{1/2} = (2.32 \times 10^5 \text{ m}^2/\text{s}^2)^{1/2} = \mathbf{482 \text{ m/s}}$$

Think about how fast this means that molecules of oxygen are moving around you.

The final units match the SI units that a velocity must be. The unit cancellation is a check that you have done the algebra correctly. Now let's solve Equation 3 for temperature.

Q5. If the molecules of neon gas in a neon sign are traveling at a v_{rms} of 635 m/s, what is the temperature of the gas in degrees Celsius? (Use R from Q4.)

WANT: $^\circ\text{C}$ The equation that relates v_{rms} and temperature finds kelvins first.

$$v_{\text{rms}} = \sqrt{\frac{3RT}{\text{MM}_{\text{kg}}}} \quad \text{(Equation 3)}$$

DATA: List the equation's symbols and their SI *base* units.

$$v_{\text{rms}} = 635 \text{ m/s}$$

$$R \text{ in SI base units} = 8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$$

$$T \text{ in K} = ?$$

$$\text{MM}_{\text{kg}} \text{ of Ne} = \frac{? \text{ kg Ne}}{1 \text{ mol Ne}} = \frac{20.2 \text{ g Ne}}{1 \text{ mol Ne}} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = \frac{\mathbf{0.0202 \text{ kg Ne}}}{1 \text{ mol Ne}}$$

SOLVE the fundamental memorized equation for T in symbols before substituting values.

$$\text{The way to get T out of the radical is to square both sides: } v_{\text{rms}}^2 = \frac{3RT}{\text{MM}_{\text{kg}}}$$

$$T = \frac{v_{\text{rms}}^2 \cdot \text{MM}_{\text{kg}}}{3R} = \frac{(635)^2 \text{ m}^2}{\text{s}^2} \cdot \frac{\mathbf{0.0202 \text{ kg Ne}}}{1 \text{ mol Ne}} \cdot \frac{\text{s}^2 \cdot \text{K} \cdot \text{mol}}{(3) 8.31 \text{ kg} \cdot \text{m}^2} =$$

$$T = 327 \text{ K} \quad \boxed{K = ^\circ\text{C} + 273} \quad ^\circ\text{C} = K - 273 = 327 \text{ K} - 273 = 54 \text{ } ^\circ\text{C}$$

Practice: Practice writing Equations (1), (2), and (3) until you can write them from memory, then try these. Try the odds first, then the evens if you need more practice. (Use $R = 8.31 \text{ J/K} \cdot \text{mol} = 8.31 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \cdot \text{K} \cdot \text{mol}$; $\text{Pa} = \text{kg}/\text{m} \cdot \text{s}^2$)

- In symbols, solve Equation (3) for R .
- Find the root mean square velocity of steam molecules at $105 \text{ } ^\circ\text{C}$.
- A gas has a velocity of 4,290 miles per hour (!) at room temperature ($25 \text{ } ^\circ\text{C}$). What must be the molar mass and the molecular formula for the gas? ($1.61 \text{ km} = 1 \text{ mile}$)
- In 2.0 moles of gas at a pressure of 0.500 atm, the average kinetic energy of the molecules is 10,200 J/mol. What is the volume of the sample in cubic meters?
- The molecules in a sample of a noble gas have an average kinetic energy of 3,910 J/mol and a $v_{\text{rms}} = 243.7 \text{ m/s}$.
 - What is the temperature of the gas in $^\circ\text{C}$?
 - Which gas must it be?
- A sample of 0.25 moles of gas has a pressure of 101 kPa and a volume of 11.2 liters. Find the average kinetic energy (J/mol) of the molecules.

Need More Practice?

On any topic, if you find you need more practice, find the topic in any standard textbook and work the examples that are included in the reading for that topic. To work the examples, cover the textbook answer with a cover sheet, try the problem in your problem notebook, then check your answer. The text may solve problems differently but will result in the same answer as the methods here. If the answer differs, you should be able to locate your error in the solution provided in the text.

Problems at the end of the chapter in a general chemistry text may be more complex than the problems in these lessons, so, even if they are not assigned, you should work a few "end of the chapter" textbook problems that have answers you can check.

ANSWERS

$$1. \quad \boxed{v_{\text{rms}} = \sqrt{\frac{3RT}{\text{MM}_{\text{kg}}}}$$

$$v_{\text{rms}}^2 = \frac{3RT}{\text{MM}_{\text{kg}}} ; \quad R = \frac{v_{\text{rms}}^2 \cdot \text{MM}_{\text{kg}}}{3T}$$

2. WANTED: v_{rms} , given t . The equation that relates v_{rms} and T is

$$\boxed{v_{\text{rms}} = \sqrt{\frac{3RT}{\text{MM}_{\text{kg}}}}$$

DATA: v_{rms} in meters/second = ?
 R in SI base units = $8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$
 T in K = $^{\circ}\text{C} + 273 = 105^{\circ}\text{C} + 273 = \mathbf{378 \text{ K}}$
 Steam is H_2O in the gaseous state.

$$\text{MM}_{\text{kg}} \text{ of } \text{H}_2\text{O} = \frac{? \text{ kg H}_2\text{O}}{1 \text{ mol H}_2\text{O}} = \frac{18.0 \text{ g H}_2\text{O}}{1 \text{ mol H}_2\text{O}} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = \frac{0.0180 \text{ kg H}_2\text{O}}{1 \text{ mol H}_2\text{O}}$$

SOLVE: If you need to take a square root, the steps are usually simplified if you find a value for the square first, then take the square root.

$$v_{\text{rms}}^2 = \frac{3RT}{\text{MM}_{\text{kg}}} = 3RT \cdot \frac{1}{\text{MM}_{\text{kg}}} = (3) 8.31 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K} \cdot \text{mol}} \cdot 378 \text{ K} \cdot \frac{1 \text{ mol H}_2\text{O}}{0.0180 \text{ kg H}_2\text{O}} =$$

$$v_{\text{rms}}^2 = 5.24 \times 10^5 \text{ m}^2/\text{s}^2 \quad \text{Taking the square root of both sides:}$$

$$v_{\text{rms}} = (5.24 \times 10^5 \text{ m}^2/\text{s}^2)^{1/2} = \mathbf{724 \text{ m/s}}$$

3. WANT: **MM**: the molar mass and gas identity. The equation that relates MM_{kg} , t , and v_{rms} is

$$v_{\text{rms}} = \sqrt{\frac{3RT}{\text{MM}_{\text{kg}}}} \quad (\text{Equation 3})$$

DATA: v_{rms} in this equation must be in SI units: meters/second. Find the m/s by conversions.

* * * * *

$$v_{\text{rms}} \text{ in } \frac{\text{m}}{\text{s}} = \frac{4,290 \text{ miles}}{\text{hour}} \cdot \frac{1.61 \text{ km}}{1 \text{ mile}} \cdot \frac{10^3 \text{ m}}{1 \text{ km}} \cdot \frac{1 \text{ hour}}{60 \text{ min.}} \cdot \frac{1 \text{ min.}}{60 \text{ s}} = \mathbf{1.92 \times 10^3 \text{ m/s}}$$

$$R \text{ in SI base units} = 8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{K} \cdot \text{mol}$$

$$T \text{ in K} = ^{\circ}\text{C} + 273 = 25^{\circ}\text{C} + 273 = \mathbf{298 \text{ K}}$$

$$\text{MM}_{\text{kg}} = ?$$

SOLVE first in symbols for the WANTED variable: **MM**. One way to solve for MM is to square both sides.

$$v_{\text{rms}}^2 = \frac{3RT}{\text{MM}_{\text{kg}}}$$

$$\text{MM}_{\text{kg}} = \frac{3RT}{v_{\text{rms}}^2} = (3) 8.31 \frac{\text{kg} \cdot \text{m}^2}{\text{s}^2 \cdot \text{K} \cdot \text{mol}} \cdot 298 \text{ K} \cdot \frac{\text{s}^2}{(1.92 \times 10^3)^2 \text{ m}^2} = \mathbf{0.00202 \text{ kg/mol}}$$

$$\text{MM}_{\text{grams}} = \frac{? \text{ g}}{\text{mol}} = 0.00202 \frac{\text{kg}}{\text{mol}} \cdot \frac{10^3 \text{ g}}{1 \text{ kg}} = \mathbf{2.02 \text{ g/mol}} \quad \text{Which gas could this be?}$$

* * * * *

The only molecule with that low molar mass is H_2 , at 2.016 g/mol. This answer means that at room temperature, molecules of H_2 gas are moving at an average speed of over 4,000 miles/hour.

4. WANT: Volume in liters. The variables that are supplied are n , P , and KE_{ave} .

The equation that relates KE_{ave} and n , P , and V is the first and last terms of Equation 1:

$$\frac{PV}{n} = \frac{2}{3} KE_{\text{ave}}$$

Write a data table that lists P, V, n, and KE_{ave} , all converted to SI *base* units.

* * * * *

DATA: n in moles = 2.0 mol

P in $\text{kg/m} \cdot \text{s}^2$ = P in Pa = 0.500 atm • 101 kPa/1 atm = $5.05 \times 10^4 \text{ kg/m} \cdot \text{s}^2$

V in m^3 = ?

KE_{ave} in SI units = 10,200 J/mol = $1.02 \times 10^4 \text{ J/mol}$ = $1.02 \times 10^4 \text{ kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2$

* * * * *

SOLVE the equality using the first and last terms of Equation 1 for the WANTED symbol in symbols first:

$$\text{If } \frac{PV}{n} = \frac{2}{3} KE_{\text{ave}}; \text{ then } V = \frac{2n \cdot KE_{\text{ave}}}{3P} = \frac{(2)(2.0 \text{ mol}) 1.02 \times 10^4 \text{ kg} \cdot \text{m}^2 \cdot \frac{\text{m} \cdot \text{s}^2}{\text{mol} \cdot \text{s}^2}}{(3) 5.05 \times 10^4 \text{ kg}} = 0.27 \text{ m}^3$$

Note that after unit cancellation, the unit is the unit WANTED.

5a. WANT: °C

DATA: KE_{ave} = 3,910 J/mol

v_{rms} in m/s = 243.7 m/s

Which equation most easily relates the WANTED variable to the DATA?

$$\frac{PV}{n} = RT = \frac{2}{3} KE_{\text{ave}}$$

The right two terms relate temperature and KE_{ave} .

Convert the DATA to SI base units. R is needed in SI base units as well.

KE_{ave} in SI units = 3,910 J/mol = $3,910 \text{ kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2$

R = 8.31 J/K • mol = $8.31 \text{ kg} \cdot \text{m}^2 / \text{s}^2 \cdot \text{K} \cdot \text{mol}$

$$T = \frac{2 KE_{\text{ave}}}{3R} = \frac{(2) 3,910 \text{ kg} \cdot \text{m}^2 \cdot \frac{\text{s}^2 \cdot \text{K} \cdot \text{mol}}{\text{mol} \cdot \text{s}^2}}{(3) 8.31 \text{ kg} \cdot \text{m}^2} = 314 \text{ K}$$

$$^{\circ}\text{C} = ? \quad T = 314 \text{ K} \quad \boxed{\text{K} = ^{\circ}\text{C} + 273} \quad ^{\circ}\text{C} = \text{K} - 273 = 314 \text{ K} - 273 = 41 ^{\circ}\text{C}$$

5b. WANT: **Identify the noble gas.** Which variable in the 3 KMT equations will identify the gas?

* * * * *

The six noble gases have very different molar masses. Find MM and you can likely identify the gas.

WANT: Molar mass. Equation (2) includes molar mass and the two DATA symbols. Equation 3 could be solved using the part 5a answer and the data. If you use Equation 2:

$$KE_{\text{average, per mole}} = 1/2 (MM_{\text{kg}}) (v_{\text{rms}})^2$$

WANT: MM_{kg} = molar mass of unknown noble gas in kg/mol

DATA: KE_{ave} in $\text{kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2 = KE_{ave}$ in $\text{J/mol} = 3,910 \text{ kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2$
 v_{rms} in $\text{m/s} = 243.7 \text{ m/s}$

SOLVE: $MM_{kg} = \frac{2 KE_{ave}}{v_{rms}^2} = 2 KE_{ave} \cdot \frac{1}{v_{rms}^2} = (2) \frac{3,910 \text{ kg} \cdot \text{m}^2}{\text{mol} \cdot \text{s}^2} \cdot \frac{\text{s}^2}{(243.7)^2 \text{ m}^2} =$

$MM_{kg} = 0.132 \text{ kg/mol}$; ? $MM_g = ? \frac{\text{g}}{\text{mol}} = \frac{0.132 \text{ kg}}{\text{mol}} \cdot \frac{10^3 \text{ g}}{1 \text{ kg}} = 132 \text{ g/mol}$

Which noble gas is this?

* * * * *

Xenon

6. WANT: KE_{ave} in joules/mole = KE_{ave} in $\text{kg} \cdot \text{m}^2 / \text{mol} \cdot \text{s}^2$

The variables that are supplied are n , P , and V .

The equation that relates KE_{ave} and n , P , and V is the first and last terms of Equation 1:

$$\frac{PV}{n} = \cancel{RT} = \frac{2}{3} KE_{ave. \text{ per mole}}$$

Write a data table that lists P , V , n , and KE_{ave} converted to SI *base* units.

* * * * *

DATA: n in moles = 0.25 moles

P in $\text{kg/m} \cdot \text{s}^2 = 101 \text{ kPa} = 101 \times 10^3 \text{ Pa} = 101 \times 10^3 \text{ kg/m} \cdot \text{s}^2$

$$V \text{ in } \text{m}^3 = 11.2 \text{ L} \cdot \frac{1000 \text{ cm}^3}{1 \text{ L}} \cdot \left(\frac{10^{-2} \text{ meter}}{1 \text{ cm}} \right)^3 = 11.2 \text{ L} \cdot \frac{1000 \text{ cm}^3}{1 \text{ L}} \cdot \left(\frac{10^{-6} \text{ m}^3}{1 \text{ cm}^3} \right) =$$

$$= 11.2 \times 10^{-3} \text{ m}^3 = 1.12 \times 10^{-2} \text{ m}^3 \text{ or } 0.0112 \text{ m}^3 = V$$

(To review distance to volume conversions, see Lesson 5F.)

* * * * *

SOLVE the equality between the first and last terms of Equation (1) in symbols first:

If $\frac{PV}{n} = \frac{2}{3} KE_{ave}$; then $KE_{ave} = \frac{3PV}{2n} = \frac{(3) 101 \times 10^3 \text{ kg}}{\text{m} \cdot \text{s}^2} \cdot \frac{(1.12 \times 10^{-2} \text{ m}^3)}{(2) 0.25 \text{ mol}} =$
 $= 6,800 \frac{\text{kg} \cdot \text{m}^2}{\text{mol} \cdot \text{s}^2} = 6.8 \times 10^3 \frac{\text{J}}{\text{mol}}$ which is the unit that was WANTED.

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Lesson 19D: Graham's Law

Timing: Complete this lesson *if* you are assigned calculations using Graham's law.

* * * * *

Diffusion and Effusion

Diffusion is the mixing of gases, and diffusion can be a slow process. At room temperature, the gas molecules released by burning toast are traveling at speeds of many hundreds of miles per hour. However, because these molecules collide with other molecules in the air as they travel, they must travel a zig-zag path to your nose, and the toast may burn a while before you notice.

Why the zig-zag? In an ideal gas, the volume of a gas molecule is presumed to be infinitesimally small, and molecules would rarely collide. In reality, the diameter of a gas molecule is significant: in a typical gas at room temperature and pressure, the gas molecules are roughly 10 diameters apart. When gas molecules are moving fast and are roughly 10 diameters apart, they will collide and bounce off each other frequently. This means that for values near room temperature and pressure, the rate of diffusion for gas molecules is much slower than their average speed.

Effusion is a term used to describe the rate at which a trapped gas escapes from its container through a small hole into a vacuum. Ideally, during such effusion, molecules would not collide, and the rate of effusion is directly proportional to the v_{rms} of the gas molecules.

Gases with different molar masses will have different rates of effusion. If we place two gases in a container with a small hole, surrounded by a vacuum, the *ratio* of the rates of diffusion will be

$$\frac{\text{Rate of effusion of gas } a}{\text{Rate of effusion of gas } b} = \frac{v_{rms} \text{ of gas } a}{v_{rms} \text{ of gas } b} = \frac{\sqrt{3RT/MM_a}}{\sqrt{3RT/MM_b}} = \frac{\sqrt{1/MM_a}}{\sqrt{1/MM_b}} = \frac{1/\sqrt{MM_a}}{1/\sqrt{MM_b}}$$

The above equation simplifies to the relationship known as

$$\text{Graham's law: } \frac{\text{Rate of effusion of gas } a}{\text{Rate of effusion of gas } b} = \frac{\sqrt{MM_b}}{\sqrt{MM_a}}$$

In words: the rate of effusion of a gas is *inversely* proportional to the square root of its molar mass. Graham's law can be used with molar masses measured in *any* units, as long as both molar masses are measured in the same units.

Let's try an example of a Graham's law calculation.

- Q. To avoid nitrogen narcosis (the bends), scuba diving tanks are often filled with a mixture of oxygen (O₂) and helium (He). Calculate the ratio of the effusion rate of helium to oxygen.

* * * * *

WANTED: The ratio (effusion rate He/effusion rate O₂).

The equation that uses a ratio of effusion rates is Graham's Law:

$$\frac{\text{Rate of effusion of gas } \mathbf{a}}{\text{Rate of effusion of gas } \mathbf{b}} = \frac{\sqrt{\text{MM}_b}}{\sqrt{\text{MM}_a}}$$

DATA: Gas A = He ; MM He = 4.0 g/mol

Gas B = O₂ ; MM O₂ = 32.0 g/mol

Either gas can be A or B. Any unit for molar mass can be used as long as the units are consistent.

$$\frac{\text{Rate of effusion of He}}{\text{Rate of effusion of O}_2} = \frac{\sqrt{\text{MM O}_2}}{\sqrt{\text{MM He}}} = \frac{\sqrt{32.0 \text{ g/mole}}}{\sqrt{4.00 \text{ g/mol}}} = \frac{5.66}{2} = 2.83$$

For helium, the rate of effusion (and v_{rms}) is 2.83 times higher than for oxygen.

Practice: Add a flashcard for Graham's law to your collection so study for this module, and practice writing the law until you can do so from memory. Then try these.

1. Uranium hexafluoride (UF₆) that contains the more common U-238 isotope has a molar mass of 352.05 g/mol. UF₆ that contains the fissionable U-235 isotope has a molar mass of 349.04 g/mol. Calculate the ratio of the rate of effusion for the lighter over the heavier molecules of UF₆.
2. A sample of chlorine gas (Cl₂) is surrounded by a vacuum. When a small hole is opened in the container, and the contents of the container are kept at constant pressure, the chlorine leaves the container at a rate of 1.50 mL/second. When a second gas is placed in the container under the same conditions, it is found to exit at a rate of 6.32 mL/s. Find the molar mass of the second gas.

ANSWERS

1. WANT: The ratio (effusion rate ²³⁵UF₆/effusion rate ²³⁸UF₆).

The equation that uses a ratio of effusion rates is Graham's Law:

$$\frac{\text{Rate of effusion of gas } \mathbf{a}}{\text{Rate of effusion of gas } \mathbf{b}} = \frac{\sqrt{\text{MM}_b}}{\sqrt{\text{MM}_a}}$$

DATA: Gas A = lighter = ²³⁵UF₆; MM ²³⁵UF₆ = 349.04 g/mol

Gas B = heavier = ²³⁸UF₆; MM ²³⁸UF₆ = 352.05 g/mol

$$\frac{\text{Rate of effusion of } \textit{lighter}}{\text{Rate of effusion of } \textit{heavier}} = \frac{\sqrt{\text{MM } \textit{heavier}}}{\sqrt{\text{MM } \textit{lighter}}} = \frac{\sqrt{352.05 \text{ g/mole}}}{\sqrt{349.04 \text{ g/mol}}} = \frac{18.763}{18.683} = \mathbf{1.0043}$$

2. WANT: **MM Unknown** (list the symbols as they are written in the equations being studied.)

DATA: MM Cl₂ = 71.0 g/mol

Effusion rate Cl₂ = 1.50 mL/s

Effusion rate Unk = 6.32 mL/s

The equation with two effusion rates is Graham's law:

$$\frac{\text{Rate effusion Cl}_2}{\text{Rate effusion Unk}} = \frac{\sqrt{\text{MM Unk}}}{\sqrt{\text{MM Cl}_2}} ; \quad \frac{1.50 \text{ mL/s}}{6.32 \text{ mL/s}} = \frac{\sqrt{\text{MM Unk}}}{\sqrt{71.0 \text{ g/mol}}}$$

Solving with the ratio upside down does not change the answer.

SOLVE for the term with the WANTED symbol.

$$\sqrt{\text{MM Unk}} = \frac{1.50}{6.32} \cdot \sqrt{71.0 \text{ g/mol}} = 0.2373 \cdot 8.426 \text{ (g/mol)}^{1/2} = \mathbf{2.00 \text{ (g/mol)}^{1/2}} \text{ Done?}$$

* * * * *

$$\text{MM Unk} = (\sqrt{\text{MM Unk}})^2 = (2.00)^2 \text{ ((g/mol)}^{1/2})^2 = \mathbf{4.00 \text{ g/mol}}$$

The solved unit must match the WANTED unit.

* * * * *

SUMMARY: Kinetic Molecular Theory

1. For squares and square roots of exponential notation:
 - a. To take exponential terms to a power, multiply the exponents.
 - b. "Taking the square root" is the same as taking the quantity to the **1/2** power.
 - c. To take the square root of an even exponential term: cut the exponent in half.
2. **Root mean square velocity** = $v_{\text{rms}} = (v^2)^{1/2} \approx \bar{v} \equiv$ average velocity of the molecules.
The v_{rms} is slightly higher than the average velocity of the molecules.

3. **Kinetic Molecular Theory (KMT).**

Based on the laws of physics for particles in motion with perfectly elastic collisions,

a.
$$\frac{PV}{n} = RT = \frac{2}{3} \text{ KE}_{\text{ave per mole}}$$

b.
$$\text{KE}_{\text{average per mole}} = 1/2 (\text{MM}_{\text{kg}}) (v_{\text{rms}})^2$$

c.
$$v_{\text{rms}} = \sqrt{\frac{3RT}{\text{MM}_{\text{kg}}}}$$

4. To simplify solving KMT equations,
- Convert all units to SI *base* units: meters, kilograms, seconds, moles, and kelvins.
 - Use R in SI base units = $8.31 \text{ J/mol} \cdot \text{K} = 8.31 \text{ kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{mol} \cdot \text{K}$
 - KE is measured in the energy unit **joules/mole**, which during KMT calculations is written as $\text{kg} \cdot \text{m}^2/\text{s}^2 \cdot \text{mol}$.
 - Molar mass is converted to **kilograms per mole** (MM_{kg}) .
 - Velocity is the root mean square velocity (v_{rms}) of the gas molecules in **m/s** .

5. Converting to SI Base Units

To write the SI *base* units for a derived quantity, or for a unit such as N, Pa, or J that is an abbreviation for a combination of SI base units ,

- Write the equation that calculates the derived quantity, or the quantity that the abbreviated unit measures. In the equation, use quantities for which the SI base units are known.
- Substitute the SI base units for the quantities with known base units into the equation for the derived quantity.

6. Graham's law

$$\frac{\text{Rate of effusion of gas } a}{\text{Rate of effusion of gas } b} = \frac{v_{\text{rms}} \text{ of gas } a}{v_{\text{rms}} \text{ of gas } b} = \frac{\sqrt{\text{MM}_b}}{\sqrt{\text{MM}_a}}$$

If two different gases are at the same temperature, the *lighter* molecules (those with the lower molar mass) have a *higher* average velocity.

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