(New activities added 8/19/18)

Mental Math for Chemistry

In this packet are activities designed to strengthen student skills in calculations without a calculator.

The resources are divided into 3 groups: For General Chemistry, for "Prep Chem", and for instructors a summary of research.

Why is Mental Math Important?

Since 2000, with assistance from new technologies, science has made rapid progress in understanding how the brain solves problems. One new finding is: When solving a scientific calculation, of critical importance is the ability to recall mathematical *facts* and *procedures* from long-term memory, and to do so *fluently* (quickly and accurately).

Activity #12 in this packet is a review of the research on the importance of mental math recall, but here's a quick summary:

- Empirical studies have found that among many factors, the best pre-course predictor of student success in college general chemistry is the ability to solve simple calculations without a calculator.
- Cognitive science has learned that the "working memory" where the brain solves problems has unlimited room for well-memorized facts, relationships, and procedures, but very little room for information that is not well-memorized. When non-memorized information such as a calculator answer is moved into working memory, it tends to bump out unique problem data; recalling memorized facts and relationships does not.
- Cognitive studies also emphasize that skill in solving calculations is most quickly achieved by "*over*-learning" math fundamentals: Practicing repeatedly over multiple days to achieve automatic, fluent recall of new facts and procedures.
- In many states prior to 2014, before the importance of automaticity in recall became a scientific consensus, math curricula adopted under state K-12 standards encouraged calculator use in elementary grades and discouraged memorization.
- Recognizing the importance of mental math for STEM success, tests including the MCAT and GRE Math and Chemistry sections require solving mathematical computations but do not allow calculator use.

The activities in this packet are designed to help students fill in gaps in their mental math skills and overlearn math and science facts and procedures.

This packet contains

- A short **summary** of each activity option, followed by
- The detailed **activity** options, and
- An **overview of research** on the importance of mental arithmetic in chemistry.

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Summary: Each "No Calculator" Activity Option

From this list, pick activities of interest, then check the detailed descriptions that follow.

For General Chemistry

#1. **Quiz on Mental Math Fluency.** This 15-minute "no calculator" quiz on math facts, exponential calculations, logarithms, decimal equivalents of fractions, and basic algebra will identify strengths and weaknesses in student preparation for the math of chemistry. Results can be used to identify gaps and plan math review.

#2 **Homework Review of Chem I.** During the first weeks of the school year, this self-study tutorial provides over 60 pages that engage students in review of numeracy, exponential calculations, the metric system, significant figures, dimensional analysis, and atomic structure.

#3. **MM # 1 – Converting Fractions to Decimal Equivalents.** Two pages of practice in recalling and applying decimal equivalents to simplify calculations.

#4. **MM # 2 - Exponentials and Fractions.** A two-page worksheet covering a variety of strategies to simplify calculations mixing numbers and exponents.

#5. **MM # 3 - Fractions, Estimates, and Exponentials.** A page of practice in estimation, fraction simplification, and exponential calculations.

For Preparatory Chemistry

#6. **Try Activity #1.** The mental math fluency test in Activity #1 – tweaked to measure math preparation for introductory chemistry.

#7. **Numeracy Tutorial.** A 6 page homework assignment reviewing the mechanics of simple calculations without a calculator. Includes lecture notes, practice problems, and worked-out answers.

#8 **Simplifying Fractions** is a page of practice designed so that instructors can provide guidance on "no calculator" calculations.

#9 **Multi-digit Multiplication and Division.** Practice with the standard algorithms for multiplication and division without a calculator to strengthen "automaticity" in recalling math facts.

#10 **Scientific Notation and Exponential Calculations.** A 17 page homework tutorial that reviews exponential basics, scientific notation, and how to simplify exponential calculations -- both with and without a calculator.

For Instructors

#11. **Other Mental Math Resources.** Contains a listing of additional mental math lessons, by topic, available for review by instructors.

#12. **How the Student Brain Solves Problems**. A summary of research on the importance of mental math skills when solving calculations in the sciences.

# # # # #

Detail: "No Calculator" Activity Options

For General Chemistry

Activity #1 – *Quiz on Mental Math Fluency*

To the Instructor

Activity #1 is a 15-minute "no calculator" quiz on math facts, decimal equivalents of fractions, exponential and logarithm calculations, and basic algebra. The goal is to identify areas of pre-requisite math that may need review as preparation for chemistry calculations.

Timing

Given in a first-week class, the quiz may indicate which math topics may need particular attention during the initial review of pre-requisite fundamentals and later in the course.

Access

The fluency quiz (in an editable format) may be downloaded at www.ChemReview.Net/FluencyQuiz.docx (click and check your downloads).

Follow-Up

Most of the quiz questions have *two* components: How well do students know the math operation, and how well could they perform the mental arithmetic of the problem?

You may want to analyze:

- On question one: What percentage of your students could correctly multiply two digits times two digits without a calculator?
- On questions 2-8, what was the percentage correct for each of these questions? For problems where a number of students had difficulty, did any difficulty appear to be with the rules of the math operation, or the mental arithmetic, or both?

If mental arithmetic (recall of math facts) is a problem, there are two possibilities. One is that students once knew their math facts *well*, but have forgotten them due to calculator use. Science tells us that in the case of *forgetting*, the information is still stored in memory, but the neural "wiring" to reach it needs to be re-grown by practice. These students will "refresh their memory" (re-connect the wiring) relatively quickly when required to practice math-fact recall.

The second possibility is that students were never required to "memorize their times tables." In some states prior to 2014, state K-12 math standards required teachers to have students use calculators to do arithmetic starting in third grade, rather than memorize facts. Since 2014, Common Core-type standards have restored some emphasis on computational fluency, but this means in many states, current students at some point went from one set of standards to another in the middle of their K-8 schooling. Those students may need substantial practice to gain the fluency in recall of math facts that science says they need to ease scientific problem solving.

If the quiz and subsequent exercises indicate that some students need help with mental arithmetic, Activities 8 and 9 will provide options for additional practice.

On the fluency quiz, questions after Question 8 concern math operations and procedures needed later in the course. If learning or "refreshing of memory" is needed for those procedures, Activity #11 lists additional available resources in mental and calculator math.

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Activity #2 – *Homework Tutorials Reviewing Chem I*

During the first weeks of the school year, posted online for free student use are over 60 pages of self-study tutorials that help students *review* and *overlearn* high school math and chemistry fundamentals that are a foundation for General Chemistry. Topics include exponential calculations, the metric system, significant figures, dimensional analysis, word problems, and atomic structure.

"Pretests" at the start of most lessons encourage students to bypass what they remember and focus where review is needed.

The tutorial content can be viewed at www.ChemReview.Net/TutorialContent.html

Suggestions for assignment of the tutorials to students are detailed at www.ChemReview.Net/GCFallReview.html

Assignment Length

Students have free access to Chapters 1-6 until September 17. Thereafter, free access is provided to Chapters 1 and 2.

It is recommended they be assigned on the first day of class. If your schedule allows students 3 or more weeks of access to Chapters 1-6, Gen Chem students should be able to complete all 6 chapters.

If students have only two weeks of access to the tutorials, you may want to limit the assignment to Chapters 1-5.

Quizzes

To encourage timely homework completion, quizzes are provided on the tutorial content. For access, click here.

At least two quizzes on the tutorials are recommended: the first at the half-way point in terms of assignment time, the second at the end.

Quizzes are available for Chapters 1, 1-2, 1-3, 1-4, 1-5, and 4-6. All quizzes except the last are cumulative, so that for these fundamentals, if they don't get a chapter studied for the first quiz, they are encouraged to complete it before the second.

If students have 3 weeks to complete the material, a short quiz each week might encourage "keeping up." These could be the provided quizzes on Chapters 1-2, 1-4, and 4-6.

Class Coverage of Topics

For students who have thoroughly covered each of the Chapter 1-6 topics in their first course in chemistry, instructors report the tutorial "refreshing of memory" works well, and minimal lecture on these topics is needed.

If some topics from Chapters 1-6 were missed for some students, you may want to provide some lecture on those topic as well as assign the tutorials. Some of the practice sheets in the Prep Chem section below might be helpful as well.

If you are not certain of the math background for your students, the fluency quiz in Activity #1, given the first week of class, should help in planning which initial topics may need special attention.

# # # # #

Activity #3

Mental Arithmetic #1 - Converting Fractions to Decimal Equivalents

To the Instructor

In tutorial Lesson 1.2, students are asked to commit to memory the decimal equivalents of eight commonly encountered fractions. Activity #3 asks students to apply those memorized values to calculate additional decimal equivalents. Those values are then applied to solve calculations.

This practice sheet may be started at any point after tutorial Lesson 3 in Chapter 1 on exponential basics (or an equivalent exercise) is completed.

Use

The activity could be started in class, with instructor guidance and feedback on selected questions, or simply assigned as homework.

Follow-Up

Activities #4 and #5 build on Activity #3.

The quizzes supplied with the tutorials do not contain questions on the content of these optional activities, but one or two similar problems could be added to a quiz (or a part of a quiz) on which "no calculator" is allowed.

If students see material from the activities on quizzes, attention to the work in class and "left for homework" may increase.

# # # # #

Mental Arithmetic #1 - Converting Fractions to Decimal Equivalents

On this exercise, do NOT use a calculator.

1. Every numeric fraction Y/X (if X # 0) has a numeric decimal equivalent. In Lesson 1.2, for the eight fractions encountered most often in scientific calculations, you were asked to commit the decimal equivalents to memory.

   From memory, write the decimal equivalents for those fractions.

   a. $\frac{1}{2}$ = 0.50　　b. $\frac{1}{3}$ =　　c. $\frac{1}{4}$ =　　d. $\frac{1}{5}$ =

   e. $\frac{2}{3}$ =　　f. $\frac{3}{4}$ =　　g. $\frac{1}{8}$ =　　h. $\frac{3}{2}$ =

2. If you know the decimal equivalent for a fraction 1/X , you can calculate the value of any decimal equivalent Y/X by *adding* the 1/X decimal equivalent Y times.

   Since. $\frac{2}{5} = \frac{1}{5} + \frac{1}{5}$ = 0.2 + 0.2 = **0.4** , then (finish) $\frac{3}{5}$ =

3. Or, knowing 1/X, you can find a decimal equivalent for Y/X using Y/X = Y(1/X) . .

   Finish the calculation below by multiplication to find the decimal equivalent.

   $\frac{4}{5} = 4 \times \frac{1}{5} =$

4. Some decimal equivalents can be solved by reducing the fraction to obtain a Y/X that can be recalled from memory or is easier to solve.

   Fill in any blanks in these fractions, then write the decimal equivalent in the form 0.XXX

   a. $\frac{1}{8}$ = **0.**　　b. $\frac{2}{8} = \frac{1}{\quad}$ =

   c. $\frac{3}{8}$ = **0.**　　d. $\frac{4}{8} = \frac{1}{\quad}$ =

5. What is the trend in the Question 4 answers?

6. Using any of the strategies above, convert these fraction to a decimal equivalent value in the format 0.XXX

   a. $\frac{5}{8}$ =　　f. $\frac{6}{8}$ =

   c. $\frac{7}{8}$ =

7. Given that 1/25 = 0.040 , convert these to a decimal equivalent in the format 0.XXX

   a. $\frac{2}{25}$ =　　b. $\frac{3}{25}$ =　　c. $\frac{5}{25}$ =

d. $\frac{9}{25} =$ e. $\frac{12}{25} =$

8. When a non-memorized fraction has an odd number in the numerator, but you know the value for 1/X, it often helps to split the numerator into two added parts where the second is 1/X. The formula is:

$$\frac{Y_{odd}}{X} = \frac{Y-1}{X} + \frac{1}{X}$$

Y – 1 is even, giving a fraction that can often be reduced to a fraction with a familiar decimal equivalent.

Follow the steps of this worked example:

$$\frac{7}{8} = \frac{6}{8} + \frac{1}{8} = \frac{3}{4} + \frac{1}{8} = 0.75 + 0.125 = \mathbf{0.875}$$

On separate paper, convert these to 0.XXXX:

a. If 1/12 = 0.0833, then 7/12 = ? b. If 1/16 = 0.0625, then 5/16 = ?

c. If 1/16 = 0.0625, then 13/16 = ?

9. If a denominator ends in a single zero before the decimal, the decimal equivalent will be the same as the equivalent for the denominator without the zero, but the equivalent will have its decimal moved one to the left.

But let's explain that with a formula and some examples.

If $\frac{Y}{X} =$ A**.**BC then $\frac{Y}{X0} =$ 0**.**ABC or

Since $\frac{1}{5} = 0.200$, then $\frac{1}{5\mathbf{0}} = 0.\mathbf{0}20$ Since $\frac{3}{4} = 0.75$, then $\frac{3}{40} = 0.075$

Follow the logic of the math:

$$\frac{3}{50} = \frac{3}{5} \times \frac{1}{\mathbf{10}} = 3 \times \frac{1}{5} \times \frac{1}{10} = 3 \times 0.200 \times 0.10 = 0.600 \times 10^{-1} = \mathbf{0.060}$$

On a separate paper, convert these to a decimal equivalent in the format 0.XXX**X**

a. $\frac{1}{40}$ b. $\frac{1}{80}$ c. 1/20 d. 13/20 e. 5/80

10. On separate paper, using any strategy you choose, convert these to the form 0.XXX

a. $\frac{1}{20}$ b. $\frac{3}{12}$ c. $\frac{7}{20}$ d. $\frac{9}{72}$ e. $\frac{8}{40}$ f. $\frac{2}{30}$ g. $\frac{3}{50}$ h. $\frac{8}{120}$

Answers: 1b. 0.333 1c. 0.25 1d. 0.20 1e. 0.667 1f. 0.75 1g. 0.125 2. 0.6 3. 4 × 0.20 = 0.80 4a. 0.125 4b. ¼ = 0.250 4c. 0.375 4d. ½ = 0.500 5. Each answer increases by 0.125 6a. 0.625 6b. 0.750 6c. 0.875 7a. 0.080 7b. 0.120 7c. 0.350 7d. 0.360 7e. 0.480 8a. 0.5833 8b. 0.8125 8c. 0.3125 9a. 0.0250 9b. 0.0125 9c. 0.0500 9d. 0.6500 9e. 0.0625 10a. 0.050 10b. 0.250 10c. 0.150 10d. 0.125 10e. 0.250 10f. 0.067 10g. 0.060 10h. 0.067

Activity #4

Mental Arithmetic #2: Exponentials and Fractions

To the Instructor

Activity #4 is 2-pages combining the decimal equivalent work in Activity #3 with the exponential notation review in tutorial Chapter 1. It can be assigned at any time after both Activity #3 and Chapter 1 have been completed.

If some students need additional mental *arithmetic* practice, you may suggest Activities #8 and #9 in the Prep Chem activities below.

# # # # #

Mental Arithmetic #2: Fractions and Exponentials

On this sheet, do NOT use a calculator.

1. Convert each term to a whole number power of 10 (such as 10^{-4}), then simplify. Write your final answer as a whole number power of 10.

   a. $\dfrac{100 \times 0.10}{0.0001 \times 0.010}$ = ________________ =

   b. $\dfrac{1}{1{,}000 \times 0.010}$ = ________________ =

   c. $\dfrac{10 \times 0.0010}{10{,}000 \times 0.010}$ =

   d. $\dfrac{0.10 \times 0.010}{1{,}000 \times 100{,}000}$ =

2. Calculate, then convert the final answer to scientific notation with 3 digits in the significand.

   a. $\dfrac{3 \times 10^{-5}}{4 \times 10^{-2}}$ =

   b. $\dfrac{2 \times 10^{9}}{8 \times 10^{-2}}$ =

   c. $\dfrac{1}{80 \times 10^{-2}}$ =

   d. $\dfrac{3 \times 0.0010}{5 \times 0.010}$ =

3. Zeros *before* the decimal but *after* all other numbers can "cancel on the top and bottom."

   Example: $\dfrac{60 \times 4{,}000}{800} = \dfrac{6\cancel{0} \times 4{,}00\cancel{0}}{8\cancel{00}} = \dfrac{2400}{8} = \mathbf{300}$

   Below, cancel zeros top and bottom, then simplify using mental arithmetic. Write the answer as a fixed decimal number.

   a. $\dfrac{600 \times 40}{10 \times 30}$ =

   b. $\dfrac{7500 \times 90}{30 \times 25}$ =

   c. $\dfrac{160 \times 300}{3200 \times 60}$ =

4. Fractions with zeros in the denominator *before* the decimal but *after* all other numbers can be separated into two parts and converted to exponential notation.

   Example: $\dfrac{3}{500} = \dfrac{3}{5 \times 100} = \dfrac{3}{5} \times \dfrac{1}{100} = 0.60 \times 10^{-2} = 6.0 \times 10^{-3}$

   Separate these into two parts, then simplify. Write the final answer in scientific notation with two digits in the significand.

   a. $\dfrac{64}{160} =$

   b. $\dfrac{144}{1200} =$

   c. $\dfrac{63}{7{,}000} =$

5. Convert these to fixed decimal values in the form 0.XXX

   a. $\dfrac{7}{28 \times 2} =$

   b. $\dfrac{70}{2 \times 50} =$

   c. $\dfrac{4 \times 45}{30 \times 9} =$

   d. $\dfrac{400}{20 \times 60} =$

6. Re-write each of the two quantities to scientific notation, multiply or divide, then convert to scientific notation with a 2-digit significand.

   a. $(7{,}000 \times 10^{-2})\ (0.080 \times 10^{-11}) =$

   b. $(0.090 \times 10^{9})\ (0.0070 \times 10^{-8}) =$

   c. $\dfrac{8 \times 10^{-3}}{200 \times 10^{-2}} =$ ________________ $=$

   d. $\dfrac{72 \times 10^{-3}}{900 \times 10^{7}} =$ ________________ $=$

Answers: 1a. 10^{7} 1b. 10^{-1} 1c. 10^{-4} 1d. 10^{-11} 2a. 7.50×10^{-4} 2b. 2.50×10^{10}
2c. 1.25×10^{0} 2d. 6.00×10^{-2} 3a. 80 3b. 900 3c. 0.25 4a. 4×10^{-1} 4b.
2b. 1.2×10^{-1} 4c. 9.0×10^{-3} 5a. 0.125 5b. 0.700 5c. 0.667 5d. 0.333
6a. 5.6×10^{-11} 6b. 6.3×10^{-4} 6c. 4.0×10^{-3} 6d. 8.0×10^{-12}

Activity #5

Mental Arithmetic #3: Fractions, Estimates, and Exponentials

To the Instructor

Activity #5 can be started in class at any time after both Activity #4 and tutorial Chapter 1 have been completed.

Mental Arithmetic #3: Fractions, Estimation, and Exponentials

On this sheet, do NOT use a calculator.

1. 1/9 = 0.1111… 2/9 = 0.22222 (you get the picture) 8/9 = 0.8888…

   Convert to the form 0.XXXX : a. 2/90 = b. 7/900 =

2. To simplify fractions composed of fixed decimal numbers, the decimals on the top and bottom can be moved the same number of places – to get an easy division problem or a fraction with a familiar decimal equivalent.

   Example: $\dfrac{0.440}{0.020} = \dfrac{0.440}{0.020} = \dfrac{44.0}{2.0} = \mathbf{22}$ (Move to get 2 in the denominator)

   Convert to either whole numbers or decimal equivalents in the form 0.XXXX :

   a. $\dfrac{0.025}{0.075}$ = ____ =

   b. $\dfrac{5.6}{0.70}$ = ____ =

3. If you see a simple division or fraction that can be gotten by moving decimals different amounts in the numerator and denominator, write the simple numbers, add a power of 10 to each to keep its value the same, and use exponential math to solve.

   Example: $\dfrac{0.0060}{2400} = \dfrac{6 \times 10^{-3}}{24 \times 10^{2}} = 0.25 \times 10^{-5} = 2.5 \times 10^{-6}$

   On separate paper, convert to scientific notation with 2 digits in the significand:

   a. $\dfrac{4000}{0.090}$ b. $\dfrac{0.56}{800}$ c. $\dfrac{60}{1.8 \times 10^{-14}}$

4. Re-write each of the two quantities to scientific notation, multiply or divide, and then convert your final answer to scientific notation with a 2-digit significand.

   a. $\dfrac{200 \times 10^{12}}{0.40 \times 10^{4}}$ = ________________ =

   b. $\dfrac{0.010 \times 10^{3}}{0.50 \times 10^{-7}}$ = ________________ =

   c. $\dfrac{0.10}{0.004}$ = ________________ =

   d. $\dfrac{0.020}{5{,}000}$ = ________________ =

e. $\dfrac{0.048}{0.008}$ = ________________ =

5. Round these value, then *estimate* to an answer with one non-zero digit.

   a. $\dfrac{183}{21 \times 2}$ =

   b. $\dfrac{24}{5 \times 53}$ =

   c. $\dfrac{4 \times 27}{525}$ =

6. Solve these in any way you choose. Convert your final answer to scientific notation with a 2-digit significand.

   a. $\dfrac{0.00062}{0.20}$ =

   b. $(4.5 \times 10^{-4})\,(0.020 \times 10^{7})$ =

   c. $\dfrac{96 \times 10^{-3}}{120 \times 10^{-1}}$ =

   d. $\dfrac{75 \times 10^{16}}{0.025 \times 10^{-4}}$ =

7. Decimal equivalents may also be solved by long division.

$$5/6 = ? \quad = \quad 5 \div 6 = \quad \begin{array}{r} 0.833\ldots \\ 6\,\overline{)\,5.000} \\ \underline{48} \\ 20 \\ \underline{18} \\ 20\ \ldots \end{array} \quad = \quad 0.83\overline{3}$$

   On separate paper, solve by long division. Round to 3 places past the decimal.

   a. 5/12 b. 4/11

Answers:

1a. 0.0222 1b. 0.0078 2a. 25/75 = 1/3 = 0.3333 2b. 56/7 = 8

3a. 4.4×10^{4} 3b. 7.0×10^{-4} 3c. 3.3×10^{15}

4a. 5.0×10^{10} 4b. 2.0×10^{8} 4c. 2.5×10^{1} 4d. 4.0×10^{-6} 4e. 6.0×10^{0}

5a. Between 3 and 6 5b. Close to 0.10 5c. Close to 0.2

6a. 3.1×10^{-3} 6b. 9.0×10^{1} 6c. 8.0×10^{-3} 6d. 3.0×10^{23}

7a. 0.417 7b. 0.364

For Preparatory Chemistry

Activity #6 – *The Mental Math Fluency Quiz*

To the Instructor

A first course in chemistry assumes students have had ten or more years of extensive practice in calculations involving arithmetic, fractions, and algebra (including exponential and logarithmic calculations). But is that assumption accurate for your students?

The fluency quiz in Activity #1 will help to find out. Questions 1-12 all involve math students are assumed to know as preparation for chemistry. The fluency quiz will help to identify what portion of that math may need review prior to when it is needed in Chem I.

In a Prep Chem course, the last two questions should be omitted.

If Questions 1-8 on the fluency quiz identify "less than mastery" of mental arithmetic and exponential operations, you may want to consider Activities #7 to #10 for homework.

For fluency quiz access and follow-up questions, see Activity #1

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Activity #7 – *The Numeracy Tutorial*

To the Instructor

Activity #7 is a 9 page tutorial homework assignment which can printed for students or accessed online. The two lessons cover math facts, standard algorithms, decimal equivalents for fractions, using mental math to simplify fractions – and how to study chemistry. Lecture notes, practice problems, and worked-out answers are included.

The tutorial content can be viewed on PDF pages 5 to 13 at this link: http://www.ChemReview.net/PrepChemFree.pdf (click and check your downloads).

Timing

The tutorial is available for free access at all times. It can be assigned at any point where you begin the "math topics" of chemistry to be completed by the 2nd or 3rd class thereafter.

For Preparatory Chemistry sections, the Numeracy Tutorial could be homework assigned on the first day of class,

Assignment Options

An assignment to students could be

> Numeracy Homework:
>
> Go to this link: www.ChemReview.Net/Chem1Assignment.html
>
> Follow the instructions and complete Chapter 1 Lessons **1.1 and 1.2**.

# # # # #

Activity #8 – *Using Mental Arithmetic to Simplify Fractions*

To the Instructor

Activity #8 is practice with the numeric simplification operations that are useful both during calculations and when estimating to check a calculator answer.

You may want to use the sheets to introduce the topics in class and provide guidance and feedback as students work the problems a few at a time. Some problems can be left to complete as homework, or as a warmup review at the start of the next class.

This activity is recommended before students begin work on calculations involving exponential notation in Activity #10. The operations in this exercise will help with "exponential calculations without a calculator."

# # # # #

Using Mental Arithmetic to Simplify Fractions

WithOUT a calculator, use your mental arithmetic skills to reduce these fractions to a one or two digit whole number. Show your work on this paper. Check your answers at the bottom.

Each problem may have multiple ways to cancel and solve. Any way to a correct answer works.

1. Example: $\dfrac{8 \times 15 \times 3}{72} = \dfrac{\cancel{8} \times 15 \times 3}{\cancel{72}\ \mathbf{9}} = \dfrac{\cancel{8} \times 15 \times \cancel{3}}{\cancel{72}\ \cancel{9}\ \mathbf{3}} = \dfrac{15}{3} = \mathbf{5}$

(When doing multiple cancellations, you may want to re-write at some point, as in the "next to last" step above, to keep your progress clear.)

Hint: It usually helps to try to reduce the larger numbers on both the top and bottom first.

2. $\dfrac{49 \times 2 \times 3}{4 \times 7} =$

3. $\dfrac{42 \times 36 \times 5}{2 \times 6 \times 7} =$

4. $\dfrac{63 \times 4 \times 42}{6 \times 7 \times 9} =$

5. $\dfrac{48 \times 6 \times 11}{4 \times 18} =$

6. $\dfrac{35 \times 2 \times 8}{40 \times 14} =$

7. $\dfrac{72 \times 4 \times 36}{8 \times 9} =$

8. $\dfrac{8 \times 12 \times 7}{2 \times 96} =$

9. $\dfrac{10 \times 18 \times 56}{8 \times 2 \times 30} =$

10. $\dfrac{8 \times 27 \times 56}{7 \times 9 \times 32} =$

11. $\dfrac{28 \times 60}{12 \times 7 \times 2} =$

12. Double these: 42 17 36 45 16 24 32 48

13. Cut these values in half: 44 98 86 38 46 78 56

Answers: 2. 12 3. 50 4. 28 5. 44 6. 1 7. 50 8. 7 9. 21 10. 6
11. 10 12. 84 34 72 90 32 48 64 96 13. 22 49 43 19 23 39 28

Activity #9 -- *Multi-digit Multiplication and Division*

To the Instructor:

The worksheet below reviews the standard algorithms for multiplication and for simple long division. This activity is recommended *if* mental arithmetic gaps were identified in the fluency quiz.

Using the multiplication "standard algorithm" requires practicing recall of multiplication and addition. The division algorithm requires practicing multiplication, division and subtraction fact recall.

The benefits of requiring students to occasionally perform these types of calculations include

- They are an easy way to practice the "overlearning" of fundamentals that science says is necessary to understand explanations of quantitative relationships;
- Problems like these can be put on the board at the start of a class throughout the term for productive student work while you tend to administrative necessities.

The explanation of the two algorithms is the same as that in Lesson 1.2 in the homework tutorials, but the problems have different numbers. This sheet will supply additional practice.

The Common Core math standards require teaching both of these standard algorithms, but until 2014, under "K-12 math standards" in some states, students were not taught long division.

# # # # #

Multi-digit Multiplication and Division

To understand the numeric relationships in science, you need to be able to do arithmetic quickly "in your head." One way to keep mental math skills sharp is to occasionally solve simple multi-digit multiplication and division without a calculator. To do this without overwhelming "working memory" (where you think), you need to apply a "standard algorithm."

1. The multiplication algorithm usually taught in the US includes these steps. For 76 × 42 = ?

Step 1:
$$\begin{array}{r} {}^{\mathbf{1}}76 \\ \times\ \underline{\ 42} \\ 152 \end{array}$$

Steps 2 and 3:
$$\begin{array}{r} {}^{\mathbf{2}}76 \\ \times\ \underline{\ 42} \\ 152 \\ \underline{304\ \ } \\ \mathbf{3192} \end{array}$$

←(putting a **0** after the 4 is an option)

With*out* a calculator, working on this paper, multiply these:

a. $\begin{array}{r} 95 \\ \times\ \underline{16} \end{array}$ b. $\begin{array}{r} 84 \\ \times\ \underline{73} \end{array}$ c. $\begin{array}{r} 39 \\ \times\ \underline{62} \end{array}$ d. $\begin{array}{r} 57 \\ \times\ \underline{48} \end{array}$

2. The "long division" algorithm usually taught in US education includes these steps:

For 2048 ÷ 8 =
$$\begin{array}{r} \underline{2\ \ \ \ \ } \\ 8\,)\,2\,0\,4\,8 \\ \underline{1\,6\ \ \ \ } \\ 4\,4\ \ \end{array} \qquad \begin{array}{r} \underline{2\,5\,6} \\ 8\,)\,2\,0\,4\,8 \\ \underline{1\,6\ \ \ \ } \\ 4\,4\ \ \\ \underline{4\,0\ \ } \\ 4\,8 \\ \underline{4\,8} \end{array}$$
= **256**

With*out* a calculator, try these "evenly divisible" cases (the answer will be a multi-digit whole number -- no decimals or remainders).

a. $6\,\overline{)\,5\,1\,6}$ b. $9\,\overline{)\,2\,1\,8\,7}$ c. $8\,\overline{)\,5\,5\,6\,0}$

Activity #10 – *The Exponential Notation Tutorial*

To the Instructor

Activity #10 is a 11 page tutorial homework assignment which can printed for students or accessed online. The three lessons cover exponential notation, converting to scientific notation, and calculations that include powers of 10.

The tutorial content can be viewed on PDF pages 13 to 25 at this link: http://www.ChemReview.net/PrepChemFree.pdf (click and check your downloads).

Timing

The Exponential Notation tutorial is available for free access at all times. It can be assigned at any time after the Numeracy Tutorial (Activity #7) is completed.

Assignment

An assignment to students could be

> Exponential Notation Homework:
>
> Go to this link: www.ChemReview.Net/Chem1Assignment.html
>
> Follow the instructions and complete Chapter 1 Lessons **1.3, 1.4, 1.5**, and the **Review Quiz**.
>
> Be ready for a quiz on the content on (date).

Quizzes

Quizzes are available to instructors on the content of Chapter One (the Numeracy and Exponential Notation tutorials).

To request the quizzes, click here.

# # # # #

For Instructors

Activity #11 – *Other Mental Math Resources*

To the Instructor

The tutorials are taken from the first two chapters of the supplementary textbook *Calculations in Chemistry - An Introduction*, available from W. W. Norton. The publisher has granted the authors of the text permission to post the first two chapters for free use by students and instructors at all times during the school year.

All instructors may request a free examination copy of all 24 chapters of the paperback or eBook. To receive a copy, click here. (it can be slow to load).

The text includes the following lessons which review math both with and without a calculator:

Chapter 3	Significant Figures
Chapter 4	Dimensional Analysis
Chapter 5	Word Problem Strategies
Lesson 12.1	Solving for Ratio Units
Lessons 12.3	Fraction and Percentage Calculations
Lesson 16.3	Cancellation of Complex Units
Lesson 17.1	Choosing Consistent Units
Lesson 17.4	Choosing the Right Equation
Lesson 21.1	Powers and Roots of Exponential Notation
Lesson 22.1	Acid-Base Math -- Review
Lesson 23.1	Base 10 Logarithms
Lesson 24.3	Natural Logarithms

These lessons may provide ideas for math review lessons if you prefer to write your own.

Lessons in *Calculations* may be assigned at points in General Chemistry for review of high school math and chem fundamentals or as homework for topics throughout Prep Chem.

# # # # #

Activity #12 – *How the Brain Solves Problems*

To the Instructor

This section is a summary of the scientific research on

The New Science of Learning

With the arrival of inexpensive calculators in about 1975, curriculum writers began to assume that memorizing math facts to use in calculations would no longer be needed. The well-intended hope was to spare students the hard work of memorizing math facts.

Similarly, after the arrival of the internet in 1995, memorization of other facts and procedures was assumed to be less necessary for problem solving.

The assumption of these reforms was that the brain, during reasoning, could apply both memorized and non-memorized information with equal facility. Unfortunately, this assumption turned out to be mistaken.

Working Memory

What cognitive studies between 2000 and 2010 discovered and confirmed was that the "working memory" where the brain solves problems can use all information that can quickly be recalled from long-term memory (LTM), but very limited information that has not been well-memorized.

Award-winning cognitive scientist Susan Gathercole writes that for information that is not-well-memorized,

> "the capacity of working memory [where the brain reasons] is limited, and the imposition of either excess storage or processing demands in the course of an on-going cognitive activity will lead to catastrophic loss of information from this temporary memory system."

Which means fundamentals that are used often in problem solving, such as math facts – and symbols for elements and ion formulas -- must be well memorized.

The Importance of Fluency

How can we help students learn to solve scientific calculations? Here's what the experts say instructors need to know.

In a U.S. Presidential Commission report, five of the nation's leading cognitive experts wrote:

> "[T]here are several ways to improve the functional capacity of working memory. The most central of these is the achievement of automaticity, that is, the fast, implicit, and automatic retrieval of a fact or a procedure from long-term memory….
>
> [T]o obtain the maximal benefits of automaticity in support of complex problem solving, arithmetic facts and fundamental algorithms should be thoroughly mastered, and indeed, over-learned, rather than merely learned to a moderate degree of proficiency (Geary et al. 2008).

“Overlearning” means practicing recall *beyond* mastery: using “retrieval” strategies such as flashcards, mnemonics, or sequence recitation (methane, ethane …) to the point of mastery, and doing so for several days in a week and occasionally thereafter (Willingham 2004).

Why Makes Math and Science Different?

To learn math and science, why is such intense study required? For many types of learning, summary (“gist”) memory is sufficient, and a species brain built for speech comprehension is good at summarizing meaning (Pinker 1994). But the relationships of math and science often require exact (verbatim) memory (6 times 7 is not “about 40” and phosphate ion isn’t “PO something”), which the brain finds more difficult to store (Geary et al. 2008).

What Else Is Needed?

Initial memorization of fundamental relationships, though necessary, is *not* sufficient. Elements of knowledge are stored in the neurons of long-term memory. Neural links among those neurons on a variety of characteristics must be constructed and weighted by solving problems in different contexts, so that elements can be fluently recalled when appropriate (Anderson et al. 2004, Willingham 2008). Demonstrations, discussions, guided inquiry, and other forms of “active learning” can provide visual, spatial, auditory, and sequential associations for new knowledge.

However, neural connections cannot form until *after* the elements being connected are stored in and recallable from LTM.

Does the Science Work in Practice?

Studies by chemistry educators have found if you teach the mental math as a part of chemistry, grades go up markedly -- consistent with what cognitive science predicts (Craig 2017).

Math Gaps for the Current Generation

In many states prior to 2014, before the importance of automaticity became a scientific consensus, math curricula adopted under state K-12 standards encouraged calculator use in elementary grades and discouraged memorization. As a result, many students may have gaps in their preparation for calculations in the sciences (Hartman and Nelson 2016).

To Summarize

According to science, to learn both math and science efficiently and effectively, students must *begin* each topic by “automating” (via “overlearning”) their recall of the fundamental relationships for the topic. This new stored information must then be applied to solving problems in a variety of distinctive contexts.

Resources

Additional articles on how instructors can help students learn science and prepare for STEM majors include

1. *“Do I Need to Memorize That?” or Cognitive Science for Chemists* (2015), posted at www.ChemReview.Net/CogSciForChemists.pdf .
2. *Addressing Math Deficits With Cognitive Science* (2017) at https://confchem.ccce.divched.org/content/2017fallconfchemp8

References for the summary above, in order cited.

Geary, D., et al.: Chapter 4 - Report of the Task Group on Learning Processes, pp. 4-2 to 4-11. (2008) https://www2.ed.gov/about/bdscomm/list/mathpanel/report/learning-processes.pdf

Willingham, D.: Practice makes perfect—but only if you practice beyond the point of perfection. *Am. Educator* 28(1), 31-33 (2004) at: http://www.aft.org/newspubs/periodicals/ae/spring2004/willingham.cfm

Pinker, S.: The Language Instinct: How the Mind Creates Language. Wm. Morrow, New York (1994)

Gathercole, S. E., Lamont, E., & Alloway, T. P. (2006). Working memory in the classroom. *Working memory and education*, 219-240.

Anderson, J. R., Bothell, D., Byrne, M. D., Douglass, S., Lebiere, C., Qin, Y.: An integrated theory of the mind. *Psychological Review* 111(4) 1036 (2004)

Willingham, D.: What will improve a student's memory? *Am. Educ*. 32(4), 17-25 (2008) at http://www.aft.org/pdfs/americaneducator/winter0809/willingham.pdf

Craig, P.: Building Student Confidence with Chemistry Computation, *Journal of Chemical Education* **2018** *95* (8), 1434-1435 https://confchem.ccce.divched.org/sites/confchem.ccce.divched.org/files/2017FallConfChemP5_0.pdf (see Figure 1)

Hartman J., Nelson E.: .*Automaticity in Computation and Student Success in Introductory Physical Science Courses* (2016) at http://arxiv.org/abs/1608.05006

Feedback

These materials are from www.ChemReview.Net/MentalMath.PDF .

Check back on occasion for updates. Version dates are noted at the top of the file.

Feedback on this material is appreciated! ChemReviewTeam@ChemReview.Net .

(End of packet)