

## **Frequency and Wavelength Calculations: Self-Study Assignment**

You will have a QUIZ on the attached pages on \_\_\_\_\_ .

Your assignment is: READ the pages attached. WORK the examples in the lesson.  
Complete the pages as homework.

To work the examples,

- use a sheet of paper to cover below the \*\*\*\*\* line,
- try the problem on your paper,
- then check your answer below the \*\*\*\*\* line.

Start early. This assignment will require 2-4 hours of work outside of class.

# Calculations In Chemistry

Modules 19 and above have been re-numbered.

Module 22 on Light and Spectra is now Module 23 in this packet

Module 23 on Electron Configuration is now Module 24 in this packet

Module 24 on Bonding is now in Module 25

If you are looking for Bonding topics, check Module 25

At [www.ChemReview.Net](http://www.ChemReview.Net)



## Modules 23 and 24 Light, Spectra, and Electron Configuration



<b>Module 23 - Light and Spectra</b> .....	<b>619</b>
Lesson 23A: Waves .....	619
Lesson 23B: Planck's Constant .....	628
Lesson 23C: DeBroglie's Wavelength .....	632
Lesson 23D: The Hydrogen Atom Spectrum.....	637
Lesson 23E: Quantum Mechanics .....	643
<b>Module 24 - Electron Configuration</b> .....	<b>648</b>
Lesson 24A: The Multi-Electron Atom.....	648
Lesson 24B: Abbreviated Electron Configurations .....	652
Lesson 24C: The Periodic Table and Electron Configuration .....	658
Lesson 24D: Electron Configurations: Exceptions and Ions .....	662

For additional modules, visit [www.ChemReview.Net](http://www.ChemReview.Net)

## Module 23 — Light and Spectra

**Timing:** Begin this module when *wavelength* and *frequency* calculations are assigned.

**Pretests:** If you believe that you know the material in a lesson, try two problems at the end of the lesson. If you can do those calculations, you may skip this lesson.

\* \* \* \* \*

### Lesson 23A: Waves

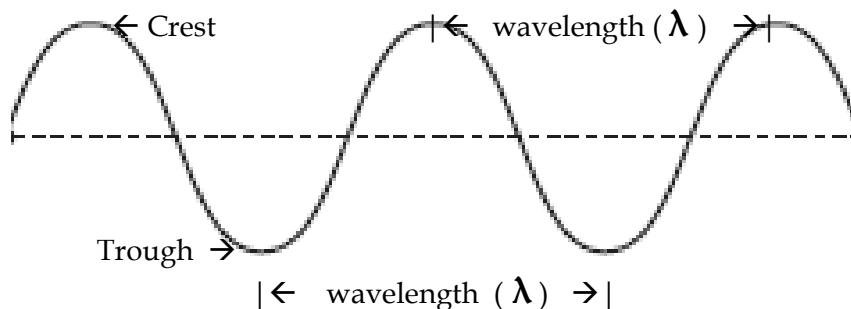
#### Waves and Chemistry

**Electromagnetic energy** includes gamma rays, x-rays, ultraviolet, visible, and infrared light, microwaves, and radio waves. Each of these types of energy occupies a different region of the **electromagnetic spectrum**.

Chemical particles can both absorb and release electromagnetic energy. This absorption and release of energy can be a powerful tool in identifying chemical particles. Exposure to certain types of electromagnetic energy can also cause chemical particles to change and react.

In some cases, the behavior of electromagnetic energy is best predicted by assuming that the energy is a particle, but in other cases, energy is best understood as a wave. Let us begin by investigating the properties of waves.

#### Wave Terminology



The following are some of the components of a wave that are important in chemistry.

1. **Wavelength** is the distance between the crests of a wave, which is equal to the distance between the troughs of a wave.
  - a. The symbol for wavelength is  $\lambda$  (the lower-case Greek letter *lambda*).
  - b. Since wavelength is a distance, the units of wavelength are distance units: meters, centimeters, nanometers.

2. **Frequency** is the number of times a wave crest passes a fixed point, per second.
- The symbol for frequency is  $\nu$  (the lower-case Greek letter *nu*).
  - The units of any frequency are events per unit of time. Though wave frequency is often expressed as “cycles per second,” wave cycles are the *entity* being measured, and 1/seconds is the *unit*. When writing wave units, the term “wave cycle” or ‘cycle’ is often included as a label in conversion calculations, but is usually omitted as understood in equation calculations. The *unit* of frequency, which must be included in both conversion and equation calculations, is always *1/time*.

The SI unit for frequency is **1/seconds** ( $s^{-1}$ ). The unit  $second^{-1}$  is also called a **hertz** (Hz). In calculations, you should either write *hertz* as  $s^{-1}$  or use the rule that *hertz* and  $s^{-1}$  are equivalent and can cancel.

3. The **speed** of a wave is equal to its **frequency** times its **wavelength**.

$$\boxed{\text{wave speed} = \lambda \nu} = (\text{lambda})(\text{nu}).$$

Memorize the equation for *wave speed* in words, symbols, and names for the symbols.

### Wave Calculations

Because wave relationships are often defined by multi-term equations, wave calculations are generally solved using equations rather than conversions. Solve the problem below using the *equation* method (for method review, see Lessons 17D or 21B).

- Q.** If ocean waves are traveling at 5.00 miles/hour and the crests pass a fixed point at a rate of 16.5 waves per minute, what is the wavelength, in feet? (1 mile = 5,280 feet)

\* \* \* \* \* (When you see \* \* \* \* \*, cover below, solve, and then check below.)

Write the one equation learned so far for waves.

$$\boxed{\text{Wave speed} = \lambda \nu}$$

List those three terms in a data table. After each term, write the data in the problem that corresponds to the term. Add a ? and the desired *unit* after the WANTED symbol.

\* \* \* \* \*

Wave speed = 5.00 miles/hr.

$\lambda$  = ? feet

$\nu$  = 16.5 wave cycles/min. = 16.5 min.<sup>-1</sup> (frequency units are 1/time)

When solving wave frequency calculations using equations, “wave cycles” is usually *omitted* as understood to be the object being measured.

To use an equation, the DATA must be converted to *consistent* units. In this data, the units are not consistent: distance is in both miles and feet, time is in hours and minutes.

If an equation does not include constants, the best units to convert the DATA to are the units of the answer (in this problem, *feet*). Since hours and minutes are not in the answer unit, *choose* one to convert the time DATA to. The following answer converts the time DATA to minutes. If needed, complete that conversion and finish the problem.

\* \* \* \* \*

(Need help with this type of conversion? See Lesson 4E.)

In the DATA table, convert all of the supplied data to the chosen consistent units.

$$\text{Wave speed} = ? \frac{\text{feet}}{\text{min.}} = \frac{5.00 \text{ miles}}{\text{hour}} \cdot \frac{1 \text{ hour}}{60 \text{ min.}} \cdot \frac{5,280 \text{ feet}}{1 \text{ mile}} = 440. \frac{\text{feet}}{\text{min.}}$$

Once the DATA is in consistent units, the units will cancel properly in the equation. Now solve the equation in symbols for the WANTED symbol, then substitute the DATA. Include the consistent units and check the unit cancellation.

\* \* \* \* \*

SOLVE:

$$\lambda \text{ (in feet)} = \frac{\text{speed}}{\nu} = \text{speed} \cdot \frac{1}{\nu} = 440. \frac{\text{feet}}{\text{min.}} \cdot \frac{1}{16.5 \text{ min.}^{-1}} = 26.7 \text{ feet}$$

Note in the unit cancellation in the denominator:

$$\text{min.} \cdot \text{min.}^{-1} = \text{min.}^1 \cdot \text{min.}^{-1} = \text{min.}^0 = 1. \text{ Anything to the zero power equals one.}$$

Wave calculations can be done in any unit system, but the above problem was complicated by the use of English and non-consistent units. We will simplify the *electromagnetic* wave calculations of chemistry by converting all DATA to the units used in the constants (most often SI units) that apply to waves of energy.

**Practice A:** Use the equation method to solve this problem. Check your answers at the end of this *lesson*.

1. Street lights containing sodium vapor lamps emit an intense yellow light at two close wavelengths. The more intense wave has a wavelength of  $589 \times 10^{-9}$  meters. If light travels at the speed of  $3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ , what is the frequency of this intense yellow wave?

## Electromagnetic Waves

The movement of electric charge creates electromagnetic waves. The waves **propagate**: they move outward from the moved charge. The energy that was added to move the charge is carried outward by the waves.

In a vacuum, all electromagnetic waves travel at the **speed of light**:  $3.00 \times 10^8$  meters/second. The speed of light is the “speed limit of the universe:” the fastest speed possible for energy or matter. In wave calculations, this important quantity is given the symbol *c*.

Electromagnetic waves slow when they travel through a medium that is denser than a vacuum, but when passing through air or other gases at normal atmospheric pressures, the speed of light does not slow sufficiently to affect most calculations in chemistry.

For *electromagnetic* waves, this relationship will be true (and must be memorized):

$$c = \lambda \nu = 3.00 \times 10^8 \text{ m/s} \quad \text{in vacuum or air}$$

Since  $c$  is a constant,  $\nu$  and  $\lambda$  are inversely proportional. As wavelength goes up, frequency must go down. If  $\nu$  goes up,  $\lambda$  must go down.

Further, as long as we work in consistent units and in air or vacuum, since  $c$  is constant, a specific value for the frequency of an electromagnetic wave will always correlate to a specific value for wavelength.

### The Regions of the Electromagnetic Spectrum

The electromagnetic spectrum goes from very high to very low wavelengths and frequencies. Regions of the spectrum are assigned different names that help in predicting the types of interactions that the energy will display. However, all of these forms of energy are electromagnetic waves. The difference among the divisions of the spectrum is the length (or corresponding frequency) of the waves.

The following table (no need to memorize) summarizes some of the general divisions of the electromagnetic spectrum.

Frequency ( $s^{-1}$ )	Wavelength (m)	Type of Electromagnetic Wave
$10^{24}$	$3 \times 10^{-16}$	Gamma Rays
$10^{21}$	$3 \times 10^{-13}$	
$10^{18}$	$3 \times 10^{-10}$	X-rays
$10^{15}$	$3 \times 10^{-7}$	Ultraviolet, Visible, Infrared Light
$10^{12}$	$3 \times 10^{-4}$	Microwaves
$10^9$	$3 \times 10^{-1}$	UHF Television Waves
$10^6$	300	Radio Waves

### Units For Frequency and Wavelength

Measurements of wavelengths and frequencies often involve very large and very small numbers. Values are often expressed using SI *prefixes* such as *gigahertz* (GHz) or *nanometers* (nm). Prefixes needed most often are those for powers of *three*.

#### Engineering Notation

*Scientific* notation writes numbers as a significant between 1 and 10 times a power of 10.

**Engineering notation** writes numbers as a significant between 1 and 1,000 times a power of 10 that is divisible by 3. In wave calculations, answers are often preferred in engineering rather than scientific notation to ease conversion to the metric prefixes based on powers of three.

Prefix	Symbol	Means
tera	T	$\times 10^{12}$
giga-	G	$\times 10^9$
mega-	M	$\times 10^6$
kilo-	k	$\times 10^3$
milli-	m	$\times 10^{-3}$
micro-	$\mu$	$\times 10^{-6}$
nano-	n	$\times 10^{-9}$
pico-	p	$\times 10^{-12}$
femto-	f	$\times 10^{-15}$

Examples: Converting scientific to engineering notation,

$$5.35 \times 10^{-4} \text{ m} = 535 \times 10^{-6} \text{ m in engineering notation ( = 535 micrometers = 535 } \mu\text{m)}$$

$$9.23 \times 10^{10} \text{ Hz} = 92.3 \times 10^9 \text{ Hz in engineering notation ( = 92.3 GHz )}$$

To convert *any* exponential notation to engineering notation, adjust the exponent and decimal position until the exponent is divisible by 3 and the significand is between 1 and 1,000.

(To review moving the decimal, see Lesson 1A). Try this example.

**Q.** Convert to engineering notation, then to metric-prefix notation:  $5.27 \times 10^{-11} \text{ m}$

\* \* \* \* \*

**A.**  $5.27 \times 10^{-11} \text{ m} = 52.7 \times 10^{-12} \text{ m}$  in engineering notation = 52.7 picometers *or* 52.7 pm

$52.7 \times 10^{-12}$  is the *only* way to write the given quantity that results in both an exponent divisible by 3 and a significand between 1 and 1,000.

Engineering notation, like scientific notation, results in one unique expression for each numeric value, and this makes answers easy to compare and check.

During calculations, work in general exponential notation. At the end, convert your answers to either scientific or engineering notation, depending on the system preferred for wave calculations in your course.

**Practice B:** Do every other question. Complete the rest during your next study session.

1. By inspection, convert these to units without prefixes and engineering notation.
 

a. 5.4 GHz	b. 720 nm	c. 96.3 MHz
------------	-----------	-------------
2. Convert these first to engineering notation, then to measurements that use metric prefixes in place of the exponential terms.
 

a. $47 \times 10^{-7} \text{ m}$	b. $347 \times 10^4 \text{ Hz}$	c. $1.92 \times 10^{-8} \text{ m}$
d. $14,920 \times 10^{-1} \text{ Hz}$	e. $0.25 \times 10^{11} \text{ Hz}$	f. 7,320 m

### Wave Calculations and the Speed of Light

To solve equations, the data must be in *consistent* units. The following rules will help in choosing and converting to consistent units.

1. If the equation has *constants*, write the constant's symbol, value and units in the DATA table *before* listing the symbols and data for the *variables* in the equation.
2. In the DATA table, write each variable in the equation *and* its chosen *consistent* unit. To choose the consistent units to write after each variable symbol, apply these steps.
  - a. If the equation has *constants*, choose as variable units the units used in the constants.

Example: If the constant  $c$  ( $3.00 \times 10^8 \text{ m/s}$ ) is in the equation needed to solve, label the symbols that measure distance as " in  $\text{m}$  = " and frequency as " in  $\text{s}^{-1}$  "

- b. If there are no constants are in the equation, label each variable with appropriate units matching the units used in the *WANTED* unit.
3. If a *WANTED* unit is not specified, pick a *WANTED* unit to match the units used in the constants of the equation. If no constants are used, write after the *WANTED* unit the *SI unit* for dimension that is *WANTED*.
4. In the *DATA*, after each symbol and its consistent units, add an = sign, write the data supplied in the problem, then *convert* that *DATA* to the consistent units if needed.
5. First solve for the *WANTED* symbol in the *consistent* unit, then convert to the specified *WANTED* unit if needed.

[Note: This method, solving in the units of the *constants*, is arbitrary. Your course may ask that you solve all problems in SI units. In most cases, constants will be stated in SI units, so both methods will solve in SI units. However, “solving in the units of the constants” will save a few steps if data is provided in kcals, electron volts, BTUs, or other non-SI units, as may be the case in some calculations.]

The problem below will help you to understand and remember the rules above.

**Q.** When neon gas at low pressure is subjected to high voltage electricity, it emits waves of light. One of the more intense waves in the visible spectrum has a frequency of  $4.69 \times 10^{14}$  Hz, perceived by the eye as red light. What is the wavelength of this light in nanometers?

Solve using the steps above.

★ ★ ★ ★ ★

### Answer

This problem involves a frequency ( $\nu$ ) and a wavelength ( $\lambda$ ) for light. We know that light travels at the speed of light ( $c$ ), a constant. So far, we know only one equation that relates those three symbols. So, to start, your paper should look like this:

$c = \lambda \nu$
DATA: $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ (list <i>constants</i> and their values first)
$\lambda$
$\nu$

For the speed of light ( $c$ ), in *conversion* calculations the unit m/s must be used as a ratio and written in the top/bottom format, but in *equations*, it will simplify unit cancellation if the units are written in the “on one line” format:  $3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ .

Now we want to add a *consistent* unit after each variable symbol. Since this equation has a constant ( $c$ ), after each variable symbol, write a unit that both measures the variable *and* matches one of the units used in the constant.

Do that step, then check below.

★ ★ ★ ★ ★

- $\lambda$  is measured in distance units, so after  $\lambda$  write “in meters = ” since meters is the distance unit used in the constant  $c$ .
- $\nu$  is measured in 1/time units, and  $c$  has time measured in seconds, so after  $\nu$  write “in  $s^{-1}$  = ”

After the = sign for each variable, write the data for that variable that is supplied in the problem. Then, in the DATA table,

- convert the supplied units to the consistent units if needed.
- Convert metric *prefixes* to consistent *base* units *without* prefixes, such as **m** and **s<sup>-1</sup>**, using conversions or by inspection.
- For the WANTED variable, after the = sign write “? WANTED”

Do those steps, and then check your answer below.

\* \* \* \* \*

$c = \lambda \nu$
DATA: $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$ (list <i>constants</i> and their values first)
$\lambda$ in <b>m</b> = ? , then convert to <b>nm</b> WANTED.
$\nu$ in <b>s<sup>-1</sup></b> = $4.69 \times 10^{14} \text{ Hz s}^{-1}$

To solve for the WANTED symbol:

- First solve the equation for the WANTED symbol in symbols.
- Substitute the DATA into the solved equation using the *consistent* units, and solve including the units.
- If needed, convert to the final unit WANTED.

Modify your work if needed and complete the problem.

\* \* \* \* \*

$$\text{SOLVE: } ? = \lambda \text{ in m} = \frac{c}{\nu} = \frac{3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{4.69 \times 10^{14} \text{ s}^{-1}} = 6.40 \times 10^{-7} \text{ m}$$

That solves in the *consistent* unit. To finish, convert to the WANTED unit.

\* \* \* \* \*

$$? = \lambda \text{ in nm} = 6.40 \times 10^{-7} \text{ m} \cdot \frac{1 \text{ nm}}{10^{-9} \text{ m}} = 6.40 \times 10^2 \text{ nm} = 640. \text{ nm}$$

\* \* \* \* \*

Using your answer to the above question, try

**Q2.** How many wavelengths of the red neon light above would it take to equal one centimeter? (one wave cycle = 1 wave = 1 wavelength)

\* \* \* \* \*

If you are not sure how to proceed, list the data, try to assign symbols, and see if the symbols fit a known equation.

\* \* \* \* \*

Here, the wanted unit is waves/cm, which is the inverse of wavelength, not wavelength. Plus, none of the data has a frequency, so the data does not match the one equation we know so far.

Note that the data includes an equality, and that all of the data can be listed as equalities or ratios. That's a hint that you should try conversions to solve.

\* \* \* \* \*

WANT: ?  $\frac{\text{waves}}{\text{cm}}$  (you want the waves *per one* cm, a ratio unit)

DATA: one wave = 1 wavelength  
 one wavelength =  $6.40 \times 10^{-7}$  meters (given a choice, pick SI base units)

Though "waves" or "wave cycles" is usually left out of wave *equation* calculations as understood, including "waves" may help when using conversions. If needed, adjust your work and then finish the problem.

\* \* \* \* \*

SOLVE: ?  $\frac{\text{waves}}{\text{cm}} = \frac{1 \text{ wave}}{\text{wavelength}} \cdot \frac{1 \text{ wavelength}}{6.40 \times 10^{-7} \text{ m}} \cdot \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 15,600 \frac{\text{waves}}{\text{cm}}$

How many waves of red light fit into a centimeter? Quite a few.

### **Summary:** Frequency and Wavelength

- Wavelength:** the distance between the crests of a wave. The symbol is  $\lambda$  (*lambda*).

The units are distance units: either the base unit meters, or nanometers, etc.

- Frequency:** the number of times a wave crest passes a fixed point, per second.

The symbol is  $\nu$  (*nu*). The units are events per unit of time ( $1/\text{time}$ ).

Frequency *units* = wave cycles per second =  $1/\text{seconds} = \text{s}^{-1} = \text{hertz (Hz)}$ .

In calculations, write *hertz* as  $\text{s}^{-1}$  so that units will cancel properly.

- The **speed** of a wave is equal to its *frequency* times its *wavelength*.

$$\boxed{\text{Wave speed} = \lambda \nu} = (\text{lambda})(\text{nu}).$$

- Electromagnetic waves travel at the **speed of light** (symbol  $c$ ).

For all *electromagnetic* waves,  $\boxed{c = \lambda \nu = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}}$  in vacuum or air.

- To simplify solving wave calculations using equations,

- In the DATA table,

- list the constants first.
- Convert the DATA to consistent units: those the *constant* of the equation if there is one (for  $c$ ,  $m$  and  $s$ ), or those used in the *answer* unit, or an SI unit.

- First solve in the *consistent* unit, then convert to the WANTED unit if needed.

---

**Practice C:** (Additional practice with  $\lambda$  and  $\nu$  will be provided in lessons that follow.)

- What are the SI units for
    - Wavelength
    - Frequency
    - Energy
    - Speed
  - If an AM radio station broadcasts a signal with a wavelength of 390 meters, what is the frequency of the signal on a radio tuner, in kHz?
  - If there are 225 waves per centimeter, what is the wavelength of the waves in meters?
- 

**ANSWERS** (Hertz and  $s^{-1}$  are equivalent and either may be used.)

**Practice A**

1.  $\boxed{\text{Wave speed} = \lambda \nu}$

$$\text{Speed} = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$$

$$\lambda = 589 \times 10^{-9} \text{ m}$$

$$\nu = ?$$

$$? = \nu = \frac{\text{speed}}{\lambda} = \frac{3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{589 \times 10^{-9} \text{ m}} = 0.00509 \times 10^{+17} \text{ s}^{-1} = 5.09 \times 10^{14} \text{ s}^{-1}$$

**Practice B**

1a.  $5.4 \text{ GHz} = 5.4 \times 10^9 \text{ Hz}$  or  $\text{s}^{-1}$     b.  $720 \text{ nm} = 720 \times 10^{-9} \text{ m}$     c.  $96.3 \text{ MHz} = 96.3 \times 10^6 \text{ s}^{-1}$

2a.  $47 \times 10^{-7} \text{ m} = 4.7 \times 10^{-6} \text{ m} = 4.7 \mu\text{m}$     (if exponent is made larger, make significant smaller)

2b.  $347 \times 10^4 \text{ Hz} = 3.47 \times 10^6 \text{ Hz} = 3.47 \text{ MHz}$     2c.  $1.92 \times 10^{-8} \text{ m} = 19.2 \times 10^{-9} \text{ m} = 19.2 \text{ nm}$

2d.  $14,920 \times 10^{-1} \text{ Hz} = 1.492 \times 10^3 \text{ Hz} = 1.492 \text{ kHz}$     (significant must be between 1 and 1,000)

e.  $0.25 \times 10^{11} \text{ Hz} = 25 \times 10^9 \text{ Hz} = 25 \text{ GHz}$     f.  $7,320 \text{ m} = 7.32 \times 10^3 \text{ m} = 7.32 \text{ km}$

**Practice C**

- Wavelength is a distance, and the SI unit for distance is the **meter** (m).
  - Frequency is defined as 1/time, the SI time unit is the second, so the unit of  $\nu$  is  $\text{s}^{-1}$ .
  - Energy The SI unit for energy is the **joule** (J).
  - Speed is defined as distance over time, so the SI units are meters/second ( $\text{m} \cdot \text{s}^{-1}$ )
- (The data is a  $\lambda$  and wanted is a  $\nu$ . The equation that relates those two variables is:)

$$\boxed{c = \lambda \nu}$$

DATA:

$$c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1} \quad (\text{list constants used in the equation first})$$

$$\lambda \text{ in m} = 390 \text{ m}$$

$$\nu \text{ in s}^{-1} = ? \text{ , then convert to kHz}$$

$$1 \text{ kHz} = 10^3 \text{ Hz} = 10^3 \text{ s}^{-1} \quad (\text{listing metric conversions is optional})$$

$$? = \nu \text{ in s}^{-1} \text{ , then kHz} = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{390 \text{ m}} = 0.77 \times 10^6 \text{ s}^{-1} \cdot \frac{1 \text{ kHz}}{10^3 \text{ s}^{-1}} = 770 \text{ kHz}$$

3. (If you are not sure how to proceed, list the data, assign symbols, then see if the symbols fit a known equation.)

WANTED:  $\lambda$  in m = ? (or meters/wave)

DATA: 225 waves = 1 cm

(If no equation seems to fit the DATA, try conversions to solve. If needed, use that hint and finish.)

\* \* \* \* \*

If the WANTED unit is re-written as **meters/wave**, a ratio is wanted, and a ratio is in the data to start from. Arrange the *given* so that one unit is where is WANTED (Lesson 11B), but any order for these two conversions works.)

$$\text{SOLVE: } ? \frac{\text{meters}}{\text{wave}} = \frac{1 \text{ cm}}{225 \text{ waves}} \cdot \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 4.44 \times 10^{-5} \frac{\text{m}}{\text{wave}}$$

The wavelength is  $4.44 \times 10^{-5} \text{ m}$ .

\* \* \* \* \*

## Lesson 23B: Planck's Constant

### Energy and Frequency

In 1900, the German physicist Max Planck, studying the *black-body radiation* emitted by objects at high temperature, discovered that *energy* is absorbed by or emitted from atoms in *bundles* of a small but constant size, or in multiples of that constant size.

Planck's discovery in equation form is written as

$$\Delta E_{\text{atom}} = \Delta n \cdot h \cdot \nu$$

where  $n$  is an integer, and

$h$  is a number with units called **Planck's constant**.

$$h = 6.63 \times 10^{-34} \text{ joule} \cdot \text{second} = \text{Planck's constant}$$

Building on Planck's work, in 1905 Albert Einstein proposed an explanation for the *photoelectric effect*: the observation that when light shines on a metal, the metal emits electrons.

Einstein postulated that light can be considered to be made of small *particles* which he called photons, and that the energy of the photons is correlated to their frequency as light. These energy bundles he called **quanta**. A single bundle is a **quantum**. In Einstein's formulation, electromagnetic energy has characteristics of *both* a wave and a particle.

The general form of Planck's equation that relates *frequency* and electromagnetic *energy* is

$$E_{\text{photon}} = h \nu \quad \text{where} \quad h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$$

Planck's constant is small but positive. This means that as the energy of a wave increases, its frequency increases. Higher *frequency* waves have higher *energy*.

Since calculations using Planck's constant involve electromagnetic waves, we can

- use our previous equation for the speed of those waves,  $c = \lambda \nu$ ,
- solve that equation for  $\nu$ :  $\nu = c / \lambda$ , then
- write the photon energy equation as  $E = h \nu$  or, substituting for  $\nu$ ,  $E = \frac{h \cdot c}{\lambda}$

These two general forms of **Planck's equation** are equivalent. The first solves for energy in terms of frequency, the second in terms of wavelength. The first should be memorized, and the second either memorized ("for wavelength, the two *constants* are on top") or derived as needed.

Together, these equations mean that for electromagnetic waves, the three variables energy, frequency, and wavelength are directly correlated: If you know any *one*, you can calculate *both* of the other two.

Further, it will always be true that as photon energies go *up*, the corresponding frequencies go *up* and wavelengths go *down*. Energy waves with high energy have high frequency and short wavelength.

### Calculations Using Planck's Equation

Planck's-equation calculations use the same rules as other equations.

- In the DATA table, list the constants first.
- List the WANTED symbols *with consistent* units: those the *constant* of the equation if there is one (for **h**, units are J and s), or those used in the *answer* unit, or *pick* an SI unit.
- In the DATA table, convert DATA to the consistent units.
- First solve in the consistent unit, then convert to the WANTED unit if needed.

Let's apply the method to a problem.

- Q.** Cosmic rays are high-energy radiation that enters the earth's atmosphere from space. The energy of a single cosmic ray photon can be as high as 50. joules. What would be the frequency of this radiation?

Try the problem, then check below.

★ ★ ★ ★ ★

Answer

To decide *which* equation is needed to solve a problem, try this method: Begin by reading the problem and writing as you go the *symbol* for each item of DATA you encounter, plus the WANTED symbol.

For long problems, you will probably want to list the data methodically and then add symbols before choosing the equations to use, but for short problems, simply listing the symbols as you read the first time will often quickly identify the equation you need.

Try that technique on this problem.

\* \* \* \* \*

50 J = E, ? =  $\nu$ . The fundamental equation that relates E and  $\nu$  is ?

\* \* \* \* \*

$E = h \nu$  Write a data table and solve.

\* \* \* \* \*

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  (list constants first, use their units)

E in J = 50 J

$\nu$  in  $\text{s}^{-1}$  = ?

$$? = \nu \text{ (in } \text{s}^{-1}\text{)} = \frac{E}{h} = \frac{50. \text{ J}}{6.63 \times 10^{-34} \text{ J} \cdot \text{s}} = 7.5 \times 10^{34} \text{ s}^{-1}$$

Photons with this is extremely high energy and frequency are produced by nuclear processes in stars. Let's try a problem with a more commonly encountered energy.

**Q2.** A microwave oven warms food by producing radiation with a typical wavelength of about 12 cm. What is the energy of this wave?

\* \* \* \* \*

Answer

The problem involves  $\lambda$  and E. The equation that uses  $\lambda$  and E is ?

\* \* \* \* \*

$$E = \frac{h \cdot c}{\lambda}$$

DATA:  $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$

$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  (list the *two* constants first, use their units)

E in J = ? (h uses joules)

$\lambda$  in m =  $12 \text{ cm} \cdot \frac{10^{-2} \text{ m}}{1 \text{ cm}} = 0.12 \text{ m}$  (c uses meters)

Adjust your work if needed, and then complete the problem.

\* \* \* \* \*

$$E = \frac{h \cdot c}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1})}{0.12 \text{ m}} = 1.7 \times 10^{-24} \text{ J}$$

## Practice

- Based on  $E = h \nu$  and the rules for unit cancellation, what must the SI unit for Planck's constant be?
- The human eye can generally see energy waves in the range of 400 to 700 nm. When hydrogen gas at low pressure is subjected to high voltage, it emits four waves of light in the visible region of the spectrum: one red, one blue-green, one blue-violet, and one violet.
  - A photon of red light from the hydrogen spectrum has an energy of  $3.03 \times 10^{-19} \text{ J}$ . What is the wavelength of this light in nanometers?
  - The blue-green line consists of waves with a frequency of  $6.15 \times 10^{14} \text{ Hz}$ . What is the energy of these waves?
  - The blue-violet line has a wavelength of 434 nm. What is the frequency of these waves in Hz?

## ANSWERS

- Since the SI unit for  $E$  is joules and for  $\nu$  is  $\text{s}^{-1}$ , the units of  $E = h \nu$  must be  $\text{J} = ? \text{ s}^{-1}$ . For unit cancellation to work, the units of  $h = ?$  must equal  $\text{J} \cdot \text{s}$  (see Lesson 21D).

(In SI base units, since Energy = work =  $\text{m} \cdot \text{a} \cdot \text{d} = (\text{mass})(\text{acceleration})(\text{distance}) = \text{J units} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ , the units of  $h = ? = \text{J} \cdot \text{s}$  also =  $\text{kg} \cdot \text{m}^2 \cdot \text{s}$ , but base units are not needed for most calculations).

- (Part (a) involves  $E$  and  $\lambda$ . The equation that relates those variables is)

$$E = \frac{h \cdot c}{\lambda}$$

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

$c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$  (list the two constants, convert DATA to those units)

$E \text{ in J} = 3.03 \times 10^{-19} \text{ J}$

$\lambda \text{ in m} = ?$  then convert to nm (c uses meters)

$1 \text{ nm} = 10^{-9} \text{ m}$  (listing fundamental metric conversions is optional in DATA)

\* \* \* \* \*

$$\lambda \text{ (in m)} = \frac{h \cdot c}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1})}{3.03 \times 10^{-19} \text{ J}} = 6.56 \times 10^{-7} \text{ m}$$

$$\lambda \text{ (in nm)} = 6.56 \times 10^{-7} \text{ m} \cdot \frac{1 \text{ nm}}{10^{-9} \text{ m}} = 6.56 \times 10^2 \text{ nm} = 656 \text{ nm}$$

2b. (Part (b) involves  $E$  and  $\nu$ . The equation that relates those symbols is)

$$E = h \nu$$

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  (list constants first. Use their units for WANTED and DATA)

$E$  in  $\text{J} = ?$

$\nu$  in  $\text{s}^{-1} = 6.15 \times 10^{14} \text{ Hz}$  (h uses seconds)

SOLVE:  $E$  (in J) =  $h \nu = (6.63 \times 10^{-34} \text{ J} \cdot \text{s}) (6.15 \times 10^{14} \text{ s}^{-1}) = 4.08 \times 10^{-19} \text{ J}$

2c. (The problem has  $\lambda$  and  $\nu$ . The equation that relates  $\lambda$  and  $\nu$  for electromagnetic waves is)

$$c = \lambda \nu$$

DATA:

$c = 3.0 \times 10^8 \text{ m} \cdot \text{s}^{-1}$  (list constants used in the equation first)

$\lambda$  in  $\text{m} = 434 \text{ nm} = 434 \times 10^{-9} \text{ m}$

$\nu$  in  $\text{s}^{-1} = ?$ , then convert to  $\text{Hz}$

$1 \text{ Hz} = 1 \text{ s}^{-1}$

$? = \nu$  in  $\text{s}^{-1}$ , then  $\text{Hz} = \frac{c}{\lambda} = \frac{3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}}{434 \times 10^{-9} \text{ m}} = 0.00691 \times 10^{17} \text{ s}^{-1} = 6.91 \times 10^{14} \text{ Hz}$

\* \* \* \* \*

## Lesson 23C: De Broglie's Wavelength

**Timing:** Do this section if you are asked to solve calculations using the De Broglie equation, and/or if you plan to take physics.)

\* \* \* \* \*

### De Broglie's Wavelength Equation

In 1923, the French physicist Louis De Broglie proposed that, just as energy has particle-like properties, particles can have wavelike properties. In the equation De Broglie derived to predict the characteristics of these "matter waves," the length of the wave associated with a particle depends on the mass of the particle and its speed (or *velocity*). The equation is

$$\lambda = \frac{h}{\text{mass} \cdot \text{speed}}$$

Though De Broglie's equation can be applied to any moving particles, it is most often applied to small particles such as electrons.

Apply De Broglie's wavelength equation to the following problem. If you need a hint, read a part of the answer below, then try again.

**Q.** Calculate the wavelength of an electron (mass of electron =  $9.11 \times 10^{-28}$  grams) that is traveling at one-tenth the speed of light.

\* \* \* \* \*

The problem involves wavelength ( $\lambda$ ), mass, and speed. The equation that relates those three variables is the De Broglie wavelength equation.

$$\lambda = \frac{h}{\text{mass} \cdot \text{speed}}$$

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

$\text{mass} = 9.11 \times 10^{-28} \text{ g}$  (list the two *constants* first)

$\text{speed} = 0.100 \times 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1} = 3.00 \times 10^7 \text{ m} \cdot \text{s}^{-1}$

$\lambda = ?$

SOLVE:  $? = \lambda = \frac{h}{\text{mass} \cdot \text{speed}} = \frac{6.63 \times 10^{-34} \text{ J} \cdot \text{s}}{(9.11 \times 10^{-28} \text{ g})(3.00 \times 10^7 \text{ m} \cdot \text{s}^{-1})} = ???$

We have a problem. The units do not cancel. You could ignore that, but if you do, you will do a lot of work to arrive at an answer that is not correct.

When units do not cancel properly, something is likely wrong. Let us see if we can fix this problem.

Joules is an abbreviation for a combination of SI *base* units used to measure distance, mass, and time. In many calculations in chemistry, joules can be left “as is” in an answer unit, as a measure of energy. The De Broglie wavelength equation, however, a case where, for the units to cancel, joules must be converted to the *base units* that joules is an *abbreviation* for.

To convert joules to its base units, you have two choices. You can *memorize* the base units that are equivalent to joules, or you can *derive* the base units from simple relationships if you know a bit of physics. Let’s review the process for converting unit abbreviations such as *newtons*, *pascals*, *watts*, and *volts* to the fundamental SI base units that those named units are abbreviating. This conversion process will be helpful to know, especially if you take future courses in chemistry, physics, or engineering.

### Converting Joules to Base Units

What is a joule of energy?

Energy is the capacity to do work. There are many forms of energy: including potential, kinetic, thermal, and electrical. There are many units that can be used to measure energy, including calories, ergs, BTUs, and kilowatt-hours. The SI unit used to measure energy is the *joule*.

Energy is a derived quantity. It can be defined in terms of the fundamental quantities of distance, mass, and time. The easiest way to do so is to define energy in terms of what in physics is termed “mechanical work.”

Work is defined as the product of a force acting over a distance. In equation form:

$$\text{Energy} = \text{work} = \text{force } \textit{times} \text{ distance} = F \cdot d$$

Since *force* is equal to mass times acceleration ( $F = ma$ ), we can write

**Energy** = work =  $F \cdot d$  = (mass times acceleration) times distance =  $m \cdot a \cdot d$  , or

$\boxed{\text{Energy} = \text{work} = m \cdot a \cdot d}$  (This relationship can be remembered as “work is mad!”)

The joule, the derived SI unit measuring energy, is defined in terms of the SI base units that measure the *fundamental* quantities. The SI base-units are derived from what was historically known as the **mks system**.

- Distance is measured in the SI-base unit of meters (**m**).
- The SI *base* unit used to measure mass is the kilogram (**kg**), not the gram. (In the SI system, base units for all *other* fundamental quantities do *not* include a metric prefix. The kilogram used for mass is the exception.)
- The base unit for time is seconds (s), and acceleration is defined as distance/time<sup>2</sup>, so acceleration is measured in base units of meters/second<sup>2</sup> (=  $m \cdot s^{-2}$ ).

Based on  $\boxed{\text{Energy} = \text{work} = m \cdot a \cdot d}$  , the SI unit measuring energy (**one joule**) is *defined* as the energy needed to accelerate **one** mass base unit (one **kg**) by **one** acceleration base unit ( $1 m \cdot s^{-2}$ ) for **one** distance base unit (1 **m**). By substituting into the definition of energy

$\boxed{\text{Energy} = m \cdot a \cdot d}$  the base units used to measure each of those four terms:

1 Joule =  $1 \text{ kg} \cdot (1 m \cdot s^{-2}) \cdot 1 m$  , this produces a definition of

$\boxed{\text{Joule} = \text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}}$  This key equality relates *joules* to its SI *base* units.

In the unit for **h**, if we *substitute* those base units in place of joules, and measure the other variables in the data in those *base* units, the units will cancel properly in De Broglie wavelength and other calculations.

To summarize: To solve the De Broglie equation, for the units of **h**, substitute for *joules* the *base* units that joules are equivalent to. Either memorize the base units for joules, or remember that “work is mad!” and substitute the SI base units (mks) that measure **m**, **a**, and **d**.

In general, SI derived units that are abbreviations for combinations of base units (such as joules, newtons, watts, and pascals) can be converted to SI base units using these steps:

**To convert an SI combined unit to its base units**

1. Write the *equation* that defines the derived quantity that the unit measures using quantities for which the SI base units are known; then
2. Substitute into the equation the *base units* used to measure those quantities.

(For more on converting unit abbreviations such as Joules to SI base units, see Lesson 19C).

\* \* \* \* \*

Now let's return to our De Broglie wavelength calculation.

In your DATA table, substitute for joules the *base* units of joules, adjust your DATA units to match these new units for the constant, solve, and then check your answer below.

\* \* \* \* \*

To arrange for the units to cancel, begin by substituting the *base* units for joules into **h**:

$$\text{DATA: } \mathbf{h} = 6.63 \times 10^{-34} \cancel{\text{J}} (\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}) \cdot (\text{s}) \quad (\text{list constants first})$$

Write after the *mass* variable in the DATA table the *units* for mass used in the constant **h**, then convert the mass DATA to those units.

$$\text{mass in kg} = 9.11 \times 10^{-28} \text{ g} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = 9.11 \times 10^{-31} \text{ kg}$$

Mass, to have consistent units that will cancel in the equation, must be **kg** and *not* g.

Speed (distance over time) is supplied in the problem in the base units used in **h**.

$$\text{speed (in m/s)} = 0.100 \times 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1} = 3.00 \times 10^7 \text{ m} \cdot \text{s}^{-1}$$

Since the unit for the WANTED wavelength is not specified, pick a unit to attach to the symbol that either uses the units in the *constant* or is an SI unit. In this and in most problems, both choices will result in the same unit. Since wavelength is a distance, solve for  $\lambda$  in the distance SI base unit: meters.

SOLVE:

$$? = \lambda \text{ (in m)} = \frac{\mathbf{h}}{\text{mass} \cdot \text{speed}} = \frac{6.63 \times 10^{-34} (\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}) \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg}) (3.00 \times 10^7 \text{ m} \cdot \text{s}^{-1})} = 2.43 \times 10^{-11} \text{ m}$$

Mark the unit cancellation on your paper carefully. *If* the units cancel to give the WANTED unit, then it is likely that the numbers were put in the right place to get the right answer.

When should you substitute *base* units for units such as joules, pascals, volts, and newtons? Do so *only* when it is necessary in order for units to cancel to obtain a WANTED unit. If the WANTED unit and/or other DATA units *include* one of those complex derived units, you will probably *not* need to convert to base units to solve.

### Summary: Equations and Constants For Electromagnetic Waves

Be sure that these 7 relationships can be recalled from memory *before* doing the following practice. Treat **practice** as a practice *test*.

1. **Wave speed =  $\lambda \nu$**

2. For electromagnetic waves, speed of light =  **$c = \lambda \nu = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$**

3. Planck's equation:  **$E = h \nu$**  and  **$E = h \cdot \frac{c}{\lambda}$**

4. Planck's constant =  **$h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$**

5. The De Broglie wavelength equation:  **$\lambda = \frac{h}{\text{mass} \cdot \text{speed}}$**

6. **Joule =  $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$**  (memorize or derive as needed)

7. When working in problems that mix derived and fundamental SI units, convert all DATA to **mks**: distance in **meters**, mass in **kg**, time in **seconds**.

## Practice

- If  $F = m \cdot a$ , and the SI unit for force (the *newton*) is defined using the SI base units for mass and acceleration, what base units are equivalent to the newton?
- If the wavelength of a moving electron is measured to be 36.3 picometers, what is the speed of the electron? (mass of electron =  $9.11 \times 10^{-28}$  grams)

## The Heisenberg Uncertainty Principle

In 1927, the German physicist Werner Heisenberg postulated the **uncertainty principle**: that for very light particles such as electrons, it is not possible to be certain of both position and velocity at the same time. Mathematically, his equation is

$$(\text{uncertainty in position}) \times (\text{uncertainty in velocity}) \geq h / (4\pi \cdot \text{mass})$$

Since the mass of a particle is constant, all of the terms on the right side of this equation are constant, and the two terms on the left are variables. The equation is therefore in the form  $xy = c$  and is an inverse proportion. As the uncertainty of one variable on the left goes *down*, the uncertainty of the other goes *up*.

This result is that if we know where an electron is, we cannot say precisely its velocity (its speed and direction, which tells us where it will be next). If we know the velocity of the electron, we cannot say precisely where it is. This means that the location of electrons must be stated in *probabilities* rather than certainties.

If you need to solve calculations using the uncertainty equation, simply memorize the equation and apply the rules for solving equations discussed above.

\* \* \* \* \*

## ANSWERS

- Since  $F = m \cdot a$ , one unit of force (one Newton) equals one base unit of mass (1 kg) times one base unit of acceleration (one meter per second<sup>2</sup> =  $1 \text{ m} \cdot \text{s}^{-2}$ ).

$$\text{newton} = \text{kg} \cdot \text{m} \cdot \text{s}^{-2}$$

- (The problem involves  $\lambda$ , speed, and mass. Use the De Broglie equation.)

$$\lambda = \frac{h}{\text{mass} \cdot \text{speed}}$$

(So that units cancel in the De Broglie equation, substitute the *base* units for joules into  $h$ .)

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  ( $\text{kg} \cdot \text{m}^2 \cdot \text{s}^{-2}$ )  $\cdot \text{s}$  (list constants first)

For consistent units that will cancel, mass must be in **kg** and *not* g.

$$\text{mass in kg} = 9.11 \times 10^{-28} \text{ g} \cdot \frac{1 \text{ kg}}{10^3 \text{ g}} = 9.11 \times 10^{-31} \text{ kg}$$

For the WANTED speed, the units are not specified, so pick units for speed (*distance per unit of time*) that are used in the constant (**h**): meters and seconds.

**speed (in m/s) = ?**

$$\lambda \text{ (in m)} = 36.3 \text{ picometers} \cdot \frac{10^{-12} \text{ m}}{1 \text{ pm}} = 36.3 \times 10^{-12} \text{ m}$$

$$\begin{aligned} \text{To SOLVE: } ? = \text{speed (in m/s)} &= \frac{h}{\text{mass} \cdot \lambda} = \frac{6.63 \times 10^{-34} \text{ (kg} \cdot \text{m}^2 \cdot \text{s}^{-2}) \cdot \text{s}}{(9.11 \times 10^{-31} \text{ kg})(36.3 \times 10^{-12} \text{ m})} = \\ &= \frac{6.63}{(9.11 \times 36.3)} \times \frac{10^{-34+31+12}}{\text{kg} \cdot \text{m}} \text{ (kg} \cdot \text{m}^2 \cdot \text{s}^{-2}) \cdot \text{s} = 0.0200 \times 10^9 \text{ m} \cdot \text{s}^{-1} \\ &= 2.00 \times 10^7 \text{ m/s} \end{aligned}$$

\* \* \* \* \*

## Lesson 23D: The Hydrogen Atom Spectrum

### Bohr and Atomic Spectra

When elements are heated to high temperatures, or vaporized and electrified, they glow: they give off light. Viewed through a prism or a diffraction grating, this light separates into thin lines of colors in the order of the colors of the rainbow: energy waves at sharply defined wavelengths. This is called the **line spectrum** of the element.

Each element has a *characteristic* line spectrum. Astronomers can analyze the light from distant stars to identify elements that are present in the stars.

In 1913, the Danish physicist Neils Bohr proposed that the light of an element's spectrum results from electrons falling from higher to lower fixed energy levels inside an atom, like marbles falling down a staircase (but one in which each step has a different rise).

Based on a simple mathematical equation,

$$E_n = \frac{-2.18 \times 10^{-18}}{n^2} \text{ J/atom}$$

Bohr's model predicted exactly the observed wavelengths of the *hydrogen* atom spectrum.

The energy levels inside the H-atom will be the basis for explaining the energy of the electrons in all other elements as well. Understanding the behavior of the electrons in atoms will help in predicting the reactions of atoms and larger particles: the central focus of chemistry.

### Bohr's Model For the Hydrogen Atom

A neutral hydrogen atom consists of one proton in the nucleus at the center of the atom and one electron outside the nucleus. (A hydrogen nucleus may also include one or two neutrons, but these neutral particles do not affect the key electrical interaction between the positive proton and the negative electron.) Nearly all of the volume of the atom is due to the space occupied by the electron's movement around the nucleus.

The hydrogen electron can be described as existing at energy levels that are similar to a staircase. The H-atom electron *must* be found on *one* of the energy levels of the staircase: it cannot rest between steps.

The steps of the H-atom staircase are uneven, but they follow a consistent pattern. The first step up from the bottom step is large, but the rise for each subsequent step is smaller.

The staircase has an infinite number of steps, numbered from  $n = 1$  to  $n = \infty$ . Step  $n = 1$  is the bottom step, and  $n = \infty$  is at the top of the staircase.

All systems tend to go to their lowest potential energy. The H-atom electron is therefore normally found at the bottom step  $n = 1$ , where it is said to be in its **ground state**.

However, if energy is added to the H-atom, such as from heat, energy waves, or high voltage, the electron can be promoted up the staircase. If the H-atom electron is above the bottom step, it is unstable and said to be in an "excited state."

An H-atom electron promoted to an upper step is unstable because it is not at its *lowest* possible potential energy. The electron will therefore tend to fall back down the staircase. It falls like a marble, either hitting every step *or* skipping some steps, until it reaches the

bottom step  $n = 1$ . The electron may “pause” on any step, but the electron cannot pause between steps.

The electron falling down the staircase must lose energy. To do so, it emits energy waves (photons). The energy waves it emits are the lines of the H-atom spectrum. To calculate the energy of the lines in the spectrum, let's add energy *values* to this H-atom model.

### The H Energy Levels

Using this equation:  $E_n = \frac{-21.8 \times 10^{-19} \text{ J}}{n^2}$  (per each atom)

calculate and fill-in each  $E_n$  value for the steps of the H-atom listed below.

To add and subtract numbers in exponential notation by hand, the numbers must have the same exponential term (see Lesson 1B). To simplify upcoming calculations, let us write all of the  $E_n$  values as “ $\times 10^{-19} \text{ J}$ .” If you need help, check the sample calculation below.

$$n = \infty \quad \underline{\hspace{2cm}} \quad E_\infty =$$

...

$$n = 6 \quad \underline{\hspace{2cm}} \quad E_6 =$$

$$n = 5 \quad \underline{\hspace{2cm}} \quad E_5 =$$

$$n = 4 \quad \underline{\hspace{2cm}} \quad E_4 =$$

$$n = 3 \quad \underline{\hspace{2cm}} \quad E_3 =$$

$$n = 2 \quad \underline{\hspace{2cm}} \quad E_2 =$$

$$n = 1 \quad \underline{\hspace{2cm}} \quad E_1 =$$

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For the energy level  $n = 2$ , :  $E_2 = \frac{-21.8 \times 10^{-19} \text{ J}}{2^2} = -5.45 \times 10^{-19} \text{ J}$

Finish calculating the remaining energies if needed.

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Your answers should match the following.

<u>The H-atom Energy Levels</u>	
$n = \infty$ _____	$E_\infty = 0 \text{ J}$
...	
$n = 6$ _____	$E_6 = -0.606 \times 10^{-19} \text{ J}$
$n = 5$ _____	$E_5 = -0.872 \times 10^{-19} \text{ J}$
$n = 4$ _____	$E_4 = -1.36 \times 10^{-19} \text{ J}$
$n = 3$ _____	$E_3 = -2.42 \times 10^{-19} \text{ J}$
$n = 2$ _____	$E_2 = -5.45 \times 10^{-19} \text{ J}$
$n = 1$ _____	$E_1 = -21.8 \times 10^{-19} \text{ J}$

(The numbers above are accurate, but the spacing between the steps is not drawn to scale.)

The movement of electric charge creates electromagnetic waves. When the H-atom electron moves falls from one level to another, it must release an energy wave with an energy equal to the *difference* in energy between those two levels.

In your notebook, calculate the *difference* in energy between energy levels  $E_3$  and  $E_2$ .

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$$E_3 \text{ minus } E_2 = -2.42 \times 10^{-19} \text{ J} - (-5.45 \times 10^{-19} \text{ J}) = +3.03 \times 10^{-19} \text{ J}$$

In Practice A, Problem 1a of Lesson 23B, you calculated the wavelength of a wave with this energy. What was the value that wavelength? According to Problem 1a, what color would your eye perceive this wave to be?

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**656 nm.** When the H-atom electron falls from level  $n=3$  to  $n=2$ , it produces an energy wave at a wavelength of 656 nm that your eye sees as **red**.

When the electron falls between other energy levels, it emits light with other colors or light in other regions of the electromagnetic spectrum that the eye cannot see.

### **Practice: The H-atom Spectrum**

Do Problems 1 and 3 below. Do more if you need more practice.

In Lesson 23B, Practice A1, and in the lesson above, we calculated these values for the

### **H-Atom Visible Spectrum**

Color	Wavelength (nm)	Frequency (Hz)	Energy (J/atom)	Transition
red	656		$3.03 \times 10^{-19}$	Step 3 $\rightarrow$ 2
blue-green	486	$6.15 \times 10^{14}$	$4.08 \times 10^{-19}$	
blue-violet	434	$6.91 \times 10^{14}$		
violet	410			

Use this table as data for the problems below. Add your answers to the table as you do the following calculations.

- As the H-atom electron falls from level  $n=5$  to  $n=2$ , based on the values for the energy levels ( $E_n$ ) that you calculated in this lesson,
  - what would be the energy of the wave emitted?
  - What would be the wavelength of the wave in nm?
  - What would be the color of the wave?
  - What information do these answers allow you to add to the table above?
- Calculate the energy of the *violet* line of the H-atom visible spectrum
  - using Planck's equation.
  - Use the energy-level diagram for the H-atom to determine which transition produces a photon with the energy of the violet line.
- If sufficient energy is added to the H-atom in its ground state (with the electron at  $n=1$ ), the electron can be ionized: it can be taken an essentially infinite distance away from the proton that is attracting the electron. The energy needed to remove the electron is the energy needed to promote the electron from level  $n=1$  to  $n=\infty$ . This energy value is termed the **ionization energy** of the atom.
  - Based on the energy level diagram for H, how much energy would be needed to be added to take away (ionize) an H-atom electron?
  - If an ionized electron falls from level  $n=\infty$  down to level  $n=1$  in one transition, what will be the energy of the wave emitted by the electron?
  - How does this energy value compare to the energy values of the lines in the visible spectrum listed in the chart above?

- d. What will be the wavelength of this energy wave in nm?
- e. If the human eye can generally see energy waves in the range of 400 to 700 nm, will you be able to see this wave when looking at an H-atom spectrum?
4. One of the lines in the ultraviolet region of the spectrum of hydrogen has a frequency of  $2.46 \times 10^{15}$  Hz. Which transition does this represent?
5. Using alternate units, the ionization energy of hydrogen is 13.6 electron volts (eV) per atom. Convert this ionization energy to joules per mole. (1 eV =  $1.6 \times 10^{-19}$  J)
6. If the ionization energy of a hydrogen atom is  $-21.8 \times 10^{-19}$  J, what is the energy of level  $n = 3$ ?

## ANSWERS

1. a.  $E_5 - E_2 = -0.872 \times 10^{-19} \text{ J} - (-5.45 \times 10^{-19} \text{ J}) = +4.58 \times 10^{-19} \text{ J}$
- b. This problem involves  $E$  (from part a) and  $\lambda$ . The equation that relates those variables is
- $$E = \frac{h \cdot c}{\lambda}$$
- DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$   
 $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$  (list the *two* constants, convert DATA to their units)  
 $E \text{ in J} = 3.03 \times 10^{-19} \text{ J}$   
 $\lambda \text{ in m} = ?$  then convert to nm WANTED (c uses meters)  
 $1 \text{ nm} = 10^{-9} \text{ m}$  (listing fundamental metric conversions is optional)
- $\lambda \text{ (in m)} = \frac{h \cdot c}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1})}{4.58 \times 10^{-19} \text{ J}} = 4.34 \times 10^{-7} \text{ m}$   
 $= 434 \times (10^{-9} \text{ m}) = 434 \text{ nm}$
- c. **Blue-violet.** See values in the table above.
- d. For the **blue-violet** wave,  $E = 4.58 \times 10^{-19} \text{ J}$ , and the electron is falling from step 5  $\rightarrow$  2.
2. a. Only  $\lambda$  is known in the table;  $E$  is WANTED. The form of Planck's equation that relates  $\lambda$  and  $E$  is

$$E = \frac{h \cdot c}{\lambda}$$

DATA:  $c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$   
 $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  (list constants first, use their units to solve)  
 $E \text{ in J} = ?$  (h uses joules)  
 $\lambda \text{ in m} = 410 \text{ nm} = 410 \times 10^{-9} \text{ m}$  (convert to the meters used in c)

$E = \frac{h \cdot c}{\lambda} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1})}{410 \times 10^{-9} \text{ m}} = 4.85 \times 10^{-19} \text{ J}$

- b. Based on the pattern in the spectrum data table, the violet line should represent the **6 → 2** transition.

$$\text{Check it: } E_6 - E_2 = -0.606 \times 10^{-19} \text{ J} - (-5.45 \times 10^{-19} \text{ J}) = +4.84 \times 10^{-19} \text{ J}$$

Allowing for rounding, this agrees with the 2a answer.

3. a. Enough energy must be added to promote the electron from  $n = 1$  to  $n = \infty$ ,

$$E_{\infty} - E_1 = 0 \text{ J} - (-21.8 \times 10^{-19} \text{ J}) = +21.8 \times 10^{-19} \text{ J}$$

- b. From  $n = \infty$  down to  $n = 1$ , the energy difference is the same:  $E_{\infty} - E_1 = +21.8 \times 10^{-19} \text{ J}$

The energy put in to pull away (ionize) the electron, starting from  $n = 1$ , must equal the total energy the electron releases when it returns to  $n = 1$ .

- c.  $+21.8 \times 10^{-19} \text{ J}$  is a **larger** energy than those for the *visible* waves listed in the table.

- d. Part (d) involves **E** (from part c) and **λ**. The equation that relates those variables is

$$E = \frac{h \cdot c}{\lambda}$$

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$

$c = 3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1}$  (list the *two* constants, convert DATA to those units)

$E \text{ in J} = 21.8 \times 10^{-19} \text{ J}$

$\lambda \text{ in m} = ?$  then convert to **nm** (for units consistent with **c**, solve in **m** first)

**1 nm =  $10^{-9} \text{ m}$**  (fundamental metric conversions are optional in DATA)

$$\lambda \text{ (in m)} = \frac{h \cdot c}{E} = \frac{(6.63 \times 10^{-34} \text{ J} \cdot \text{s})(3.00 \times 10^8 \text{ m} \cdot \text{s}^{-1})}{21.8 \times 10^{-19} \text{ J}} = 0.912 \times 10^{-7} \text{ m} \\ = 91.2 \times (10^{-9} \text{ m}) = 91.2 \text{ nm}$$

- e. **No.** This wave has a shorter wavelength (and a higher frequency and energy) than the eye can see. 91.2 nm is in the **ultraviolet** (UV) region of the spectrum.

The high-energy ultraviolet lines of the H-atom spectrum can be seen by special films.

Prolonged exposure to UV radiation is dangerous to the eyes and skin. The sun produces high amounts of UV radiation, but most is absorbed by the ozone layer in the earth's upper atmosphere before it can reach the earth's surface. If the ozone layer decays, the incidence of skin cancer on earth would likely increase, among other harmful effects.

4. To find the transition using the energy-level diagram, the energy of the line is needed.

The equation that relates **E** and **ν** is  $E = h \nu$

DATA:  $h = 6.63 \times 10^{-34} \text{ J} \cdot \text{s}$  (list constants first, use their units for WANTED and DATA)

$E \text{ in J} = ?$

$\nu \text{ in s}^{-1} = 2.46 \times 10^{15} \text{ Hz} \text{ s}^{-1}$  (**h** uses seconds)

SOLVE:  $E \text{ (in J)} = h \nu = (6.63 \times 10^{-34} \text{ J} \cdot \text{s})(2.46 \times 10^{15} \text{ s}^{-1}) = 1.63 \times 10^{-18} \text{ J}$

Which transition has this energy?

\* \* \* \* \*

Convert this E to  $16.3 \times 10^{-19} \text{ J}$  for easier comparison to the numbers in the energy-level diagram.

\* \* \* \* \*

$$E_2 - E_1 = -5.45 \times 10^{-19} \text{ J} - (-21.8 \times 10^{-19} \text{ J}) = +16.4 \times 10^{-19} \text{ J}$$

That matches the energy value calculated from the frequency above, allowing for rounding.

For *all* of the transitions where the electron falls to  $n = 1$ , the H-atom will emit UV radiation.

5. WANTED: ?  $\frac{\text{kJ}}{\text{mol}}$

DATA:  $13.6 \text{ eV} = 1 \text{ atom}$

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

A ratio is WANTED. The data is two ratios/equalities/conversions. Try conversions (Lesson 11B).

$$? \frac{\text{kJ}}{\text{mol}} = \frac{1.6 \times 10^{-19} \text{ J}}{1 \text{ eV}} \cdot \frac{1 \text{ kJ}}{10^3 \text{ J}} \cdot \frac{13.6 \text{ eV}}{1 \text{ atom}} \cdot \frac{6.0 \times 10^{23} \text{ atoms}}{1 \text{ mole}} = 1.3 \times 10^3 \frac{\text{kJ}}{\text{mol}}$$

6. The ionization energy is the energy need to promote the electron from  $n = 1$  to  $n = \infty$ .

That energy is also the value in the energy-level equation  $E_n = \frac{-21.8 \times 10^{-19} \text{ J}}{n^2}$  (per atom)

For energy level  $n = 3$ ,  $E_3 = \frac{-21.8 \times 10^{-19} \text{ J}}{3^2} = -2.42 \times 10^{-19} \text{ J}$

\* \* \* \* \*

## Lesson 23E: Quantum Mechanics

### Schrödinger's Wave Equation

Though the Bohr model explained the spectrum of hydrogen, other aspects of the model were less successful at explaining the behavior of the hydrogen electron.

In 1926, the German physicist Erwin Schrödinger developed equations which described the electron as if it were a wave, similar to the “standing waves” created by stringed instruments. Solutions to these equations are termed the **quantum mechanical (wave) model**. This model remains today our best explanation for the behavior of the hydrogen electron, and it is the basis for predicting the behavior of electrons in all other elements as well.

Solutions to the wave equation generally involve complex mathematics, and one question often has multiple solutions. In many respects, however, the wave equation produces a model for the hydrogen atom based on mathematical patterns that are elegant in their simplicity.

The following points are part of the description of the hydrogen atom based on quantum mechanics.

## Predictions of the Wave Equation For Hydrogen

1. Inside the hydrogen atom are the levels described by Bohr:  $n = 1, 2, 3, \dots, \infty$ . When used with the wave equation, these numbers are called **principal quantum numbers**. Higher quantum numbers represent higher energy levels. These energy levels have the  $E_n$  values calculated by Bohr's  $E_n$  equation.
2. Around the H-atom nucleus are **orbitals** that describe the space where an electron is likely to be found. The orbital has an energy and a shape, but, consistent with the uncertainty principle, the location of an electron in an orbital is described in terms of probability. The *shape* of the orbital describes where the electron will be 90% of the time.
3. At each principal quantum number  $n$ , there are  $n^2$  total orbitals, and  $n$  different *types* of orbitals.
  - a. At level  $n = 1$ , there is one orbital, the *1s* orbital. An *s* orbital has a *spherical* symmetry around the nucleus: in an *s* orbital, at a given distance from the nucleus in all directions, there is an equal chance of finding an electron.
  - b. At level  $n = 2$ , there are two types of orbitals and four total orbitals: one *2s* orbital with spherical symmetry, and three *2p* orbitals. The three *p* orbitals are *perpendicular* to each other, and can be described as falling on *x*, *y*, and *z* axes around the nucleus.
  - c. At level  $n = 3$ , there are three types of orbitals and nine total orbitals: one spherical *3s* orbital, three perpendicular *3p* orbitals, and five *3d* orbitals. Most (but not all) of the *d* orbitals are *diagonal* to the *p* orbitals.
  - d. At level  $n = 4$ , there are four types of orbitals and 16 total orbitals: one *4s*, three *4p*, five *4d*, and seven *4f* orbitals.

(It may help to remember the *spdf* order of the orbitals as “stupid pirates die fighting.”)
4. The H-atom electron in its ground state is in the *1s* orbital. If sufficient energy is added to an H-atom, its electron can be promoted into one of the higher energy orbitals.

The above points can be summarized by a diagram. Below is the model for the H-atom predicted by the wave equation for the first *four* principal quantum numbers. Each line ( \_\_\_ ) represents an orbital.

### The Hydrogen Atom: Orbitals For the First Four Energy Levels

4s ___	4p ___ ___ ___	4d ___ ___ ___ ___ ___	4f ___ ___ ___ ___ ___ ___ ___	( = 16 orbitals )
3s ___	3p ___ ___ ___	3d ___ ___ ___ ___ ___		( = 9 orbitals )
2s ___	2p ___ ___ ___			( = 4 orbitals )
1s ___				( = 1 orbital )

## Quantum Numbers

In the wave equation for the H-atom, each electron in an orbital can be identified by a series of **quantum numbers** that predict the characteristics of the orbital and of an electron in that orbital.

- Principal quantum numbers** are the **n** values: the integers 1, 2, 3, ... The value of **n** will predict the size and energy of the orbital. For higher **n** values, the orbitals occupy more volume and are at higher potential energy. An electron in orbitals with a higher **n** will on average be further from the nucleus than electrons in orbitals with a lower **n** in the same atom.
- Angular momentum quantum numbers** (symbol  $\ell$  - a lower-case script L) at each **n** are numbered from 0 to **n**-1. Each  $\ell$  value correlates with one of the types (*s*, *p*, *d*, or *f*) of orbitals.

The *s* orbital is  $\ell = 0$ , *p* orbitals are  $\ell = 1$ , *d* orbitals are  $\ell = 2$ , and *f*'s are  $\ell = 3$ .

- Magnetic quantum numbers** (symbol  $m_\ell$ ) have values from  $-\ell$  to 0 to  $+\ell$ . These numbers identify the multiple *p*, *d*, and *f* orbitals at each **n**.
- Electron spin quantum numbers** (symbol  $m_s$ ) identify the **spin** of an electron in an orbital. An electron must have a spin of either  $+\frac{1}{2}$  or  $-\frac{1}{2}$ . In these lessons, we will represent an electron that has a positive spin as  $\uparrow$  and one with a negative spin as  $\downarrow$ .

A way to remember these rules for quantum numbers is to memorize the H-atom orbital diagram above *plus* the quantum number diagram below for **n = 4**. Note the patterns of the numbers going from the bottom up.

<u>Relating Quantum Numbers and Orbitals</u>																				
	4s	—	4p	—	—	—	4d	—	—	—	—	—	4f	—	—	$\uparrow$	—	—	—	—
$m_\ell =$	0		-1	0	1		-2	-1	0	1	2		-3	-2	-1	0	1	2	3	
$\ell =$	0			1				2							3					
<b>n =</b>	4																			

The electron shown above in the *4f* level would be described as having the quantum numbers  $n = 4$ ,  $\ell = 3$ ,  $m_\ell = -1$ , and  $m_s = +\frac{1}{2}$ .

The diagram for level  $n = 3$  is similar to  $n = 4$  above, except that it will lack the *f* orbitals. The diagram for level  $n = 2$  will have only *s* and *p* orbitals. The diagram for level  $n = 1$  will have only the single *s* orbital.

\* \* \* \* \*

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**Practice**

Memorize the two diagrams in this lesson, then do the problems below.

- At level  $n = 4$  of the hydrogen atom are
  - how many types of orbitals?
  - How many total orbitals?
  - Write the number and letter used to identify the types of orbitals, and list the number of orbitals there will be of each type, at  $n = 4$ .
- At level  $n = 5$ , the H atom has orbitals designated  $5s$ ,  $5p$ ,  $5d$ ,  $5f$ , and  $5g$ . How many orbitals are there of each type?
- Add numbers to complete the chart below.

**3s** — **3p** — — — **3d** — — — —

**$m_l$**  =

**$l$**  =

**$n$**  =

- At level  $n = 5$ , what quantum numbers are permitted for
    - $l$  ?
    - $m_l$  ?
    - $m_s$  ?
  - Write the diagram for the first four (the lowest four) energy levels of the hydrogen atom, then add to the diagram one electron that has quantum numbers
    - $n = 3$ ,  $l = 1$ ,  $m_l = 1$ , and  $m_s = -1/2$ .
    - $n = 4$ ,  $l = 2$ ,  $m_l = -2$ , and  $m_s = +1/2$ .
  - Write symbols and values for the four quantum numbers that characterize the hydrogen atom electron when it is in the following orbitals.
    - 2s**  $\uparrow$  **2p** — — —
    - 3s** — **3p** — — — **3d** —  $\downarrow$  — — —
    - 4s** — **4p** — — — **4d** — — — — **4f**  $\uparrow$  — — — — —
-

**ANSWERS**

1. a. 4 types of orbitals    b. 16 total orbitals    c. 1 4s, 3 4p, 5 4d, and 7 4f orbitals

2. At  $n = 4$ , there are  $n^2 = 16$  total orbitals: 1 s, 3 p, 5 d, and 7 f.

At level  $n = 5$ , there must be  $n^2 = 25$  total orbitals.  $25 - 16 = 9$  additional orbitals at  $n = 5$ .

At level  $n = 5$ , there must be one 5s, three 5p, five 5d, seven 5f, and nine 5g orbitals.

3.

	3s	—	3p	—	—	—	3d	—	—	—	—	—
$m_\ell =$	0		-1	0	1		-2	-1	0	1	2	
$\ell =$	0		1				2					
$n =$	3											

4. a.  $\ell$ : 0 1 2 3 4    b.  $m_\ell$ : -4 -3 -2 -1 0 1 2 3 4    c.  $m_s$ : Always  $+\frac{1}{2}$  and  $-\frac{1}{2}$ .

5. a.

4s	—	4p	—	—	—	4d	—	—	—	—	—	4f	—	—	—	—	—	—	(= 16 total)
3s	—	3p	—	—	↓	3d	—	—	—	—	—								(= 9 total)
2s	—	2p	—	—	—														(= 4 total)
1s	—																		(= 1 total)

b.

4s	—	4p	—	—	—	4d	↑	—	—	—	—	4f	—	—	—	—	—	—	(= 16 total)
3s	—	3p	—	—	—	3d	—	—	—	—	—								(= 9 total)
2s	—	2p	—	—	—														(= 4 total)
1s	—																		(= 1 total)

6. a.  $n = 2$ ,  $\ell = 0$ ,  $m_\ell = 0$ , and  $m_s = -\frac{1}{2}$ .

b.  $n = 3$ ,  $\ell = 2$ ,  $m_\ell = -1$ , and  $m_s = -\frac{1}{2}$ .

c.  $n = 4$ ,  $\ell = 3$ ,  $m_\ell = -3$ , and  $m_s = +\frac{1}{2}$ .

\* \* \* \* \*

# # # # #