Calculations
In Chemistry

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Module 39 – Nuclear Chemistry

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Module 39 – Nuclear Chemistry

Prerequisites: This module may be started at any point after Lessons 6A and 6B.

Timing: This module covers alpha decay, beta decay, fission, and fusion reactions. If you are assigned calculations involving the *half-life* of radioactive isotopes, you should also complete the sections on *first-order* reactants in Lesson 27H.

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Lesson 39A: The Nucleus – Review

Pretest: If you recall the rules for the structure of the nucleus, try the problems in the practice set at the end of this lesson. If you can do those problems, you may skip this lesson.

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Chemistry and Nuclear Reactions

The rules for nuclear reactions are quite different from those that govern chemistry. For example,

- During *chemical* reactions, the nucleus is not changed in any of the reacting atoms. Since the number of protons in the nucleus determines the identity of the atom, the number and the kind of atoms must remain the same in the reactants and products during chemical reactions.

  In *nuclear* reactions, nuclei can combine or divide to form different atoms.
- Chemical reactions such as the explosion of TNT (trinitrotoluene) release large amounts of energy, but the energy added or released per atom in nuclear reactions is nearly always much higher than in chemical reactions. An atomic bomb is an example of a nuclear reaction.
- On Earth, chemical reactions occur with relative ease. In a biological organism, millions of chemical reactions occur each minute. Nuclear reactions occur naturally on earth during radioactive decay, and may occur when cosmic rays enter our atmosphere, but on earth, compared to chemical reactions, nuclear reactions are relatively rare. (In a star, however, nuclear reactions are common.)

Because of this difference in rules, the detailed investigation of *most* nuclear reactions is generally assigned to physics. In chemistry, we limit our study to three nuclear reactions: radioactive decay, which is a powerful tool in the study of chemical reactions; plus fission and fusion. These three types of reactions explain how atoms are formed.

A Model for the Nucleus

Science has only a partial understanding of the nature of the atomic nucleus. When understandings in science are limited, *models* are developed that may be simplified, incomplete, or even speculative, but allow us to predict how systems will behave.

To explain the nuclear reactions that are important in chemistry, the model for the nucleus that we utilize in chemistry is simplified compared to the models of physics, but this simplified model can nearly always predict the impact of the structure of the nucleus on processes of interest in chemistry and biology.

Our chemistry model for the atom and its nucleus was introduced in Lesson 6B. To briefly review:

1. **Nuclear Structure**

   Atoms are composed three *subatomic particles***.** In standard chemistry, our primary focus is on the behavior of the electrons in atoms. In *nuclear* chemistry, our focus is on the protons and neutrons in the nucleus of atoms.

   - **Protons**
     - Protons have a **+1** electrical charge (1 unit of positive charge). Each proton has a mass of 1.007 amu (atomic mass units) which is equivalent to 1.007 grams per mole.
     - The number of protons is the **atomic number** of a nucleus or atom. The number of protons determines the *name* (and thus the *symbol*) of a nucleus or atom. The number of protons determines the **nuclear charge** of an atom's nucleus.
     - The number of protons is a major factor in the atom's behavior.
     - The number of protons in an atom is never changed by *chemical* reactions, but can change during *nuclear* reactions.
   - **Neutrons**
     - Neutrons have an electrical charge of zero. A neutron has about the same mass as a proton: 1.009 amu.
     - Neutrons, like protons, are never gained or lost in *chemical* reactions, but the number of neutrons and protons in an atom can change in a *nuclear* reaction.
     - Unlike the number of protons, the number of neutrons in an atom has little to no influence on the types of *chemical* reactions that substances containing that atom will undergo. However, nuclei with the same number of protons but different numbers of neutrons will undergo different *nuclear* reactions.
     - In addition, atoms with the same numbers of protons but differing numbers of neutrons have different masses. As a result, some *physical* properties of a substance, such as their densities and the relative speeds at which the particles move, may differ measurably if its atoms have differing numbers of neutrons.

2. **The Nucleus**

   All of the protons and neutrons in an atom are found at the center of the atom: in the nucleus. The diameter of a nucleus is roughly 100,000 times smaller than the effective diameter of most atoms. However, the nucleus contains all of an atom's positive charge and nearly all of its mass. Electrons are located outside the nucleus and are much lighter than protons and neutrons.

3. Types of Nuclei

Only certain combinations of protons and neutrons form a nucleus that is stable. In a nuclear reaction, if a combination of protons and neutrons is formed that is unstable, the nucleus will decay. In terms of stability, nuclei can be divided into three types.

- **Stable** nuclei are combinations of protons and neutrons that do not change in a planetary environment such as Earth over many billions of years.
- **Radioactive** nuclei are *somewhat* stable. Some radioactive nuclei exist for only a few seconds, and others exist on average for several billion years, but they fall apart (**decay**) at a constant and characteristic rate.
- **Unstable** nuclei, if formed in nuclear reactions, decay within a few seconds.

Nuclei that exist in the earth's crust include all of the stable nuclei plus some radioactive nuclei. All atoms with between one and 82 protons [except technetium (Tc) and promethium (Pm)] have at least one nucleus found in the earth's crust that is stable. Atoms with 83 to 92 protons exist in the earth's crust but are always radioactive. Atoms with 93 or more protons exist on earth only when they are created in manmade nuclear reactions.

Radioactive atoms comprise a very small percentage of the matter on earth. Over 99.99% of the earth's atoms have stable nuclei that have not changed since atoms came together to form the Earth billions of years ago.

4. Terminology

Protons and neutrons are termed the **nucleons**. The combination of a certain number of protons and neutrons is called a **nuclide**. The set of nuclides that have the same number of protons (so they are the same atom) but differing numbers of neutrons are called the **isotopes** of the atom.

Isotopes

Some atoms have only one stable nuclide; others have as many as 10 stable isotopes.

Examples: All atoms with 1 proton are called hydrogen. Two kinds of hydrogen nuclei are stable: those with

- 1 proton; an isotope that is named protium or hydrogen-1, and
- 1 proton and 1 neutron, an isotope that is referred to as **hydrogen-2**, **deuterium,** or **heavy hydrogen**.

Most hydrogen atoms found on earth are the isotope containing one proton and no neutrons: only 1 H atom in about 6,400 contains a deuterium nucleus. However, deuterium can be separated from the majority isotope, and it has many important uses in chemistry. Deuterium is often represented in substance formulas by its own "atomic" symbol: a **D.** When both hydrogens in a water molecule contain a deuterium nucleus, its formula may be written as D_2O (instead of H_2O). Water that has a substantially higher percentage of deuterium than normal is termed **heavy water**.

An isotope of hydrogen consisting of one proton and *two* neutrons, called **tritium**, is not found in the earth's crust, but it can be produced in nuclear reactors. Unlike deuterium, tritium is radioactive: about half of the nuclei in a sample of tritium will decay in 12 years.

Nuclide Symbols

Each nuclide has a **mass number** which is the *sum* of its number of protons and neutrons.

$$\boxed{\textbf{Mass Number} \text{ of a nucleus } = \textbf{Protons + Neutrons}}$$

Example: All nuclei with 6 protons are carbon. If a carbon nucleus has 8 neutrons, the mass number of the carbon isotope is 14.

A nuclide can be identified in two ways,

- by its number of protons and number of neutrons, or
- by its **nuclide symbol** (also termed its **isotope symbol**).

The nuclide symbol for an atom has two required parts: the *atom symbol* and the *mass number.* The *mass number* is written as a $^{\text{super}}$script in front of the atom symbol.

Example: The three isotopes of hydrogen can be represented as

- 1 proton + no neutrons *or* as $^{1}\mathbf{H}$ (a nuclide named "hydrogen-1");
- 1 proton + 1 neutron *or* as $^{2}\mathbf{H}$ (termed hydrogen-2 or deuterium); and
- 1 proton + 2 neutrons *or* as $^{3}\mathbf{H}$ (called hydrogen-3 or tritium).

Uranium has two isotopes that are important commercially and historically.

- $^{238}\mathbf{U}$, the most common naturally occurring isotope, contains 92 protons and 146 neutrons.
- $^{235}\mathbf{U}$, the isotope that is *split* in atomic bombs and nuclear power plants, contains 92 protons and 143 neutrons.

Using a table of atoms that includes atomic numbers, try this question.

Q. A nuclide with 47 protons and 62 neutrons has what nuclide symbol?

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A. Atoms with 47 protons must be named silver, symbol **Ag**. The mass number of this nuclide is 47 protons + 62 neutrons = **109** . This isotope is called silver-109 and its symbol is $^{109}\mathbf{Ag}$.

Nuclide symbols may also be written with the **nuclear charge** below the mass number. This is called **A-Z notation**, illustrated for tritium at the right. **A** is the symbol for mass number and **Z** represents nuclear charge.

$$\boxed{^{3}_{1}\mathrm{H}}$$

Any nucleus that includes protons is by definition an atom, and since the atom symbol also identifies the number of protons in the nucleus, Z values are not required to identify a nucleus. However, for many subatomic particles, the nuclear charge is not the same as the number of protons, and showing the nuclear charge is helpful in clearly identifying these particles. In addition, for problems in which we must *balance* nuclear reactions, knowing the nuclear charge (Z) is necessary and showing Z is helpful.

Practice: First learn the rules above, then complete these problems.

1. The charge on a nucleus is determined by its number of _________________.
2. The mass number of a nucleus is determined by its number of _____________________.
3. Isotopes have the same number of ____________ but different numbers of ___________.
4. Of the three sub-atomic particles, the two with the highest mass are _________________ and _________________.
5. Write the nuclide (isotope) symbol for a single proton using A-Z notation.
6. Consulting a table of atoms or periodic table, fill in the blanks below.

Protons	Neutrons	Atomic Number	Mass Number	Nuclide Symbol
2	2			
	118	79		
82			206	
				^{242}Pu

ANSWERS

1. **Protons** 2. **Protons + Neutrons** 3. Same number **Protons**, different number of **Neutrons**

4. **Protons and Neutrons**. 5. $^{1}_{1}H$ (A particle with one proton is always given the symbol H.)

6.

Protons	Neutrons	Atomic Number	Mass Number	Nuclide Symbol
2	2	**2**	**4**	$\mathbf{^{4}He}$
79	118	79	**197**	$\mathbf{^{197}Au}$
82	**124**	**82**	206	$\mathbf{^{206}Pb}$
94	**148**	**94**	**242**	^{242}Pu

In writing the nuclide (isotope) symbols for atoms, the nuclear charge below the mass number is *optional*.

Lesson 39B: Radioactive Decay Reactions

Pretest: If you have previously studied nuclear reactions, try the problems in the practice set at the end of this lesson. If you can do those problems, you may skip this lesson.

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Stable Nuclei

Protons have a positive electrical charge. If there is more than one proton in a nucleus, the like charges of the protons repel and the nucleus will have a tendency to fly apart.

Neutrons have a zero electrical charge and they do not repel other types of particles or each other. Neutrons act as the "glue" of the nucleus: if the *right* number of neutrons is mixed with the protons, the repelling protons remain in the nucleus and the nucleus is stable.

For small nuclei, the neutron to proton ratio that results in a stable nucleus is about one to one. As the number of protons in nuclei increases, the number of neutrons needed to form a stable nucleus increases slightly faster: the n^0/p^+ ratio gradually increases.

For example:

- All stable helium nuclei have 2 protons and 2 neutrons.
- Chlorine has two stable nuclei: both have 17 protons, while one has 18 and the other 20 neutrons.
- All lead atoms have 82 protons. The four stable isotopes of lead have 122, 124, 125, or 126 neutrons.

However, forming a stable nucleus is more complex than just adding more neutrons as "glue" or being in the right range of ratios. Certain combinations of protons and neutrons are stable, but others are not.

Radioactive Decay

A nucleus that is **radioactive** is between stable and unstable: it will have a tendency to *gradually* expel particles until a stable neutron and proton combination is achieved. The process of expelling particles from the nucleus is termed **radioactive decay**. Depending on the nucleus, radioactive decay may occur on average in seconds or gradually for up to billions of years.

Radioactive decay can be a powerful tool in the study of chemical reactions.

For example, for most atoms with less than 84 protons, stable isotopes exist, but radioactive isotopes also exist: the radioactive nuclei can either be found in nature or synthesized in nuclear reactors. A radioactive atom will undergo *decay* that we can detect, but in chemical reactions the radioactive and non-radioactive forms of the atom have essentially the same behavior.

By substituting a radioactive nucleus for a stable nucleus, we can "tag" the atoms in substances. Because we can detect the location of radioactive nuclei as they decay, we can track how they behave during chemical reactions, including reactions in biological systems. The use of *radioactive dyes* in medical imaging is one example of the importance of nuclear chemistry.

There are several types of radioactive decay, but the two types that describe the decay of most radioactive nuclei are **alpha (α) decay** and **beta (β) decay**.

Alpha Decay

When a particle with 2 protons and 2 neutrons is ejected from a nucleus, it is termed an **alpha particle**. Because an alpha particle has the same structure as a helium-4 nucleus, it can be given the same isotopic symbol.

$$\alpha \text{ particle} = {}^{4}_{2}\text{He}$$

The process of ejecting an alpha particle from a nucleus is termed *alpha decay*.

Example:

The isotope U-238 undergoes radioactive decay by emitting an alpha particle. This nuclear reaction is written as

$${}^{238}_{92}\text{U} \xrightarrow{\alpha} {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

Alpha decay always lowers the atomic number (and nuclear charge) of a nucleus by 2 and its mass number by 4.

Balancing Nuclear Reactions

Nuclear reactions balance differently than chemical reactions, but they balance relatively easily. The rule is:

In nuclear reactions, mass numbers and nuclear charge must be conserved.

In a balanced *nuclear* reaction, on both sides of the arrow,

- The *sum* of the *mass numbers* (A, on top) must be the same, and
- The sum of the *nuclear charge*s (Z, on the bottom) must be the same.

The result is that nuclear reactions can be balanced by simple addition and subtraction.

Example: In the case of the alpha decay of U-238, the mass numbers on both sides of the reaction equation total 238 and the nuclear charges total 92.

$${}^{238}_{92}\text{U} \xrightarrow{\alpha} {}^{234}_{90}\text{Th} + {}^{4}_{2}\text{He}$$

Apply the nuclear balancing rule to this problem.

Q. Use the nuclear balancing rule to write below the symbol for the nucleus remaining after the alpha decay of plutonium-242.

$${}^{242}_{94}\text{Pu} \xrightarrow{\alpha} \underline{\hspace{3cm}} + {}^{4}_{2}\text{He}$$

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The nuclear charges on the bottom must total 94 on both sides. The nuclear charge on the missing particle must be 92 and the atom is uranium. The mass numbers on top add up to 242 on both sides, so the mass number of the missing nucleus must be 238. The balanced nuclear reaction is:

$$^{242}_{94}\text{Pu} \xrightarrow{\alpha} {}^{238}_{92}\text{U} + {}^{4}_{2}\text{He}$$

Try one more.

Q. Which isotope is produced by the alpha decay of radium-226?

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A key to balancing nuclear reactions is to write the isotope symbols in A-Z notation. Start there for Ra-226.

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Radium by definition has a nucleus with 88 protons, so this reaction begins

$^{226}_{88}\text{Ra} \xrightarrow{\alpha}$ Fill in the missing symbols.

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In alpha decay, one product is always an alpha particle. Add its symbol on the right.

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$$^{226}_{88}\text{Ra} \xrightarrow{\alpha} {}^{4}_{2}\text{He} +$$

Use the balancing rule to write the isotopic formula for the remaining particle: the nucleus left behind after the alpha particle is expelled.

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$$^{226}_{88}\text{Ra} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{222}_{86}\text{Rn}$$

After the decay, the nucleus has 86 protons, so it must be radon (Rn). For the mass numbers to balance, the Rn nucleus must have a mass number of 222.

Practice A

1. Write a balanced equation for the alpha decay of radon-219.
2. How many protons and how many neutrons are in ^{219}Rn??
3. How many protons and how many neutrons are in the nucleus left behind after the alpha decay of radon-219? How many protons and neutrons are lost in the decay?
4. Lead-206 can be formed by the alpha decay of which radioactive isotope?

Beta Decay

Beta decay is another type of radioactive decay. In beta decay, a neutron decays into a proton and an electron, and the electron is expelled from the nucleus at high speed. An electron formed in this manner is termed a **beta particle**.

- Because an electron has no protons and neutrons, its mass number is zero.
- Because an electron has a negative charge, when it is formed in the nucleus its "nuclear charge" is negative one.

In nuclear reaction equations, a beta particle can be represented in these ways:

$$\beta \text{ particle} = {}^{0}_{-1}\beta \quad or \quad {}^{0}_{-1}\text{e}$$

Atoms must have a positive nuclear charge, but on a *sub*atomic particle (one without protons, including neutrons and electrons), the charge may be may be +1, 0, or −1. In the special case of an electron formed in the nucleus by beta decay, briefly, before the electron is expelled, it is a nuclear particle with a negative one charge.

In beta decay, the number of neutrons in a nucleus *decreases* by one, but the number of protons increases by one, so the mass number of the isotope stays the same.

Example: The equation for the beta decay of the radioactive isotope lead-210 can be written as

$${}^{210}_{82}\text{Pb} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{210}_{83}\text{Bi}$$

Is the above equation balanced?

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Before and after the reaction, the mass numbers total 210 and the nuclear charges total 82, so this is a balanced nuclear equation.

Note that the process is decay, but that the nucleus remaining after decay gains a proton: its atomic *number* increases.

Using the rules for nuclear balancing, we can predict the structure and symbol for the products of beta decay. Apply the rules to this question.

Q. For the beta decay of carbon-14, write the isotope symbols for the two nuclear particles formed in the reaction.

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In nuclear balancing, begin by converting to A-Z notation: ${}^{14}_{6}\text{C} \xrightarrow{\beta}$

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One product must be a beta particle. For its symbol, you may use a β or an e .

${}^{14}_{6}\text{C} \xrightarrow{\beta} {}^{0}_{-1}\text{e} +$ Complete the balancing.

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$${}^{14}_{6}\text{C} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{14}_{7}\text{N}$$

The isotope formed by the beta decay of carbon-14 is nitrogen-14.

Practice B

1. From memory, write the symbols for an alpha particle and a beta particle.
2. Write balanced equations for these decay reactions.

a. $^{40}\text{K} \xrightarrow{\beta}$ b. $^{239}\text{Pu} \xrightarrow{\alpha}$

3. The isotope ^{131}I is used for the treatment of hyperthyroidism: a condition in which the thyroid gland produces too much thyroid hormone. In the body, iodine is absorbed by the thyroid gland. If iodine-131 is administered to a patient, its beta decay kills cells in the thyroid, resulting in a reduced level of thyroid hormone without surgery. Write the symbol for the nucleus produced by the beta decay of iodine-131.
4. Lead-206 can be produced by the beta decay of which nucleus?
5. In a part of what is termed a **radioactive decay series**, nucleus A with 88 protons and 140 neutrons can beta decay to form nucleus B. B can emit a high speed electron to form nucleus C, which can α decay to form nucleus D. Write isotopic symbols for A, B, C, and D.
6. Radioactive atom A with 82 protons and 132 neutrons emits a beta particle to become atom B. Atom B emits an alpha particle to become atom C, which can emit a high speed electron from the nucleus to form Atom D. Write the isotopic formulas for A, B, C, and D.

ANSWERS

Practice A

1. $^{219}_{86}\text{Rn} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{215}_{84}\text{Po}$ 2. **86 Protons** and **133 neutrons.**

3. **84 protons** and **131 neutrons: 2 protons and 2 neutrons** are always lost in alpha decay.

4. $\boxed{^{210}_{84}\text{Po}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{206}_{82}\text{Pb}$

Practice B

1. α particle = $^{4}_{2}\text{He}$; β particle = $^{0}_{-1}\beta$ *or* $^{0}_{-1}\text{e}$

2a. $^{40}_{19}\text{K} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{40}_{20}\text{Ca}$ 2b. $^{239}_{94}\text{Pu} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + {}^{235}_{92}\text{U}$

3. $^{131}_{53}\text{I} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \boxed{^{131}_{54}\text{Xe}}$ 4. $\boxed{^{206}_{81}\text{Tl}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{206}_{82}\text{Pb}$

5. $\underset{\textbf{A}}{\boxed{{}^{228}_{88}\text{Ra}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{B}}{\boxed{{}^{228}_{89}\text{Ac}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{C}}{\boxed{{}^{228}_{90}\text{Th}}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + \underset{\textbf{D}}{\boxed{{}^{224}_{88}\text{Ra}}}$

6. $\underset{\textbf{A}}{\boxed{{}^{214}_{82}\text{Pb}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{B}}{\boxed{{}^{214}_{83}\text{Bi}}} \xrightarrow{\alpha} {}^{4}_{2}\text{He} + \underset{\textbf{C}}{\boxed{{}^{210}_{81}\text{Tl}}} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + \underset{\textbf{D}}{\boxed{{}^{210}_{82}\text{Pb}}}$

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Lesson 39C: Fission and Fusion

Pretest: If you have previously studied fission and fusion, try the problems in the practice set at the end of this lesson. If you can do those problems, you may skip this lesson.

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Fission

Nuclear fission is a reaction in which a large nucleus divides into two smaller nuclei, both of which contain more than two protons. If a fission reaction is accompanied by the creation of free neutrons, those neutrons can collide with other nearby fissionable nuclei and cause them to split.

If this "splitting of atoms" begins in a sample of fissionable nuclei that is large enough to have **critical mass**, the result can be a **chain reaction** which releases large amounts of energy. If the chain reaction is not controlled, the result is an **atomic bomb**.

In a **nuclear power plant** (a type of **nuclear reactor),** a chain reaction is controlled by adding materials that absorb some of the free neutrons. The large amount of energy produced by a chain reaction is then released gradually, and the resulting heat can be harnessed to drive turbines that produce electricity.

One example of nuclear fission is the splitting of uranium-235. ^{235}U can fission in multiple ways, but a typical reaction is

$$ {}^{235}_{92}\text{U} + {}^{1}_{0}\text{n} \rightarrow {}^{236}_{92}\text{U} \rightarrow {}^{140}_{54}\text{Xe} + {}^{94}_{38}\text{Sr} + 2\ {}^{1}_{0}\text{n} + 2 \times 10^{10}\ \text{kJ/mol} $$

In this reaction, the U-235 nucleus is struck by a free neutron. The neutron is at first absorbed, but this unstable nucleus then splits into two smaller nuclei plus 2 free neutrons. Those two neutrons can collide with other fissionable nuclei to create a chain reaction.

Fission reactions produce amounts of energy per mole that are millions of times larger than that produced by chemical reactions such as the burning of fossil fuels, and processes that can produce energy at a controlled rate are valuable to society. However, a disadvantage of using fission for electricity generation is that the products include highly radioactive isotopes. Exposure to the radiation released by radioactive decay can cause cancer, and some of the waste products of fission remain significantly radioactive for thousands of years. A major issue in nuclear power generation is how to store the waste products so that they will not escape into the earth's biological environment.

Isotopic Separation

Both U-235 and U-238 can be split, but in practice only U-235 is an effective fuel for chain reactions. For use in nuclear power plants or weapons, naturally occurring uranium must **enriched**: meaning that the percentage of nuclei that are U-235 must be increased. In mined uranium ore, 99.3% of nuclei are U-238 and 0.7% are U-235. To generate nuclear power, uranium must be enriched to at least 3% U-235. For nuclear weapons, uranium must be enriched to at least 7%% U-235, and over 50 kilograms of this "weapons grade" uranium must be collected.

Since all isotopes, including U-235 and U-238, have the same tendency to react chemically, chemical reactions cannot effectively separate isotopes. However, particles with the lighter isotopes will be less dense, and they will on average move faster at a given temperature. Gas molecules containing U-235 atoms therefore diffuse slightly faster than those containing U-238, and gaseous diffusion is one method that is used to separate uranium isotopes. Because particles that are more dense tend to be moved toward the outside when spun in a circle at high speed, a centrifuge can also be employed to separate uranium isotopes.

Both gaseous diffusion and centrifugation of uranium-containing substances are slow and expensive processes. This makes it difficult (thankfully) to obtain the amount of highly enriched uranium needed to build a nuclear weapon.

Practice A

1. From memory, write the isotopic symbol for a free neutron.
2. Fill in the one missing isotopic symbol in this nuclear fission reaction.

$$ {}^{235}_{92}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{87}_{35}\mathrm{Br} + \qquad\qquad + 6\ {}^{1}_{0}\mathrm{n} $$

3. Fill in the missing isotopic symbol that is the first reactant in this fission equation.

$$ + {}^{1}_{0}\mathrm{n} \rightarrow {}^{91}_{38}\mathrm{Sr} + {}^{146}_{56}\mathrm{Ba} + 3\ {}^{1}_{0}\mathrm{n} $$

Fusion

Nuclear fusion is a reaction that combines two small nuclei to make a larger one. When a small nucleus such as helium is the product of fusion, this reaction produces large amounts of energy. An example of a fusion reaction is

$$ {}^{2}_{1}\mathrm{H} + {}^{3}_{1}\mathrm{H} \rightarrow {}^{4}_{2}\mathrm{He} + {}^{1}_{0}\mathrm{n} + 2 \times 10^{9}\ \mathrm{kJ/mol} $$

In this reaction, two hydrogen nuclei are fused, and one product is a heavier helium nucleus. Fusion is the reaction that produces energy in stars, including our sun, and in hydrogen bombs.

The primary reaction that causes stars to "burn" (release energy) is the conversion of hydrogen to helium. At the extremely high temperatures and pressures found in stars, lighter nuclei can fuse to form heavier nuclei, and those nuclei can undergo successive fusion reactions. After long periods of making heavier nuclei, some stars become unstable and explode. The atoms scattered into space from exploded stars can accumulate over time due to gravitational attraction to form new stars and planets. The atoms that coalesced to form our own planet billions of years ago are nuclei, or the decay products of nuclei, that were originally formed by fusion in a star.

When fusion combines hydrogen isotopes, the products are generally stable nuclei rather than long-lived radioactive isotopes. If the fusion of hydrogen could be slowed and contained in a nuclear reactor, the result could be energy generated without greenhouse gas production or radioactive waste. To produce the energy needed for our society, such fusion reactors could replace the burning of fossil fuels and fission-based nuclear power plants, both of which form products that can harm our environment. However, all of the nuclear reactors currently in use are based on nuclear fission. No way has yet been discovered to engineer a gradual release of the energy of nuclear fusion.

Summary: So far in this module, the rules you need *in memory* are

1. α particle = ${}^{4}_{2}\mathrm{He}$; β particle = ${}^{0}_{-1}\beta$ *or* ${}^{0}_{-1}\mathrm{e}$, neutron = ${}^{1}_{0}\mathrm{n}$
2. To balance nuclear reactions, use A-Z notation: mass number on top, nuclear charge on the bottom.
3. In nuclear reactions, the mass numbers and nuclear charges must be conserved: both must *add* to give the same number on both sides of the arrows.
4. Fission splits a nucleus, fusion combines nuclei.

As needed, design flashcards that assist in moving these rules into long-term memory.

Practice B

1. Briefly describe the difference between fission and fusion.
2. If a single nucleus is formed as the product of this reaction, write its isotope symbol.

   $${}^{2}_{1}\mathrm{H} + {}^{2}_{1}\mathrm{H} \rightarrow$$

3. In stars that are *red giants*, helium-4 can fuse with beryllium-8 to form a single nucleus. Write the equation for this reaction.
4. Assuming that *one* isotope symbol is missing from these equations, fill in the missing isotope, then write the name for this *type* of reaction.

Type:

a. ${}^{235}\mathrm{U} + {}^{1}_{0}\mathrm{n} \rightarrow {}^{92}\mathrm{Kr} + \qquad + 3\ {}^{1}_{0}\mathrm{n}$ ____________

b. ${}^{238}\mathrm{U} \rightarrow {}^{4}\mathrm{He} +$ ____________

c. $^{1}H + {}^{3}H \rightarrow$ ____________

d. $^{234}Th \rightarrow {}^{0}_{-1}e +$ ____________

ANSWERS

Practice A

1. $^{1}_{0}n$ 2. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{87}_{35}Br + \boxed{^{143}_{57}La} + 6\ {}^{1}_{0}n$

3. $\boxed{^{239}_{94}Pu} + {}^{1}_{0}n \rightarrow {}^{91}_{38}Sr + {}^{146}_{56}Ba + 3\ {}^{1}_{0}n$

Practice B

1. Fission splits a nucleus, fusion combines nuclei.

2. $^{2}_{1}H + {}^{2}_{1}H \rightarrow \boxed{^{4}_{2}He}$ 3. $^{4}_{2}He + {}^{8}_{4}Be \rightarrow {}^{12}_{6}C$

4a. $^{235}_{92}U + {}^{1}_{0}n \rightarrow {}^{92}_{36}Kr + \boxed{^{141}_{56}Ba} + 3\ {}^{1}_{0}n$ **fission**

4b. $^{238}_{92}U \rightarrow {}^{4}_{2}He + \boxed{^{234}_{90}Th}$ **alpha decay**

4c. $^{1}_{1}H + {}^{3}_{1}H \rightarrow \boxed{^{4}_{2}He}$ **fusion**

4d. $^{234}_{90}Th \rightarrow {}^{0}_{-1}e + \boxed{^{234}_{91}Pa}$ **beta decay**

* * * * *

Lesson 39D: Radioactive Half-Life

Timing: Complete this lesson when you are asked to solve radioactive half-life calculations or half-life calculations for first-order reactants.

Prerequisites: Before starting this lesson you must complete Lesson 9A (on fractions and percentages) and Lessons 27D and 27E (on base 10 and base ***e*** logarithm calculations). If you have not already done so, do those lessons now.

* * * * *

Radioactive Decay and First-Order Kinetics

In a reaction, if a reactant is used up at a rate that is proportional to its concentration, the rate of its reaction can be described by a *differential rate law* equation as

$$\text{Rate} = k[A]^{\mathbf{1}} = k[A]$$

The reactant is said be "first order" due to the exponent 1 in the rate equation.

If this first-order rate law is expressed in terms of time, the reaction rate can also be expressed by the following mathematically equivalent equation, termed the *integrated rate law* for a first-order reactant:

$$\ln[A]_t = -kt + \ln[A]_0$$

In the above equations, **t** represents time in any units, 0 is the time at the start of the measurements, and **k** is the symbol for the rate constant for the reaction.

Radioactive decay, a nuclear reaction, is always first-order. However, reactants in standard chemical reactions can also be first order as well. In this lesson, we will learn to solve half-life calculations for radioactive nuclei, but the same equations can be applied to standard chemical reactions that involve first-order reactants.

Half-Life

The half-life of any reactant (symbol $t_{1/2}$) is the time that is required for half of the particles of the reactant to be used up in a reaction. First-order reactants are a special case in which the half-life of the reactant is not dependent on its initial concentration. In chemical reactions, at a given temperature, a first-order half-life is constant.

In chemical reactions, the rate of reaction changes with temperature, and the half-life of a first-order reactant changes as well. However, in radioactive decay, a nuclear reaction, the time of a half-life does not change significantly at temperatures even over temperature changes of 1,000 K.

This means that each radioactive nucleus has a *characteristic* half-life. Some radioactive nuclides have a half-life of a few seconds; others have a half-life of billions of years. It is not possible to predict when any one nucleus will decay. However, in any sample of more than a few hundred of a given nucleus, the time in which *half* of the nuclei decay is always the same. If we can calculate (or look up) the characteristic half-life, we know how long it will take for half of the nuclei to decay. By solving half-life equations, we can also calculate how long it will take for any percentage of the nuclei in a sample to decay.

Half-Life Calculations For Simple Multiples

In calculations involving half-life, the two variables will generally be the *time* over which a given nuclide in a sample decays and the *percentage* of those nuclei that remain. If the time period for the decay is equal to either the half-life or a simple multiple of the half-life, calculations can be answered by mental arithmetic.

- If a sample of a given nucleus has decayed for a time equal to one half-life, 1/2 of the original nuclei have decayed and half remain.

  If the nuclei are in a sample that has a constant volume (which should be assumed unless other conditions are stated), 1/2 of the original *concentration* of the reactant remains after one half-life.

- After two half-lives (double the time of the half-life), the number of nuclei remaining is half *of* the half that remained after the first half-life: half of 1/2 = 1/4 (25%) of the original nuclei remain and 75% have decayed.
- At triple the half-life, 1/2 of 1/4 = 1/8 (12.5%) of the original nuclei remain.

In decay calculations, you must also distinguish between quantities *decayed* and *remaining*.

- 100% − percentage decayed = percentage remaining

  Example: If 25% remains, 75% has decayed.
- 1.000 − fraction decayed = fraction remaining

  Example: If the fraction decayed is 0.35, the fraction remaining is 0.65.

Apply the logic above to the following problem.

Q. Fluorine-18, a radioactive isotope used in nuclear medicine, has a 1.8 hour half-life. How long will it take for 87.5% of the F-18 nuclei in a sample to decay?

* * * * *

A. If 87.5% has decayed, 12.5% remains. How many half-lives are required? How much time would this be?

* * * * *

After two half-lives, 25% remains, so after three half-lives, 25% x 1/2 = 12.5% remains. 3 half-lives x 1.8 hours/half-life = **5.4 hours**

To solve radioactive decay calculations for these "simple multiple" cases, given the headings in the table below, you will need to be able to fill in the rest of the table from memory. This should not be difficult: note that in the two middle columns, each number is simply half of the one above.

For Radioactive Nuclei, at time =	Fraction Remaining	Percent Remaining	Percent Decayed
0	1	100%	0%
One half-life	1/2	50%	50%
Two half-lives	1/4	25%	75%
Three half-lives	1/8	12.5%	87.5%

For half-life calculations that are not easy multiples, we will use the logic of this chart to make *estimates* of answers.

Practice: Write the table above until, given the top row, you can fill in four rows below from memory. Then complete these problems.

1. If 90.% of a sample has decayed, what fraction remains?
2. If the fraction of a sample that has decayed is 0.40, what percent remains?
3. The nucleus of plutonium-239 undergoes radioactive decay with a half-life of 24,400 years. In a sample of constant volume containing ^{239}Pu,
   a. After how many years will 25% of the original Pu-239 nuclei remain?
   b. After how many half-lives will the [Pu-239] be 1/16th of its original concentration?
   c. What percentage of the Pu-239 has decayed after exactly 4 half-lives?

ANSWERS

1. If 90.% has decayed, 10.% remains, and fraction remaining = 10% / 100% = **0.10**
2. If fraction decayed = 0.40, percentage decayed = 0.40 x 100% = 40.%, and percentage remaining = 100% — 40.% = **60.%**

3a.. First-order half-life is constant. Half remains after one half-life, half of that half (25%) remains after two half-lives. Two half-lives = 2 x 24,400 years = **48,800 years.**

3b. Half remains after one half-life, 1/4th after two, 1/8th after three, 1/16th after **four** half-lives.

3c. Half remains after one half-life, 1/4th after two, 1/8th after three; 1/16th after **four** half-lives.
1/16 = 0.0625 = 6.25% remains, so **93.75% has decayed.**

* * * * *

Lesson 39E: Radioactive Half-Life Calculations

Rate Constants for Radioactive Decay

In half-life calculations that do not involve simple multiples, we can solve using rate equations and the math of natural logs.

Each radioactive nucleus has a rate constant for decay (***k***) that is characteristic: a value that is constant. Different radioactive nuclei will have different values for ***k*** .

One way to write the equation that predicts the decay rate for radioactive nuclei is

$$\ln\left[\frac{[A]_t}{[A]_0}\right] = -kt \quad \text{which can be abbreviated as} \quad \boxed{\ln(\textit{fraction remaining}) = -kt}$$

In the first equation, $[A]_t\,/\,[A]_0$ is the fraction of nuclei remaining.

For example: after one half-life, half (50%) of the original sample remains and the *fraction remaining* is **0.50**

After two radioactive half lives, the fraction remaining would be?

* * * * *

After two half-lives, the percentage remaining is 25% (1/4), and the fraction remaining is therefore **0.25**

Since the reactant is being used up over time, the value of [A] after time = **t** will be less than it was at time = **0**, and the value of the fraction will be less than one. This means that in decay calculations, the decimal equivalent of the fraction must have values between 0 and 1.00 (such as 0.25).

The units used to calculate the fraction can be concentration *or* any consistent units that are proportional to concentration, including the mass or number of particles in a sample that has a fixed volume.

Radioactive half-life calculations often involve fractions or percentages, and in those cases the form of the equation above that includes (*fraction remaining*) will be the most convenient to use.

Apply $\boxed{\ln(\textit{fraction remaining}) = -kt}$ to the following problem.

Q. For the decay of a radioactive isotope with a short half-life, if the value for its rate constant is $+ 0.00500\ s^{-1}$, what fraction of this isotope remains after 45 seconds?

* * * * *

A. To solve using an equation, make a data table listing the terms in the equation. To solve this equation for the *fraction remaining*, you will need to solve for **ln**(fraction remaining) first.

DATA**:**

ln(fraction remaining) = WANTED

$k = 0.00500\ s^{-1}$

$t = 45\ s$

SOLVE: $\textbf{ln(fraction remaining)} = -(0.00500\ s^{-1})(45\ s) = \mathbf{-0.225}$

From ln(fraction) = -0.225 , how can you calculate the fraction?

* * * * *

fraction remaining $= e^{\ln(\text{fraction})} = e^{-0.225} = \mathbf{0.80}$

As one *check* on your answer in a decay calculation, recall that a *fraction remaining* must always have a decimal equivalent value between 0 and +1, which 0.80 does.

<u>Solving the Rate Equation With Percentages</u>

In the equation $\boxed{\ln(\textit{fraction remaining}) = -kt}$,

the term *fraction remaining* is an abbreviation for the fraction $[A]_t / [A]_0$.

The units used to calculate this fraction can be units of concentration *or* any consistent units that are proportional to concentration, including the mass or number of particles in a sample that has a fixed volume.

However, to use the equation with the term *(*fraction remaining*),* you must calculate using fractions and not percentages. This means:

- If a percentage is WANTED, you will need to solve for the *fraction first*.
- If a percentage is *given*, you will need to convert to its decimal equivalent to use in the equation.

When using fractions and percentages, recall that

- A fraction can be expressed as x/y or as a decimal equivalent value that is x divided by y.
- Percentage = fraction x 100% *and* decimal equivalent = percent/100%

- Be careful as well to distinguish between quantities *decayed* and *remaining*.

  Example: If 55% remains, 45% has decayed.

(For practice in converting between percentages and fractions, see Lesson 9A.)

Practice A: Commit to memory the equation above that includes *(fraction remaining)*, the complete each of these.

1. If the value for **ln**(fraction of sample remaining) is -1.386 ,
   a. What is the fraction of the sample remaining?
   b. What percentage has decayed?
2. For the decay of a radioactive nucleus, if the rate constant of the reaction is $k = 0.04606\ \text{hours}^{-1}$, what percentage remains after 50.0 hours?
3. The plutonium-238 used in "nuclear power supplies" has a half-life of 87.7 years: at that time, 50.0% of the original nuclei remain. Calculate the rate constant for the decay of this isotope.

Half-Life Calculations For Non-Simple-Multiples

For a radioactive nucleus, after a time equal to one half-life ($t_{1/2}$), half of a sample has decayed and *half* remains. Substituting into the equation

$\boxed{\ln(\textit{fraction remaining}) = -kt}$ at a time equal to one half-life,

we can write $\boxed{\ln(1/2) = -k\, t_{1/2}}$ Solve this equation in symbols for half-life.

* * * * *

One way of several to write the equation is

$\boxed{t_{1/2} \equiv -\ln(1/2) / k}$ This equation is one way to *define* radioactive half-life.

In these equations, the rate constant (k) and half-life ($t_{1/2}$) are variables: their numeric values will differ for different radioactive nuclei. The term **ln(1/2)** is a constant: it can be converted to a numeric value.

Use your calculator to convert ln(1/2) to a fixed decimal number.

ln(1/2) = __________________

* * * * *

$\ln(1/2) = \ln(0.500) = \mathbf{-0.693}$

Substitute this numeric value into the equation above defining half-life, then simplify.

* * * * *

$t_{1/2} = -(-0.693)/k$ which simplifies to $\boxed{t_{1/2} \equiv 0.693/k}$

This last equation above is a form often listed in textbooks as a definition of radioactive half-life. From this form, it is clear that if you know the half-life, you can find the rate constant k, and if you know the rate constant you can find the half-life.

To solve decay calculations, we need equations that relate the fraction remaining, half-life, time, and rate constant. Several combinations of the equations above can be used, but the best equations are those that are easy to remember. In these lessons, we will follow this rule.

Radioactive Decay Prompt

If a *radioactive decay* or any first-order rate calculation includes or *half-life* and a *fraction* or *percentage* of a sample, and the answer cannot be calculated using simple multiples, write in the DATA:

$\boxed{\ln(\text{fraction } remaining) = -kt}$ and $\boxed{\ln(1/2) = -k\,t_{1/2}}$

Note that the second equation is simply a special case of the first: when the fraction remaining is 1/2, the time is equal to the half-life.

When using the radioactive decay prompt, we solve for the common variable k .

- First solve the equation that can be solved for k given the data provided, then
- Use that k to solve for the variable WANTED.

Commit the radioactive decay prompt to memory, then apply it to solve this problem.

Q. Iodine-131, a radioactive isotope used to treat thyroid disorders, has a half-life of 8.1 days. What percentage of an initial [I-131] remains after 48 hours?

* * * * *

WANT: Percent $[\text{I-131}]_{48\text{ hrs.}}$ = % remaining

DATA: 8.1 **days** = radioactive half-life = $t_{1/2}$

48 hours = 2.0 **days** = t (choose any *consistent* time unit)

Strategy: Write the equations that relate the symbols in the problem.

For radioactive decay calculations that include half-life and fraction or *percentage*, write and use

$\boxed{\ln(\text{fraction } remaining) = -kt}$ and $\boxed{\ln(1/2) = -k\,t_{1/2}}$

Percentage remaining = fraction remaining x 100%

If needed, adjust your work and solve from here.

* * * * *

The variable that links the prompt equations is k

Since we know the half-life, the second equation will find k

Knowing k and t, the first prompt equation will find **ln(**fraction remaining**)**.

Knowing **ln**(fraction remaining), the fraction remaining can be found using

Fraction = $e^{\ln(\text{fraction})}$ and Percentage = Fraction x 100%

Apply those steps and solve.

* * * * *

$$\boldsymbol{k} = \frac{-\ln(1/2)}{t_{1/2}} = \frac{-(-0.693)}{t_{1/2}} = \frac{+0.693}{8.1 \text{ days}} = \mathbf{0.0856\ day^{-1}}$$

$$\ln(\text{fraction remaining}) = -kt = -(0.0856 \text{ day}^{-1})(2.0 \text{ days}) = \mathbf{-0.171}$$

$$\text{Fraction} = e^{\ln(\text{fraction})} = e^{-0.171} = 0.843 = \boxed{\textbf{84 \%} \text{ of I-131 } \textit{remains} \text{ after 2 days}}$$

Practice B: Add the equations of the decay prompt to your flashcards and practice the cards for this chapter, then try the problems below. Save one problem for your next practice session.

1. The rate constant for the decay of the tritium isotope of hydrogen is 0.0562 years^{-1}. Calculate the half-life of tritium.

2. Strontium-90 is a radioactive nuclide found in **fallout**: dust particles in the cloud produced by the atmospheric testing of nuclear weapons. In chemical and biological systems, strontium behaves much like calcium. If dairy cattle consume crops exposed to dust or rain containing fallout, dairy products containing calcium will also contain ^{90}Sr. Similar to calcium, ^{90}Sr will be deposited in the bones of dairy product consumers, including children. In part for this reason, most (but not all) nations conducting nuclear tests signed a 1963 treaty which banned atmospheric testing.

   Strontium-90 undergoes beta decay with a half-life of 28.8 years. What percentage of an original [^{90}Sr] in bones will remain after 40.0 years?

   a. Estimate the answer.
   b. Calculate the answer.
   c. Write the equation for the beta decay of strontium-90.

3. The element Polonium was first isolated by Dr. Marie Sklodowska Curie and named for her native Poland. Radioactive ^{210}Po is found in significant concentrations in tobacco. If 20.0% of ^{210}Po remains in a sample after 321 days of alpha decay,

   a. Estimate the half-life of ^{210}Po.
   b. Calculate a precise half-life of ^{210}Po. Compare it to your *part a* estimate.

4. In a sample of radon-222, 10.0% remains after 12.6 days of alpha decay.
   - a. What is the composition of the radon-222 nucleus?
   - b. Write the decay reaction.
   - c. Estimate the half-life for ^{222}Rn.
   - d. Calculate a precise half-life of ^{222}Rn. Compare to your *part a* estimate.
5. If the half-life of carbon-14 is 5,730 years, what fraction of the original carbon-14 in a sample has decayed after 1650 years? Estimate, then calculate.

ANSWERS

Practice A

1a. WANT: fraction of sample remaining

DATA: ln(fraction remaining) = — 1.386

Knowing a value for **ln**(fraction remaining), to find fraction remaining, use

Fraction remaining = $e^{\ln(\text{fraction remaining})}$

* * * * *

= $e^{-1.386}$ = **0.250**

1b. If the fraction remaining is 0.250, the percentage remaining is 25.0% and the percentage decayed is **75.0%.**

2. WANT: % of sample remaining . To find percentage, find *fraction* first.

DATA: 0.04606 hours^{-1} **=** ***k***

50.0 hours = ***t***

Strategy: The equation that relates the three terms in the data is

$\boxed{\ln(\text{fraction remaining}) = -kt}$

Knowing **k** and **t**, **ln**(fraction remaining) and then fraction remaining can be calculated.

* * * * *

ln*(fraction remaining)* **= —*kt*** = — (0.04606 hours^{-1})(50.0 hours) **= — 2.303**

Fraction remaining = $e^{\ln(\text{fraction remaining})}$ = $e^{-2.303}$ = **0.100**

The percentage remaining is fraction x 100% = 0.100 x 100% = $\boxed{\textbf{10.0\%}}$

3. WANTED: *k* = the rate constant

Equation: ln(fraction remaining) = —*kt* (This is the equation we know that uses these variables.)

DATA: fraction remaining = 50.0 % = **0.500** (The percent must be converted to its fraction.)

t = 87.7 years

Solve the equation for the WANTED variable.

$$\boldsymbol{k} = \frac{-\ln(\text{fraction remaining})}{t} = \frac{-\ln(0.500)}{87.7\text{ years}} = \frac{+0.693}{87.7\text{ years}} = \mathbf{0.00790\ years^{-1}}$$

Practice B

1. WANTED: $t_{1/2}$ for tritium

DATA: $0.0562\text{ yr}^{-1} = k$

Strategy: In radioactive decay calculations that include half-life and fraction or percentage, write

$\ln(\text{fraction remaining}) = -kt$ and $\ln(1/2) = -k\,t_{1/2}$

In this problem, only the second equation is needed to relate the symbols in the WANTED and DATA.

SOLVE:

$$\boldsymbol{t_{1/2}} = \frac{\ln(1/2)}{-k} = \frac{-0.693}{-0.0562\text{ yr}^{-1}} = \mathbf{12.3\ years}$$

Answer unit math: $1/\text{yr}^{-1} = (\text{yr}^{-1})^{-1} = \text{yr}$

2a. WANTED: *Estimate* of % [Sr-90] remaining after 40 years. If one half-life is about 30 years and 50% remains, and two half-lives is about 60 years and 25% remains, then at 40 years, about…. 40% remains?

2b. WANTED: % [Sr-90] remaining at $t = 40.0$ years

Strategy: In radioactive decay calculations that include half-life and fraction or percentage, write

$\ln(\text{fraction remaining}) = -kt$ and $\ln(1/2) = -k\,t_{1/2}$

DATA: $28.8\text{ yr} = t_{1/2}$

$40.0\text{ yr} = t$

Percentage = Fraction x 100%

From half-life, *k* can be found. From *k* and *t*, ln(fraction) and then fraction can be found.

SOLVE: Starting from the equation that includes half -life: $\ln(1/2) = -k\,t_{1/2}$

$$\boldsymbol{k} = \frac{-\ln(1/2)}{t_{1/2}} = \frac{-(-0.693)}{t_{1/2}} = \frac{0.693}{28.8\text{ yr}} = \mathbf{0.02406\ yr^{-1}}$$

$$\mathbf{ln}(\text{fraction remaining}) = -kt = -(0.02406\text{ yr}^{-1})(40.0\text{ yr}) = \mathbf{-0.9624}$$

$$\mathbf{Fraction} = \boldsymbol{e}^{\ln(\text{fraction})} = \boldsymbol{e}^{-0.9624} = 0.382 = \boxed{\mathbf{38.2\ \%}\text{ Sr-90 remains after 40 years}}$$

Compare this to your estimate.

2c. $${}^{90}_{38}\text{Sr} \xrightarrow{\beta} {}^{0}_{-1}\text{e} + {}^{90}_{39}\text{Y}$$

3a. Estimate: We know that 20% remains after 321 days. We also know that 50% remains after one half-life, and 25% after 2 half -lives.

20% is close to 25%, and when 25% remains, it would be about 300 days, and 25% is 2 half-lives, so one half-life is **about 150 days**.

3b. WANTED: $t_{1/2}$

DATA: 20.0 % Po-210 remains

fraction remaining = 20% / 100% = 0.200

t = 321 days.

See radioactive decay, half-life and fraction or percentage? Write

$$\ln(\text{fraction remaining}) = -kt \quad \text{and} \quad \ln(1/2) = -k\,t_{1/2}$$

Percentage = Fraction x 100%

Knowing the fraction and t, k can be found from the first prompt equation. Half-life can then be found from the second equation. Solve for k in symbols first:

$$\boldsymbol{k} = \frac{-\ln(\text{fraction})}{t} = \frac{-\ln(0.200)}{321\text{ days}} = \frac{-(-1.61)}{321\text{ days}} = \mathbf{5.01 \times 10^{-3}\ days^{-1}}$$

$$\boldsymbol{t_{1/2}} = \frac{-\ln(1/2)}{k} = \frac{+0.693}{5.01 \times 10^{-3}\text{ days}^{-1}} = \boxed{\mathbf{138\ days} = \text{half-life of Po-210}}$$

Is this answer close to the estimate in *part a*?

4a. A radon-222 nucleus has **86 protons** and **136 neutrons**.

4b. $^{222}_{86}\text{Rn} \rightarrow {}^{4}_{2}\text{He} + {}^{218}_{84}\text{Po}$

4c. Estimate: 10% remains after 12.6 days. 12.5% remains after three half –lives, which is close to 10%.

If three half-lives is about 12 days, then one half-life would be **about 4 days.**

4d. WANTED: $t_{1/2}$

DATA: 90.0% Rn-222 has decayed, so 10.0% remains

t = 12.6 days

$$\ln(\text{fraction remaining}) = -kt \quad \text{and} \quad \ln(1/2) = -k\,t_{1/2}$$

Fraction remaining = 10% / 100% = **0.100**

SOLVE: Knowing the fraction and t, k can be found from the first equation. Half-life can then be found from the second equation.

$$\boldsymbol{k} = \frac{-\ln(\text{fraction})}{t} = \frac{-\ln(0.100)}{12.6\text{ days}} = \frac{-(-2.30)}{12.6\text{ days}} = \mathbf{0.183\ days^{-1}}$$

$$\boldsymbol{t_{1/2}} = \frac{-\ln(1/2)}{k} = \frac{+0.693}{0.183\text{ days}^{-1}} = \boxed{\mathbf{3.79\ days} = \text{half-life of Rn-222}}$$

Is this close to your estimate in *part a*?

5. Estimate: 0.50 is the fraction decayed after about 6,000 years, so **about 0.15** is the fraction decayed in about a third of that time?

Calculate:

WANTED: fraction [C-14] decayed = 1.000 — fraction remaining

DATA: $t_{1/2}$ = 5,730 years

t = 1,650 years

In radioactive decay calculations that include half-life and fraction or percentage, write

$$\ln(\text{fraction remaining}) = -kt \quad \text{and} \quad \ln(1/2) = -k\,t_{1/2}$$

From half-life, *k* can be found. From *k* and *t*, ln(fraction) and then fraction can be found.

SOLVE: $$\boldsymbol{k} = \frac{-\ln(1/2)}{t_{1/2}} = \frac{-(-0.693)}{t_{1/2}} = \frac{+0.693}{5730\text{ yr}} = \mathbf{1.21 \times 10^{-4}\ yr^{-1}}$$

$$\mathbf{ln}(\text{fraction remaining}) = -kt = -(1.21 \times 10^{-4}\text{ yr}^{-1})(1{,}650\text{ yr}) = \mathbf{-0.1996}$$

$$\mathbf{Fraction}\text{ remaining} = \mathbf{e}^{\ln(\text{fraction remaining})} = e^{-0.1996} = \mathbf{0.819}$$

If 0.819 = fraction remaining, 1.000 — 0.819 = $\boxed{\mathbf{0.181} = \text{fraction C-14 } \textit{decayed}}$

Compare to your estimate.

* * * * *

Summary: Nuclear Chemistry

1. α particle = ${}^{4}_{2}\mathrm{He}$; β particle = ${}^{\;0}_{-1}\beta$ *or* ${}^{\;0}_{-1}\mathrm{e}$, neutron = ${}^{1}_{0}\mathrm{n}$
2. To balance nuclear reactions,
   a. Always use A-Z notation: mass number on top, nuclear charge on the bottom.
   b. Both mass numbers and nuclear charge must be conserved; each must *add* to give the same number before and after the reaction.
3. Fission splits a nucleus, fusion combines nuclei.
4. In the decay of radioactive isotopes, the half-life is constant.
   - At the half-life, 1/2 of the original number of nuclei remain;
   - At double the time of the half-life, 1/4 of the original nuclei remain;
   - At triple the half-life, 1/8 of the original nuclei remain.
5. <u>**Radioactive Decay Prompt**</u>

   If a *radioactive decay* calculation includes *half-life* and a *fraction* or *percentage* of a sample, and the answer cannot be calculated using simple multiples,

   write in the DATA: $\boxed{\mathbf{ln(fraction}\ \textit{remaining}\mathbf{) = -kt}}$ and $\boxed{\ln(\mathbf{1/2}) = \mathbf{-k\ t_{1/2}}}$

   and use the math of natural logs to solve.

6. When using the radioactive decay prompt,

   - To use the equation with *(fraction* remaining*),* you must work in fractions, not percentages.
   - Percentage = fraction x 100%
   - Percentage remaining = 100% − percentage decayed
   - Fraction remaining = 1.000 − fraction decayed
   - The fractions in decay calculations must have a value between 1.00 and 0 (such as 0.25).

# # # # #

* * * * *

NOTE on the Table of Atoms

The atomic masses in this Table of Atoms use fewer significant figures than most similar tables in college textbooks. By "keeping the numbers simple," it is hoped that you will use "mental arithmetic" to do easy numeric cancellations and simplifications before you use a calculator for arithmetic.

Many calculations in these lessons have been set up so that you should not need a calculator at all to solve, if you look for *easy cancellations* first.

After any use of a calculator, use mental arithmetic and simple cancellations to *estimate* the answer, in order to catch errors in calculator use.

# # # # #

The ATOMS –

The **third** column shows the atomic number: The **protons** in the nucleus of the atom.

The **fourth** column is the molar mass, in **grams/mole**. For radioactive atoms, **( )** is the molar mass of most stable isotope.

Actinium	Ac	89	(227)
Aluminum	Al	13	27.0
Americium	Am	95	(243)
Antimony	Sb	51	121.8
Argon	Ar	18	40.0
Arsenic	As	33	74.9
Astatine	At	84	(210)
Barium	Ba	56	137.3
Berkelium	Bk	97	(247)
Beryllium	Be	4	9.01
Bismuth	Bi	83	209.0
Boron	B	5	10.8
Bromine	Br	35	79.9
Cadmium	Cd	48	112.4
Calcium	Ca	20	40.1
Californium	Cf	98	(249)
Carbon	C	6	12.0
Cerium	Ce	58	140.1
Cesium	Cs	55	132.9
Chlorine	Cl	17	35.5
Chromium	Cr	24	52.0
Cobalt	Co	27	58.9
Copper	Cu	29	63.5
Curium	Cm	96	(247)
Dysprosium	Dy	66	162.5
Erbium	Er	68	167.3
Europium	Eu	63	152.0
Fermium	Fm	100	(253)
Fluorine	F	9	19.0
Francium	Fr	87	(223)
Gadolinium	Gd	64	157.3
Gallium	Ga	31	69.7
Germanium	Ge	32	72.6
Gold	Au	79	197.0
Hafnium	Hf	72	178.5
Helium	He	2	4.00
Holmium	Ho	67	164.9
Hydrogen	H	1	1.008
Indium	In	49	114.8
Iodine	I	53	126.9
Iridium	Ir	77	192.2
Iron	Fe	26	55.8
Krypton	Kr	36	83.8
Lanthanum	La	57	138.9
Lawrencium	Lr	103	(257)
Lead	Pb	82	207.2
Lithium	Li	3	6.94
Lutetium	Lu	71	175.0
Magnesium	Mg	12	24.3
Manganese	Mn	25	54.9
Mendelevium	Md	101	(256)
Mercury	Hg	80	200.6
Molybdenum	Mo	42	95.9
Neodymium	Nd	60	144.2
Neon	Ne	10	20.2
Neptunium	Np	93	(237)
Nickel	Ni	28	58.7
Niobium	Nb	41	92.9
Nitrogen	N	7	14.0
Nobelium	No	102	(253)
Osmium	Os	76	190.2
Oxygen	O	8	16.0
Palladium	Pd	46	106.4
Phosphorus	P	15	31.0
Platinum	Pt	78	195.1
Plutonium	Pu	94	(242)
Polonium	Po	84	(209)
Potassium	K	19	39.1
Praseodymium	Pr	59	140.9
Promethium	Pm	61	(145)
Protactinium	Pa	91	(231)
Radium	Ra	88	(226)
Radon	Rn	86	(222)
Rhenium	Re	75	186.2
Rhodium	Rh	45	102.9
Rubidium	Rb	37	85.5
Ruthenium	Ru	44	101.1
Samarium	Sm	62	150.4
Scandium	Sc	21	45.0
Selenium	Se	34	79.0
Silicon	Si	14	28.1
Silver	Ag	47	107.9
Sodium	Na	11	23.0
Strontium	Sr	38	87.6
Sulfur	S	16	32.1
Tantalum	Ta	73	180.9
Technetium	Tc	43	(98)
Tellurium	Te	52	127.6
Terbium	Tb	65	158.9
Thallium	Tl	81	204.4
Thorium	Th	90	232.0
Thulium	Tm	69	168.9
Tin	Sn	50	118.7
Titanium	Ti	22	47.9
Tungsten	W	74	183.8
Uranium	U	92	238.0
Vanadium	V	23	50.9
Xenon	Xe	54	131.3
Ytterbium	Yb	70	173.0
Yttrium	Y	39	88.9
Zinc	Zn	30	65.4
Zirconium	Zr	40	91.2