Calculations in Chemistry

Modules 19 and above have been re-numbered.

Module 19 – Graphing is now Module 20.

Module 20 – Spectra is now Module 21

If you are looking for Spectra topics, check Module 21

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Module 20 — Graphing

Module 20 — Graphing

Timing: Begin this module when you are asked to make a line graph of data as part of a lab report or other assignment.

Prerequisites: It will be helpful to do Lessons 17A, 18A and 18B prior to this module.

Provisions: You will need a pencil, eraser, and about 5-10 sheets of graph paper (quarter-inch grid preferred), but you can begin the lessons without graph paper.

Pretests: If you feel confident about your graphing ability, try the last graph in the problem set at the end of each *lesson*. If you can do the last graph easily, you should not need to do the others. If the last graph is difficult, complete the lesson.

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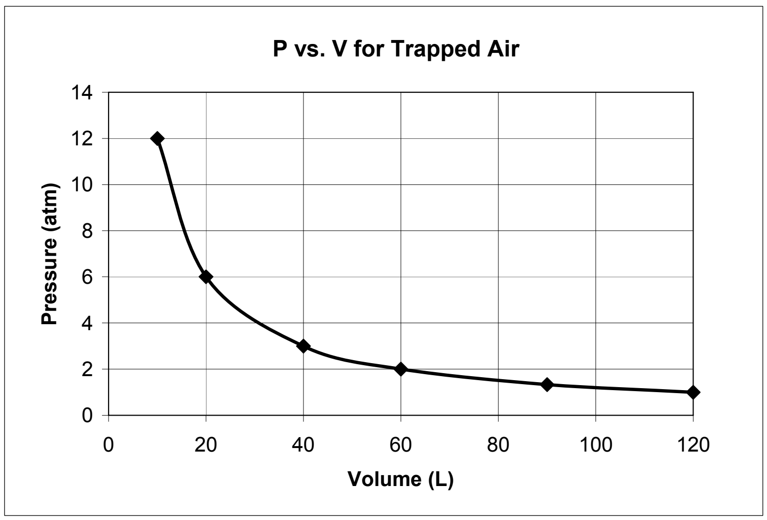
Lesson 20A: Graphing Fundamentals

Graphs

A **graph** is a way to display numbers visually.



There are many types of graphs, including bar graphs, pie charts, and histograms. In these lessons, our interest will be limited to **line graphs:** a type of graph often used to display experimental results. An example of a line graph is at the right.

Computer software can also create graphs. However, in order to create a proper software graph, it is important to be able to do the basic graphing operations without a computer.

Graphing Exercises

In the following lessons you will "learn by doing" several types of line graphs. The exercises will proceed from relatively easy to difficult. The early examples can be solved in easier steps, but the steps we practice on simple cases will make more complex and computer graphs easier. Please try the rules suggested here.

Tip #1: Graph In Pencil

When making a graph by hand, use a pencil and eraser. Simple or rough graphs may be sketched in ink, but complex graphs may require draft numbers that are later erased.

Graphing Numbers Near 0,0

In science experiments that study two variables at a time, the data are generally a series of *pairs* of numbers and their units. A relationship between two variables can often be determined from a graph of the points in two dimensions.

In such experiments, two measurements are recorded after each change in conditions. To simplify our initial study, we will begin with "math graphs" of numbers without units. Table 1 at the right lists an example of these data points.

Table 1

Data Points	
x	y
20	297
10	177
4	105
–3.5	15
8	153

Our focus will be two-dimensional graphs in **Cartesian coordinates** with a horizontal ***x-scale*** and a vertical ***y*-scale**. The ***x-axis*** is the heavy line perpendicular to the *y*-scale at $y = 0$. The ***y*-axis** is drawn perpendicular to the *x*-scale at $x = 0$. Graphs with *x* and *y*-scales may or may not include the *x*- and/or *y*-axis.

In science, data for related variables usually plot as points that fall close to *smooth curves* or *straight lines*. Based on the shape and characteristics of the graph, our goal is to develop an *equation* that describes the relationship between the two variables.

To graph data, use the following steps. Complete these steps using the data in Table 1.

1. **Decide which column will be plotted on the *x*-scale.** For this data, that's been done.
2. **Fill-in a *range chart* for each scale.** In Cartesian coordinates, each scale is
   - *numbered* to start lower than the lowest data value on that scale and end higher than the highest value, and
   - *evenly numbered* so that the lines along each scale increase by the same value.

   To number the scales, begin by filling in the *range chart* below. Use the data in Table 1. The lowest value in the *x* column goes in the first blank.

   Range Chart:

   x-scale: Low #: __________ High #: __________ Minor unit: ___ Major: ____

   y-scale: Low #:___________ High #: __________ Minor unit: ___ Major: ____

3. **Consider adding zero to each range** if the range does not include 0.

   Some graphing exercises will specify a range to use on the scales. If not, you will need to decide if the *origin* (0,0) should be shown on the graph. On most graphs, the answer will be *yes*. Including (0,0) will help you to recognize the type of equation that the data represents.

   For a range does *not* include zero, to include zero in the range, change one of the numbers in the range to *zero*. Change the number that *increases* the range.

   The number changed to zero will *usually* be the lower number (but if both of the range numbers are negative, the higher value is changed to zero).

For this current graph, include zero on both scales. Apply the rule above to the numbers in the range chart, then check your answer below.

* * * * * (← means *cover* below the * * * * *, do the work *above*, *then* check below.)

<u>*x*-scale</u>: Low #: **—3.5** High #: **20** Minor Unit: ____ Major: ____

<u>*y*-scale</u>: Low #: ~~**15**~~ **0** High #: **297** Minor Unit: ____ Major: ____

The *x*-scale range already includes 0, so no change is needed. For the *y*-scale, changing the 15 to 0 increases the range of the numbers on the scale.

4. **Mark the boundaries of the plot area** on your graph paper.

   Decide how much space will be used to plot your points.

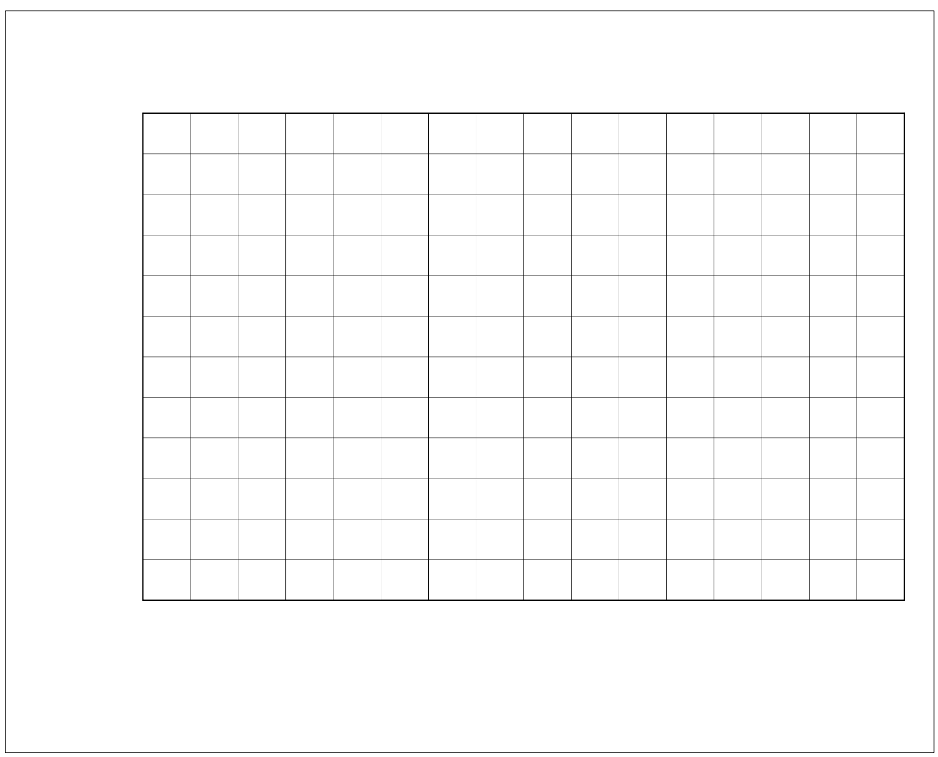
   In science, graphs must include a *title*, the *numbers* on each scale, and a *label* for each axis. We will include room for those science labels on this math graph even though the labels of the two current columns are simply *x* and *y*.

   Graphs may be done in the portrait or landscape mode. Using the steps for graphing suggested here, data can be plotted to use all of a sheet of graph paper, or a half or quarter sheet.

   How large to make a graph depends on how it will be used. A small graph is useful for a quick check of relationships. Using an entire sheet of paper is better for calculating an accurate slope of a line, or if you want to include zero on a scale but the data is clustered away from zero.

For this problem, *either* graph on the sample layout provided at the right, *or* on a copy of this page, *or* use a half-sheet of your own graph paper.

If using your own paper, try to use the same number of squares (12 up and 16 across) as at the right.



Lightly sketch the borders of the box of gridlines which you will plot the points inside. Leave room at the top, bottom, and left for the title, numbers and scale labels.

5. **Calculate the scale *minor unit.* Round UP.**

The **minor unit** is the *spacing number* for the scale. When numbering the scale, we increase the value of each line by this number. To number each scale so that all of the data will fit, use this equation.

$$\textit{Scale minor unit} = \frac{\text{(High \# on scale) minus (Low \# on scale)}}{\text{The count of the grid lines on the scale } \mathbf{-2}}$$

Then ***round UP*** to the next *easy* number to *count by* and *divide* into pieces.

The **—2** is a factor that helps to assure that the data will fit if we *write* the numbers on every *other* line along the scale. To write numbers less often, this factor would need to have a larger number after the negative sign.

What does the *round up to the next easy number* rule mean?

Counting by 2's means 2, 4, 6, 8, 10, ….

1's, 2's, 4's, 5's, 10's, 20's, 50's, and 100's are easy to count by *and* divide into parts. You could count by 3's, but in graphing, we need to be able to plot values *between* numbered lines. Estimating where 4.8 falls on a line between 4 and 6 is probably easier than between 3 and 6.

On occasion, the division in the equation above results exactly in a round number that is easy to count by and count between. If that is the case, *use* that round number as the minor unit. If it does *not*, round up.

For the current graph, on your paper, calculate the *x*-scale minor unit. Use the equation above.

- In the numerator, enter the numbers from the *range chart* for the *x-scale*.
- In the denominator, count the number of lines on the scale in the box you sketched to plot in. Start with **0** on the bottom left, and count the lines on the graph paper that cross that scale, including the line at the far border.

<u>For example</u>, in the *sample layout* above, count of grid lines along the *x*-scale. (The *x*-scale is the horizontal scale ("the horizon is horizontal" = -------- .)

* * * * *

The count of the *x*-scale grid lines is 16.

Finish the math of the *x*-scale *minor unit* equation, then check your answer below.

* * * * *

$$x\text{-scale minor unit} = \frac{20 - (-3.5)}{16 \text{ grid lines } \mathbf{-2}} = \frac{23.5}{14} = 1.68 \quad \textit{Round up} \text{ to } \mathbf{2.}$$

Counting by 2's is easy, and plotting points between 0 and 2 is easy.

6a. **Decide the *major unit.***

The **major unit** is the value by which you increase the numbers that *show* on the scale. In these lessons, we will *write* numbers on the scale at every *other* line. This means the major unit will be *double* the minor unit.

In the *x*-scale *range* chart above, write the minor and major unit for the scale.

* * * * *

<u>*x-scale*</u>: Low #: −3.5 High #: 20 Minor: **2** Major: **4**

Now calculate the *y*-scale minor unit, then check your answer below.

* * * * *

$$y\text{-scale minor unit} = \frac{297 - 0}{12\text{ lines} - \mathbf{2}} = 29.7$$ *Round up* to **40.**

40 is familiar to count by. (You could round to 50, but for this example choose 40.) In the range chart above, for the *y*-scale, fill in the minor and the major unit.

6b. **Make both ranges *slightly wider* to be *evenly divisible* by the *major* unit.**

Change the numbers in the range chart using these rules.

a. *Do not* change a 0. Treat zero as evenly divisible by all numbers.

b. *Do not* change numbers that are already evenly divisible by the major unit for that scale.

c. Cross out numbers that are not zero and not evenly divisible by the *major* unit. Increase the range by substituting the *next* number that is *evenly divisible* by the major unit for that scale. Make the high number higher and the low number lower.

For this current graph, try that step for the *x-scale*, then check below.

* * * * *

<u>Range Chart</u>:

<u>*x-scale*</u>: Low #: ~~−3.5~~ **−4** High #: **20** Minor: **2** Major: **4**

The major unit for the *x-scale* is **4**. −3.5 is not evenly divisible by 4, so change −3.5 to the next number lower that is evenly divisible by **4.** Since the high value of 20 is already evenly divisible by **4**, no change is needed.

Try those steps above for the *y-scale*, then check below.

* * * * *

<u>*y-scale*</u>: Low #: ~~15~~ **0** High #: ~~297~~ **320** Minor: **40** Major: **80**

The minor unit on *y* is **40**; make the range wider and divisible by **80**. Do not change 0. Change 297 to the next higher number that is evenly divisible by 80, which is 320.

(Making the higher number divisible by the major unit may not change how scales are numbered in graphs done by hand, but it's good preparation for software graphing.)

7. **Number and label each scale.**

When graphing by hand, numbering the scales may involve some trial and error, but the following rules will minimize erasing.

- The *y*-**axis** is a line perpendicular to *x* = 0.
- The *x*-**axis** is a line perpendicular to *y* = 0.

If the *x*- or *y*-scale includes an axis, we number *next* to the axis. If not, we number on the left or bottom edge of the grid. Some graphs will have one or both axes in the middle of the plot. Some graphs will omit *x* = 0, *y* = 0, or both.

A relatively easy way to number the scales is

a. At the *bottom left* corner of the grid, start with the *lowest* number in the range for each scale. Lightly number the *x*-scale on the bottom edge, and the *y*-scale on the left edge.

b. If a scale includes zero, draw the *axis* line all the way across the graph perpendicular to that zero. Put the tick marks and numbers *below* an *x*-axis or scale, and to the *left* of a *y*-axis or scale. Then move the numbers to the axis, writing firm, final numbers with the same spacing as the light numbers. Erase the light numbers.

c. If a scale does not include an axis, simply replace the light numbers on the bottom or left edge for that scale with heavier final numbers.

d. For each line on the scale, increase the count by the scale *minor* unit, but write the count on every *other* line. The difference in value between the lines is the *minor* unit, and the difference in value between the numbered lines is the *major* unit.

These steps will be more clear with an example. Using the range chart for Table 1 and rules above, start with the *x*-scale. Number *either* the sample grid provided on a previous page *or* the graphing box on your own paper. Then check your answer below.

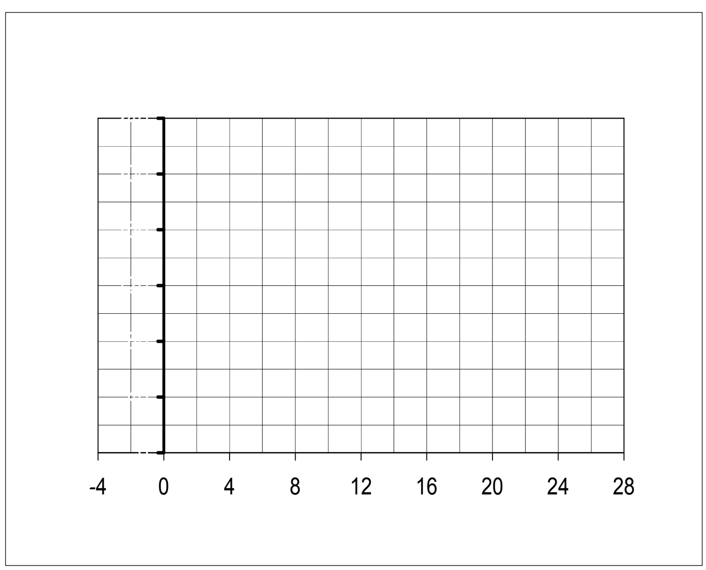
* * * * *

Since the *x*-scale includes zero, draw the heavy line perpendicular to zero.

Now number the *y*-scale. Since you already have the *y*-axis marked, write the numbers to the left of the axis, instead of on the left edge of the grid.

If the *y*-scale includes *y* = 0, draw the perpendicular heavy line for the *x*-axis. Replace the light *x*-scale numbers with final numbers below the axis.

Then, label each scale in words. Use the labels for the column plotted on the scale. Add a title to the graph. Base the graph title on the title of the data table for the graph.



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Your result should look like the graph at the right.

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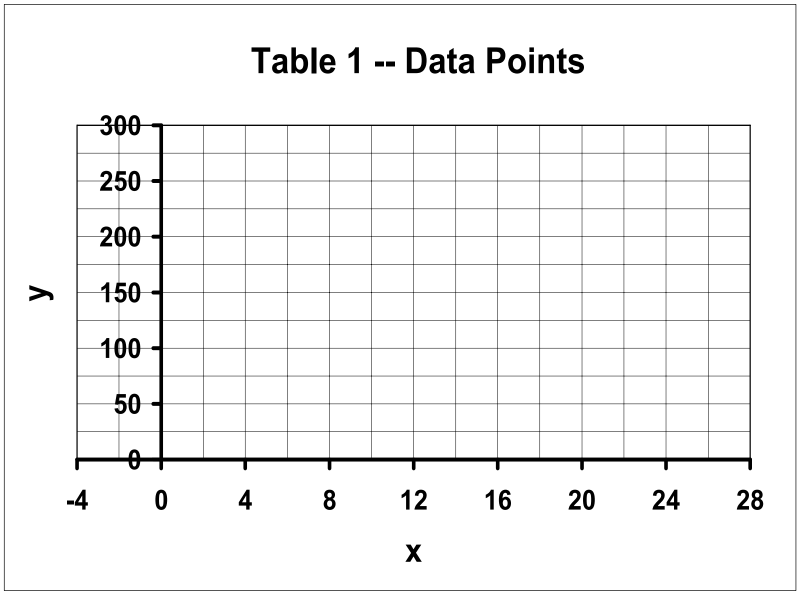


8. **Plot the points.** Use your own graph paper *or* the grid at the right.

   Using the numbers in Table 1, go out the *x*-scale by the number in the first column, then up by the number in the 2nd column, and make a dot. For visibility, either make the dot somewhat thick or circle it.

   Add a point for each row in the data table.

Do that step, and then check your answer below.

* * * * *

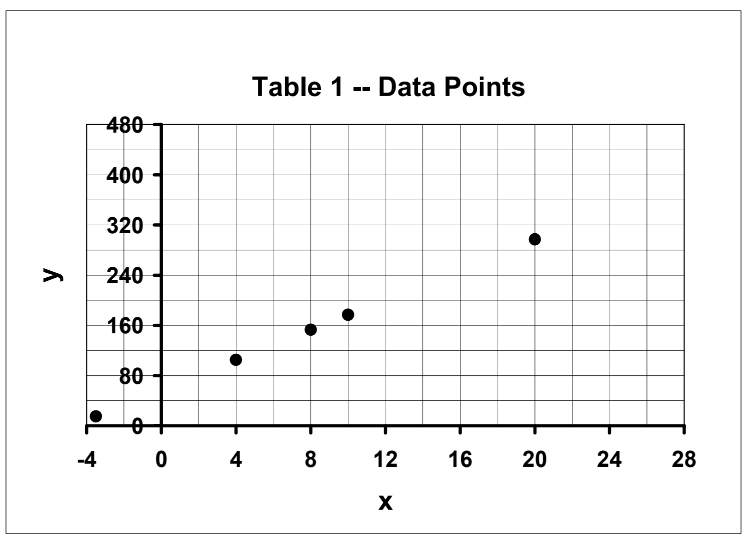
9. **Draw the function.**

   For natural phenomena, two related quantities will often graph to produce a *smooth curve* or a *straight line.*

   Add a line or curve to your graph that represents where you think all measurements for the two variables would fall if the data were without experimental error.

   In drawing the line or curve, do *not* change the data based on where the points plot. You may, however, want to check the plotting of points that are far off the line or curve.

   Do *not* "connect the dots." The function drawn should be a smooth curve or line that passes *close* to *most* of the points, through or between them, representing the *best fit* between the data and a smooth function.



   If the points plot close to a line, use a *straight edge* to add the best-fit line to the graph.

   Complete step 9, then check your answer in the next ***lesson***.

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Summary of Initial Rules: Graphing In Two Dimensions

1. **Decide which variable to plot on *x*.**
2. **Write the *range chart* for each scale.**

   x-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___

   y-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___
3. **Consider adding 0 to each range, increasing the range.**

   In most cases, if a range does *not* include zero, change one number in the range to *zero*. Change the number that *increases* the range.
4. **Mark the boundaries of the grid plot** on the graph paper.
5. **Calculate spacing number for each scale. Round UP.** Use this equation:

   $$\textit{Scale minor unit} = \frac{\text{(High \# on scale) minus (Low \# on scale)}}{\text{(The count of the grid lines on the scale)} - \mathbf{2}}$$

   and then *round UP* to the next *easy* number to count by and count between.
6. **Decide the major unit for each scale, then make both ranges slightly wider and evenly divisible by the major unit** for that scale. To number every second line on a scale, make the major unit double the minor unit. Add the minor and major units to the range chart.
7. **Number the scales or axes.** Label the scales. Title the graph.
8. **Plot the points.**
9. **Draw the function:** either a smooth curve or straight line that "best fits" the points.

Practice: For more practice in graphing, do these.

1. Graph the data in Table 2. Include zero on both scales.

   Use your own graph paper. Plot on a grid that includes 15 lines across the page and 8 lines up, counting from zero and including the lines on the far borders of the grid.
2. Graph the data in Table 3. Include zero on both scales.

   Use your own graph paper. Plot on a grid that includes 16 lines across the page and 10 lines up, counting from zero and including the lines on the far borders of the grid.

Table 3

Data Points	
x	y
100	0.030
225	0.0050
325	–0.015
475	–0.045

Table 2

Data Points	
x	y
1.5	4.1
4.2	5.2
6.3	6.0
2.7	4.6

ANSWERS

1. At Step 3, the range chart is

x-scale: Low #: ~~1.5~~ **0** High #: **6.3** Minor unit: ____ Major: ____

y-scale: Low #: ~~4.1~~ **0** High #: **6.0** Minor unit: ____ Major: ____

At Step 5:

$$x\text{-scale minor unit} = \frac{6.3 - 0}{15 \text{ lines on } x\text{-scale} - \mathbf{2}} = \frac{6.3}{13} = 0.48$$ **Round up to 0.5**

$$y\text{-scale minor unit} = \frac{6.0 - 0}{8 \text{ grids on } y\text{-scale} - \mathbf{2}} = 1$$ **1 is round. Use it.**

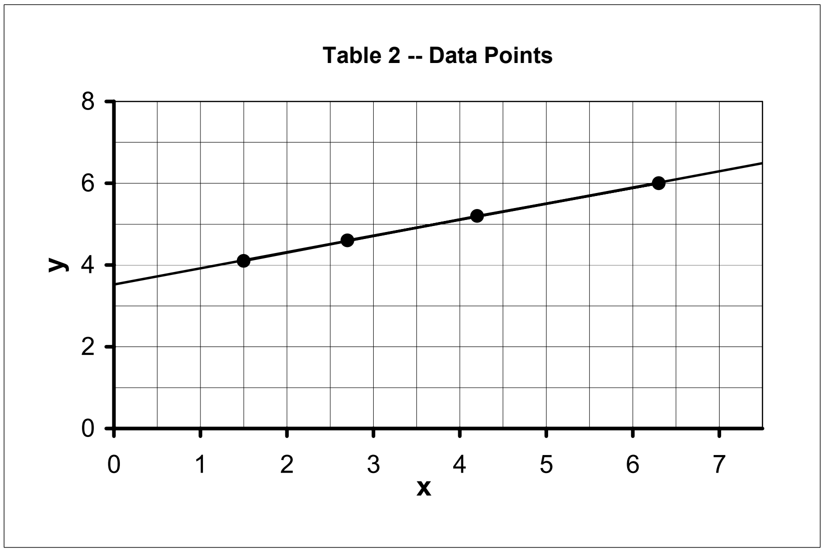
At Step 6, the range chart is

x-scale: Low #: ~~1.5~~ **0** High #: ~~6.3~~ **7** **Minor** unit: **0.5** **Major: 1**

y-scale: Low #: ~~4.1~~ **0** High #: **6.0** **Minor** unit: **1** **Major: 2**

If the major unit is double the minor unit, number every other line. Change each range number to increase the range, if needed, to be evenly divisible by the *major* unit. Zero is always evenly divisible.

For this graph, at Step 9, all points should fall exactly on a straight line.



2. At Step 3, the range chart is

x-scale: Low #: ~~100~~ **0** High #: **475** Minor unit: ____ Major: ____

y-scale: Low #: **—0.045** High #: **0.030** Minor unit: ____ Major: ____

The *y*-scale, in going from negative to positive, already includes zero.

* * * * *

Count the grid lines starting from 0. At Step 5, the spacing numbers are:

$$\boldsymbol{x}\text{-scale minor unit} = \frac{475 - 0}{16 \text{ lines } - \mathbf{2}} = \frac{475}{14} = 33.9$$ **Round up to 40**

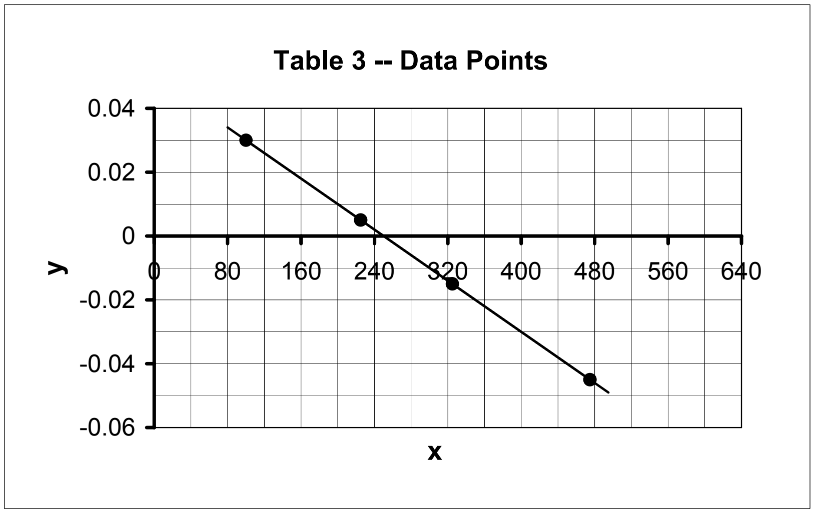
$$\boldsymbol{y}\text{-scale minor unit} = \frac{0.030 - (-0.045)}{10 \text{ lines } - \mathbf{2}} = \frac{0.075}{8} = 0.0094$$ **Round up to 0.01**

At Step 6, the range chart is

x-scale:	Low #: ~~100~~ **0**	High #: ~~475~~ **480**	**Minor** unit: **40**	**Major: 80**
y-scale:	Low #: ~~—0.045~~ **—0.060**	High #: ~~0.030~~ **0.040**	**Minor** unit: **0.01**	**Major: 0.02**

To write numbers on every other line, make the major unit is double the minor unit. Each number in a range is moved away from the other, if needed to be evenly divisible by the *major* unit. Zero is always evenly divisible.

At Step 9 for this graph, all points should fall very close to a straight line.



Lesson 20B: The Specific Equation For a Line

Timing: Do this lesson *if* you are asked to *derive an equation* from graphed data in which the data points fall close to a straight line.

Pretest: If you can do the last problem in this lesson, you may skip the lesson.

* * * * *

Graphs of Straight Lines

If two variables graph as a straight line, a specific equation relating the variables can be written using the **general equation for a line** (also called the **slope-intercept formula**):

$$y = \text{m}x + \text{b}$$

The equation $y = \text{m}x + \text{b}$ has two *variables* ($\boldsymbol{y}$ and $\boldsymbol{x}$) and two *constants* (**m** and **b**):

$\boldsymbol{y}$ = the value of the data point on the *y-scale*

$\boldsymbol{x}$ = the value of the data point on the *x-scale*

m = the **slope** of the line, and

b = the ***y*-intercept** = the value of y at $x = 0$ (where the line crosses the y-axis).

Calculating the Slope

On a straight line, the slope is the *same* between any two points. The slope can therefore be calculated between *any* two points. Each point is identified by its coordinates (x, y). The point with the *lower x* value is designated (x_1, y_1), and the other point (x_2, y_2).

The equation that defines slope must be memorized.

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

The symbol Δ means *change in*.

When an equation is needed to solve a problem, the steps are: Write the equation, make a data table using the *symbols* in the equation, then solve for the WANTED symbol.

Using those rules, solve the following problem and then check your answer below. Assume "math" numbers without units are exact; round if needed to 2 or 3 places.

Q. Calculate the slope between the points (2, 3) and (5, 18).

* * * * *

1. WANTED: **m** When a slope is needed, first write the equation for slope.

   Writing an equation each time you need it helps to set up your data table and helps to store the equation in your long-term memory.

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

2. DATA:

   m = ? x_1 = ________ y_1 = ________ x_2 = ________ y_2 = ________

   To solve for the slope, the point with the *lower x* value is designated (x_1, y_1), and the other point as (x_2, y_2).

   Identify x_1, then complete the data table above.

3. SOLVE: Substitute into the equation and solve for the WANTED unit.

If needed, finish those steps and then check your answer below.

* * * * *

Answer

WANTED: **m**

DATA:

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

m = ? x_1 = **2** y_1 = **3** x_2 = **5** y_2 = **18** (the lower x is x_1 = 2)

SOLVE: $\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{18 - 3}{5 - 2} = \frac{15}{3} = \boxed{5 = \mathbf{m}}$

The Slope of a Line on a Graph

A slope can be a positive number, negative number, zero, or infinity.

When a straight line on a graph is

- shaped **/**, the slope is a positive number;
- shaped ****, the slope is a negative number;
- shaped **—**, the slope is zero, and
- shaped **|**, the value of the slope is infinity (∞).

Work this example, then check your answer below.

Table 1	
Data Points	
x	***y***
20	297
10	177
4	105
–3.5	15
8	153

Q. In the previous lesson, for the data in Table 1 at the right, all points plotted were on a straight line.

a. Calculate the slope between the points in Table 1 that have the highest and lowest *x* values.

b. Calculate the slope between any other two points in Table 1.

* * * * *

WANTED: **m**

DATA:

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

a. **m = ?** lower $x = x_1 =$ **–3.5** $y_1 =$ **15** $x_2 =$ **20** $y_2 =$ **297**

SOLVE: $\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{297 - 15}{20 - (-3.5)} = \frac{282}{23.5} =$ **12 = m** for the Table 1 data

b. Between any other two points, the slope must be the same, allowing for uncertainty.

* * * * *

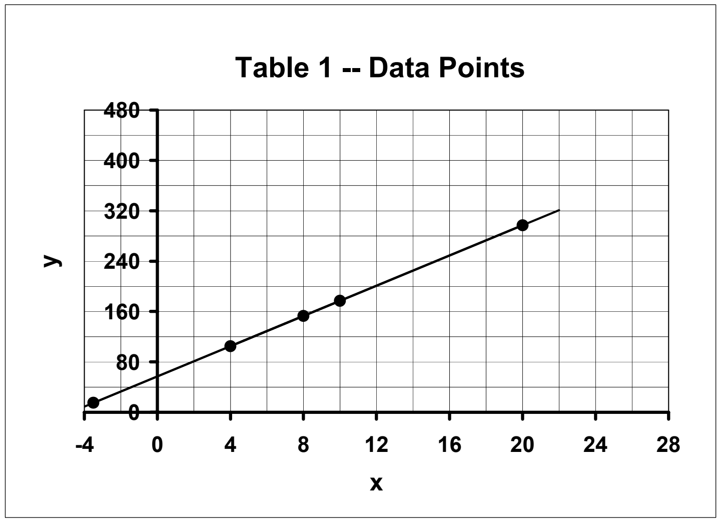
Calculating the *y*-Intercept

In the slope-intercept formula $y = \mathbf{m}x + \mathbf{b}$, the value of the *y*-intercept (**b**), a constant, can be found in two ways:

- By reading the graph and estimating the value of *y* at the point where the line crosses the *y*-axis, *or*
- By calculating the *y*-intercept using the *slope* value and the coordinates of any *one point* on the line.

At the right is a graph for the Table 1 data. Looking at the graph, estimate a value for the *y*-intercept.

b ≈ _______



The value of the *y*-intercept can also be calculated mathematically. The general equation for a line is

$y = \mathbf{m}x + \mathbf{b}$

The values for *y* and *x* will vary for each point on the line, but **m** and **b** are constants: they stay the same for all points on the line. If you know the values for *y* and *x* for any *two* points on the line, you can find the slope (**m**) of the line, as was done in the calculations above.

Once the value for the slope (**m**) of the line and the coordinates (***x,y***) for any *one* point on the line are known, you know values for three of the four symbols in ***y*** **= m*x*** + b. When values are known for all of the symbols in an equation except one, the missing value, in this case the *y*-intercept (**b**), can be found using algebra.

For the graph of the Table 1 data, all points fell exactly on the line. In that case, any points in the data can be used to calculate numeric values for **m** and **b**.

Use the slope for the Table 1 data that was calculated above, plus the data for the highest *x* value in the data, calculate a value for **b**, then check your answer below.

* * * * *

WANT: **b**

DATA: Writing the WANTED symbol helps to identify the possible equations that may solve the problem. At this point, we only know one equation that uses **b**. Write that equation, then make a data table using the symbols in the equation.

* * * * *

$\boxed{y = \mathbf{m}x + \mathbf{b}}$

m = **12** (solved above)

For the point with the highest *x* value in the Table 1 data,

x = 20 and ***y*** = 297 **b = ?**

Solve $\boxed{y = \mathbf{m}x + \mathbf{b}}$ for the WANTED symbol.

* * * * *

$$\mathbf{b} = \mathbf{?} = y - \mathbf{m}x = 297 - (12)(20) = 297 - 240 = \boxed{57 = \mathbf{b}}$$

Compare the calculated **b** value to your **b** value *estimated* from the graph above. Are they close? Which is likely to be the more accurate value?

The Specific Equation for the Line

The *general* equation for a line is $y = \mathbf{m}x + \mathbf{b}$.

The *specific equation* for a line is obtained by substituting into $y = \mathbf{m}x + \mathbf{b}$ the numeric values for the two constants. The *specific* equation for the line on the graph above is

$$y = \mathbf{12}x + \mathbf{57}$$

The specific equation can be used to calculate what the values would be at other points on the line. For any *y*, using the specific equation, you can calculate what *x* must be. For any *x*, you can calculate *y*.

In science, this is a key benefit of graphing data. Based on a data *sample* that you find by experiment, if you can find an equation that fits the data, you can predict results for other cases without having to test every case.

Testing the Equation

To be certain that you have solved for the specific equation correctly, test the equation: substitute into the specific equation for the line either the *x* or *y* value for a point on the line where both *x* and *y* are known. Then use the specific equation to calculate a value for the other variable, and compare this *equation* prediction to the known *data*.

Try it. Substitute $x = 4$ into the *specific* equation for the graphed line above. Calculate a value for *y*. Compare that answer to the *y* value for $x = 4$ in Table 1.

* * * * *

$$y = \mathbf{12}x + \mathbf{57} = 12(4) + 57 = 48 + 57 = \mathbf{105 = y}$$

That value matches the value for *y* in the table at $x = 4$.

Try another. Predict the value for *x* when $y = 15$.

* * * * *

$$y = \mathbf{12}x + \mathbf{57}\ ;\ \mathbf{15} = 12x + 57\ ;\ -42 = 12x\ ;\ \mathbf{x = -3.5}$$

This matches the *x* value at $y = 15$ in the data table.

Summary

1. The general equation for a line is $y = \mathbf{m}x + \mathbf{b}$, where **m** and **b** are constants. The slope of the line is **m**, **b** is the *y*-intercept, and *x* and *y* are variables that are the coordinates of the points on the line.

2. $$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

3. Substitute the *values* of the constants **m** and **b** into the general equation This gives the *specific* equation for the line. If any one of the *x* or *y* coordinates for a point on the line is substituted into the specific equation, the equation will predict the value for the other coordinate.

4. To find the *specific* equation for a line,

   a. Find the value of the constant slope (**m**) between any two points on the line.

   b. Find the x and y coordinates of any point on the line, substitute those values plus the value for **m** into $y = \mathbf{m}x + \mathbf{b}$, and solve for the constant value of **b**.

   c. Substitute the numeric values of **m** and **b** into the general equation $y = \text{m}x + \text{b}$.

Practice: On Problems 1 and 2, do every other part. On the rest, do the odd problems. Do the evens if you need more practice. Assume these "math" numbers are exact; round answers to 2 or 3 places.

1. Calculate the slope between these points. Try these without a calculator.

   a. (40., 20.) and (10., 30.) b. (20., 50.) and the origin.

2. Two points determine a line. Calculate the specific equation for the line that passes through the two points

   a. in Problem 1a. b. In Problem 1b.

3. If the slope of a line is —2 and one point on the line is (1, 7),

   a. calculate the y-intercept. b. Write the specific equation for the line.

   c. What is the value for y on the line at $x = 12$?

4. Look at the two graphs below labeled Problem 5 and Problem 6. In which case will the calculated slope be a negative number?

5. Try this problem without a calculator.

   For the graph at the right,

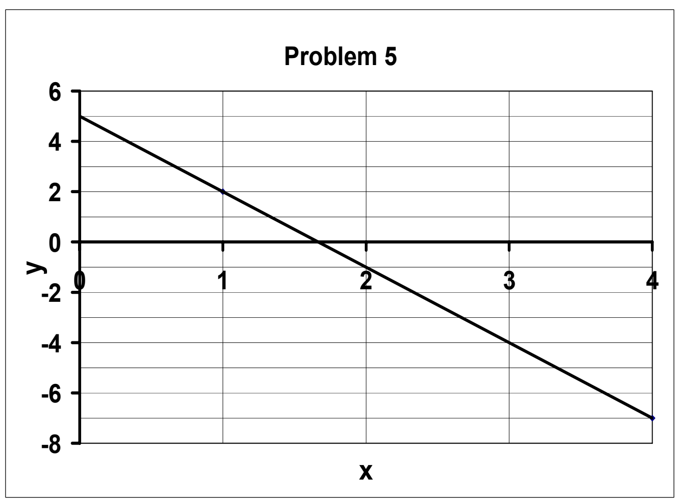
   a. Fill in this chart.

      $x_1 = 0$ $y_1 =$ _____

      $x_2 = 4$ $y_2 =$ _____

   b. Calculate the slope.

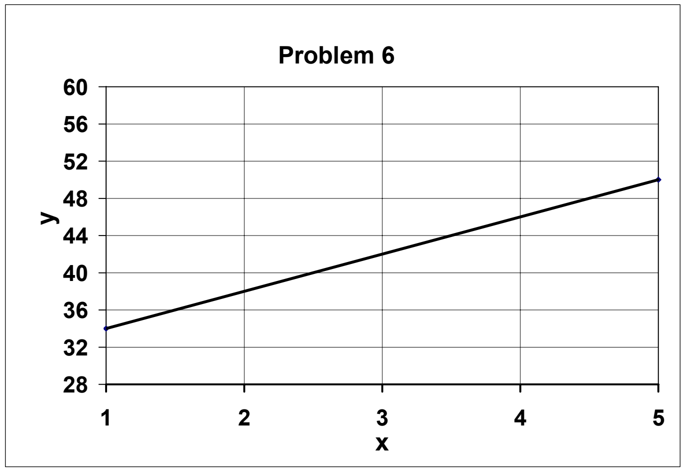
   c. Read the y-intercept.



   d. Write the specific equation for the line.

   e. On the line, at $x = 25$, $y = ?$

6. For the graph at the right,
   a. Calculate the slope.
   b. Calculate the y-intercept.
   c. Write the specific equation for the line.
   d. On this line, at $y = 78$, $x = ?$



7. The data in Table 2 at the right was graphed in the problems at the end of the previous lesson. The data graphed as a straight line, with each data point on the line.
   a. Calculate the slope for the line.
   b. Calculate the value of the y-intercept.
   c. Write the specific equation for the line.
   d. Test the equation by calculating a predicted value of x when $y = 5.2$. Compare that calculated value to the value for x at $y = 5.2$ in the data table.

Table 2

Data Points	
x	***y***
1.5	4.1
4.2	5.2
6.3	6.0
2.7	4.6

8. The data in Table 3 at the right was graphed in the problems at the end of the previous lesson. The data graphed as a straight line, with each data point very close to the line.
   a. Use the data in the table or the graph to calculate the specific equation for the line.
   b. Test the equation by calculating a predicted value of y when $x = 225$.

Table 3

Data Points	
x	***y***
100.	0.030
225	0.0050
325	–0.015
475	–0.045

ANSWERS

1. WANT **m**. The equation that uses m and data for *two* points is

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

1a. Define the lower x value as $\boldsymbol{x_1}$, so $x_1 = 10.$, $y_1 = 30.$, $x_2 = 40.$, $y_2 = 20.$

Plug those numbers into the equation for **m** and solve.

* * * * *

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{20. - 30.}{40. - 10.} = \frac{-10}{30} = \mathbf{-0.33}$$

1b. Define the lower x as $\boldsymbol{x_1}$, so $\boldsymbol{x_1} = 0$, $y_1 = 0$, $x_2 = 20.$, $y_2 = 50.$

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50. - 0}{20. - 0} = \mathbf{2.5}$$

2. To find the equation for a line, first calculate the slope (**m**), then use the coordinates of any point on the line plus the slope to find the y-intercept (**b**), using $y = mx + b$. To write the specific equation, substitute the values for m and b into $y = mx + b$.

   a. For problem 1a, the slope is **— 0.33** (see above).

   Two points were used to determine the line, so by definition both points are exactly on the line.

   $y = \mathbf{m}x + \mathbf{b}$ is the only equation we know that uses b.

   Any point (x,y) exactly on the line can be used to calculate **b**. If we use the first point,

   $y = 30.$ $\quad$ m = — 0.33 $\quad$ $x = 10.$ $\quad$ Solving in symbols first

   b = y — m x = 30 — (— 0.33(10)) = 30 + **3.3**

   b = 33.3 The specific equation for the line is $\boldsymbol{y = -0.33(x) + 33.3}$

   b. Only two points were used to determine the line, so by definition both points are exactly on the line.

   Any point exactly on the line can be used to calculate **b**. If we use the second point,

   $x = 20.$, $y = 50.$ m = 2.5 (see 1b).

   $y = mx + \mathbf{b}$

   50 = 2.5(20) + **b**

   50 = 50 + **b**

   b = 0 The specific equation for the line is $\boldsymbol{y = 2.5(x)}$

   The value for **b** can also be solved by inspection. The y-intercept is the value of y when $x = 0$.

   For any line that passes through the origin, **b = 0**.

3a. WANT: **b** The only equation we know that uses **b** is

DATA: $y = \mathbf{m}x + \mathbf{b}$

$\mathbf{m} = -2$, $x = 1$, $y = 7$.

SOLVE: $y = mx + \mathbf{b}$; $7 = -2(1) + \mathbf{b}$; $7 = -2 + b$; $\mathbf{b = 9}$

3b. WANT: specific equation for line. Use $y = \mathbf{m}x + \mathbf{b}$ with the values substituted for the constants **m** and **b.**

From 3a above, b = 9 **,** m = —2 , $\mathbf{y = -2(x) + 9}$

3c. WANT: ***y***

DATA: $x = 12$

SOLVE: from part b, $\mathbf{y = -2(x) + 9}$ = $-2(12) + 9$ = $-24 + 9$ = $\mathbf{-15 = y}$

4. **Problem 4.** If the data plots to give a **** slope, the slope (m) must have a **negative** value.

In the graph for problem 5, the / slope means a positive **m** value.

5a. $x_1 = 0$ $y_1 = \mathbf{5}$ $x_2 = 4$ $y_2 = \mathbf{-7}$

5b. $\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-7 - 5}{4 - 0} = \frac{-12}{4} = \mathbf{-3 = m}$

The slope is negative, consistent with the slant of the graphed line.

5c. The line crosses the *y*-axis at **+5** . $\mathbf{b = 5}$

5d. To get the specific equation, substitute the specific values for the constants **m** and **b** into $y = \mathbf{m}x + \mathbf{b}$

From above, m = —3 , b = + 5 **,** $\mathbf{y = -3(x) + 5}$

5e. The specific equation calculates values for points on the line.

On this line, at $x = 25$, **?** = $\mathbf{y = -3(x) + 5}$ = $-3(\mathbf{25}) + 5$ = $\mathbf{-70 = y}$

6a. WANT: **m** Write the equation for **m**, and a data table with the symbols needed to solve **m.**

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

DATA: If the lowest and highest *x* values on the graph are used,

m = ? $x_1 = 1$ $y_1 = 34$ $x_2 = 5$ $y_2 = 50$

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{50 - 34}{5 - 1} = \frac{16}{4} = \mathbf{4.0}$$

The slope is positive, consistent with the / slant of the graphed line.

6b. The graph does not show $x = 0$, so the y-intercept does not show on the graph and must be calculated.

WANTED: **b** Write the one equation that you know that uses b: $y = mx + b$.

DATA: b = ? At x = 1, y = 34 ; y = mx +b ; 34 = 4.0(1) + b ; **b = 30**

6c. WANT: specific equation for line. Use y = **m**x + **b** with the values substituted for the constants **m** and **b.**

From above, m = 4 , b = 30 , ***y* = 4*x* + 30**

6d. Substitute y = 78 into the specific equation: ***y* = 4*x* + 30** ; 78 = 4x + 30 ; 4x = 48 ; ***x* = 12**

7a. WANT: **m** Write the equation for **m**, and a data table with the symbols needed to solve **m.**

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

DATA: If we use the lowest and highest x values in the data table,

m = ? x_1 = 1.5 y_1 = 4.1 x_2 = 6.3 y_2 = 6.0

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.0 - 4.1}{6.3 - 1.5} = \frac{1.9}{4.8} = \mathbf{0.40}$$

The slope is positive, consistent with the slant of the graphed line.

7b. WANTED: **b** Write the one equation that you know that uses b: y = mx +b

DATA: b = ? , m = 0.40, If we use point x = 2.7, y = 4.6,

y = mx + **b**

4.6 = 0.40(2.7) + **b**

4.6 = 1.08 + **b**

b = 3.52 **= 3.5**

7c. To write the specific equation, substitute the calculated values for **m** and **b** into y = mx + b .

The specific equation for the line is **y = 0.40(x) + 3.5**

7d. Substitute y = 5.2 into the specific equation ***y* = 0.40(*x*) + 3.5** to find x .

5.2 = 0.40(x) + 3.5

1.7 = 0.40(x) ; x = 1.7/0.40 = 0.425 = **4.2 = *x*** This answer matches Table 2 for y = 5.2

8a. WANT: the *specific* equation for the line. Start from the general equation for a line: y = mx + b .

To find the specific equation,

- calculate the slope (**m**) between any two points on the line, then
- use the coordinates of any point on the line plus **m** to find the y-intercept (**b**) using y = **m**x + **b** ;
- substitute the values for **m** and **b** into y = **m**x + **b** .

All of the points plotted very close to the line, so any two points may be used. If we use the lowest and highest x values in the table,

x_1 = 100. , y_1 = 0.030 , x_2 = 475 , y_2 = − 0.045

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-0.045 - 0.030}{475 - 100.} = \frac{-0.075}{375} = \mathbf{-0.00020} = \boxed{\mathbf{-2.0 \times 10^{-4} = m}}$$

Note that the slope is negative, consistent with the \ slant of the graphed line.

Any point on the line can be used to calculate **b**. If we use the data point $x = 325$, $y = -0.015$,

$y = mx + \mathbf{b}$

$-0.015 = (-2.0 \times 10^{-4})(325) + \mathbf{b}$

$-0.015 = -0.065 + \mathbf{b}$

b = + 0.050 The specific equation for the line is $\boxed{\mathbf{y = -2.0 \times 10^{-4}(x) + 0.050}}$

8b. WANT: ***y*** Substitute $x = 225$ into $\mathbf{y = (-2.0 \times 10^{-4})(x) + 0.050}$

$\mathbf{y} = (-2.0 \times 10^{-4})(225) + 0.050 = -0.045 + 0.050 = \boxed{\mathbf{+0.005 = y}}$

This answer matches Table 3 for $x = 225$.

* * * * *

Lesson 20C: Graphing Experimental Data

Pretest: If you can do the last problem at the end of this lesson, you may skip the lesson.

* * * * *

Graphs in Science

Graphs of experimental data collected in laboratory experiments are similar to the "math" graphs studied in the previous two lessons, but graphs in science differ in some respects.

- Most math graphs are assumed to have exact numbers. In science, graphs of experimental data must allow for *uncertainty* in the data.
- Math graphs are usually done with numbers. Graphs in science are usually done using measurements: numbers with *units* and *labels*.

Graphs in science explore how the physical world works. When the rules for our environment can be expressed as equations, predictions about the behavior of natural phenomena become easier, and the results can be applied to improve the quality of our lives.

Studies of Two Variables

In science, to investigate the relationships among a number of variables, one strategy is to design an experiment that holds all but two variables *constant*. For the two remaining variables, we measurably change one variable and measure what happens to the other.

The two variables studied in experiments can often be classified as dependent and independent.

- An **independent** variable is one that is changed in a controlled way or is measured at a regular interval.
- A **dependent** variable responds to the change in the independent variable.

For example, in experiments we often measure a variable at specific points of time. Time is then the *independent* variable: we control the interval at which we record measurements.

However, for an experiment in which we change a solution concentration by regular amounts, then measure the time a reaction takes, time is the *dependent* variable. Which quantities are dependent and independent depend on the experimental design.

In many experiments, there will be no controlled or regular interval at which the value of a variable is measured, and neither variable will be considered to be dependent or independent.

Data Tables

A graph must be accompanied by the data table upon which it is based. In a data table, it is easier to see the numeric *measurements* recorded in an experiment. Graphs assist in finding the *relationships* among the variables.

A data table must have the following elements.

- A *title* that identifies what quantities are being studied;
- *columns* of measurements; and
- *labels* for each column showing the quantity being measured and its consistent units.

In some cases, the quantity is clear from its unit and is omitted.

If one of the variables in an experiment is independent, it is generally listed in the first column and graphed on the *x*-scale.

Graphs of Data Near 0,0

In an experiment, for a collection of solid (not hollowed) cylinders that are all made of the same metal, mass and volume is recorded for each cylinder. The data is at the right.

We would like to know: are the numbers mathematically related? Can we answer questions such as: if a similar cylinder had a mass of 8,532 grams, what would be its volume?

To answer these questions, begin by graphing the data. The steps listed below are similar to the steps used in math graphs As we solve, we will address how science graphs may differ.

Table 4

Mass and Volume for Metal Cylinders

Volume in mL	Mass in grams
2.7	21.2
4.0	29.5
9.2	72.4
12.6	99.2
7.6	61.9

1. **Decide which variable to plot on *x*.**

   In Table 4, no variable is more controlled than the other or recorded at a regular interval. Either unit can therefore be plotted on the *x*-scale. For this example, plot mL on *x*.

2. **Fill in the *range chart* for each scale.**

   The data in science graphs includes units. All of the data on the same scale must have the same units. In the range chart, including the units is optional, but doing so will serve as a check that the numbers are placed correctly. Fill in the range chart using the data in Table 4.

x-scale: Low #: ________	High #:________	Minor unit: ___	Major: ___
y-scale: Low #:________	High #:________	Minor unit: ___	Major: ___

3. **Consider adding 0 to each range** if the range is not specified in the problem.

   For this graph, add the *origin* to the range, then check your answer below.

* * * * *

x-scale: Low #: ~~2.7 mL~~ **0 mL** High #: **12.6 mL** Minor Unit: _____ Major: _____

y-scale: Low #: ~~21.2 g~~ **0 g** High #: **99.2 g** Minor Unit: _____ Major: _____

For the origin (0,0) to show on the graph, zero must be a part of both ranges.

For Table 4, to include the origin in the graph, zero will need to be added to *both* ranges.

4. **Mark the boundaries of the plot area** on your graph paper.

   For this problem, either graph on the sample grid provided below or on a printed copy of this page, or use a half-sheet of your own graph paper. Leave room at the top, bottom, and left for the title and scales. Use the same number of boxes up and across that are shown in the sample grid below.

5. **Calculate the scale minor unit. Round UP.**

$$\textbf{\textit{Scale minor unit}} = \frac{\text{(High \# on scale) minus (Low \# on scale)}}{\text{(The count of the grid lines on the scale)} - \mathbf{2}}$$

and then ***round UP*** to the next *easy* number to count by and count between.

   Calculate the scale minor unit for the *x-scale*, then check your answer below.

* * * * *

$$x\text{-scale minor unit} = \frac{12.6 - 0}{14 \text{ grid lines } - \mathbf{2}} = \frac{12.6}{12} = 1.05 \quad \textit{Round up} \text{ to } \mathbf{2.}$$

   Calculate the minor unit for the *y*-scale, then check your answer below.

* * * * *

$$y\text{-scale spacing} = \frac{99.2 - 0}{12 \text{ grid lines} - \mathbf{2}} = \frac{99.2}{10} = 9.92 \quad \text{Round up to } \mathbf{10.}$$

   Enter both minor units to the range chart. Enter the major units: double the minor.

6. **Make both ranges slightly wider and evenly divisible by the scale *major* unit.**

   For this example, first adjust the range for the *x-scale*, then check below.

* * * * *

x-scale: Low #: ~~2.7 mL~~ **0 mL** High #: ~~12.6~~ mL **16** Minor Unit: **2** Major: **4**

Zero is always evenly divisible. Make the high number higher. Move 12.6 to the next higher number divisible by the scale major unit (4), which is 16.

Try that step for the *y-scale*, then check below.

* * * * *

y-scale: Low #: ~~21.2 g~~ **0 g** High #: ~~99.2~~ g **100** Minor Unit: **10** Major: **20**

Adjust the *y*-scale high number higher, from 99.2 to **100**, since 100 is divisible by 20.

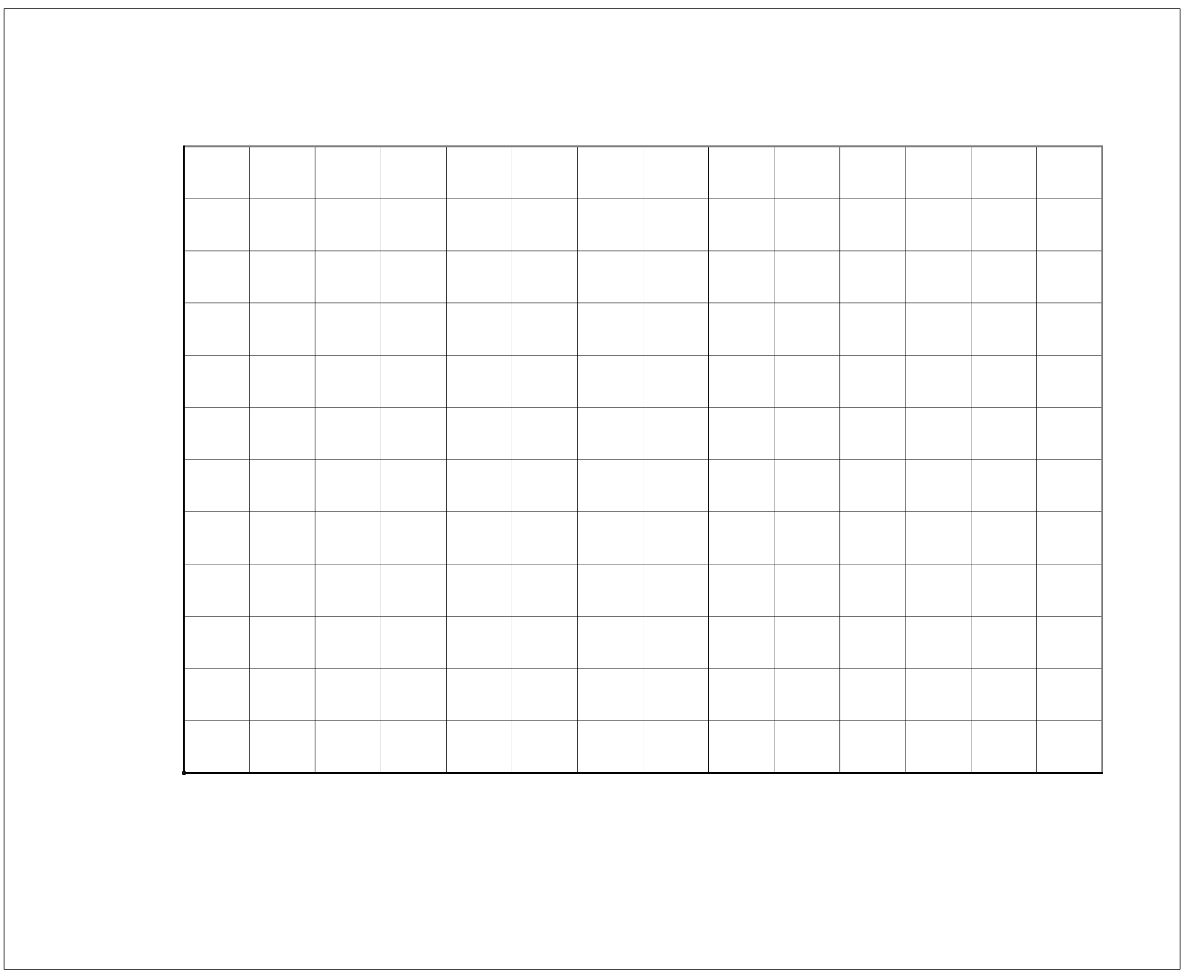
7. **Number and label each scale.**

   The low number on each scale is 0. That results in the most familiar type of scale, where the bottom left corner is the origin (0,0).

   For this example, number each scale, then add a label to the scale. For the label, use the heading of the data-table column that you are plotting on that scale.

   Add a title at the top of the graph, based on the title of the data table.

8. **Plot the points.** As one indication that the data has uncertainty, either make each point somewhat thick, or circle it.



9. **Draw the function.**

   Add a line or curve to the graph. In natural phenomena, two related quantities will usually graph to produce a *smooth curve* or a *straight line.*

   The function should be a smooth curve or line that passes *close* to *most* of the points, representing your best estimate of where the points would be if there were no experimental error (do not "connect the dots").

Do those steps and then check your answer on the second page of the *next* lesson.

* * * * *

The position of the points on your graph, relative to the line, should match the graph below. Note that the data does not fit exactly on a straight line.

Should the line for *this* graph go through the origin? Let's work through the logic of the experiment.

- If the volume of the metal cylinders approaches zero, what should their mass approach?

If the mass of the cylinders approaches zero, what should their volume approach?

* * * * *

If the mass of a solid metal cylinder approaches zero, its volume must approach zero as well. The line showing the relationship of mass to volume for the cylinders *should* approach the origin (0,0).

However, including (0,0) as a point on the line would be questionable science. We did not record data at (0,0), and if the instruments used in measurements had a systematic error, or the scale on the instrument was read with a consistent error, the line might not pass though (0,0). Graphing can help us to find such systematic errors.

That said, if the line representing the smooth function passes through (0,0), it would fit the *theoretical* results for this experiment.

* * * * *

Practice

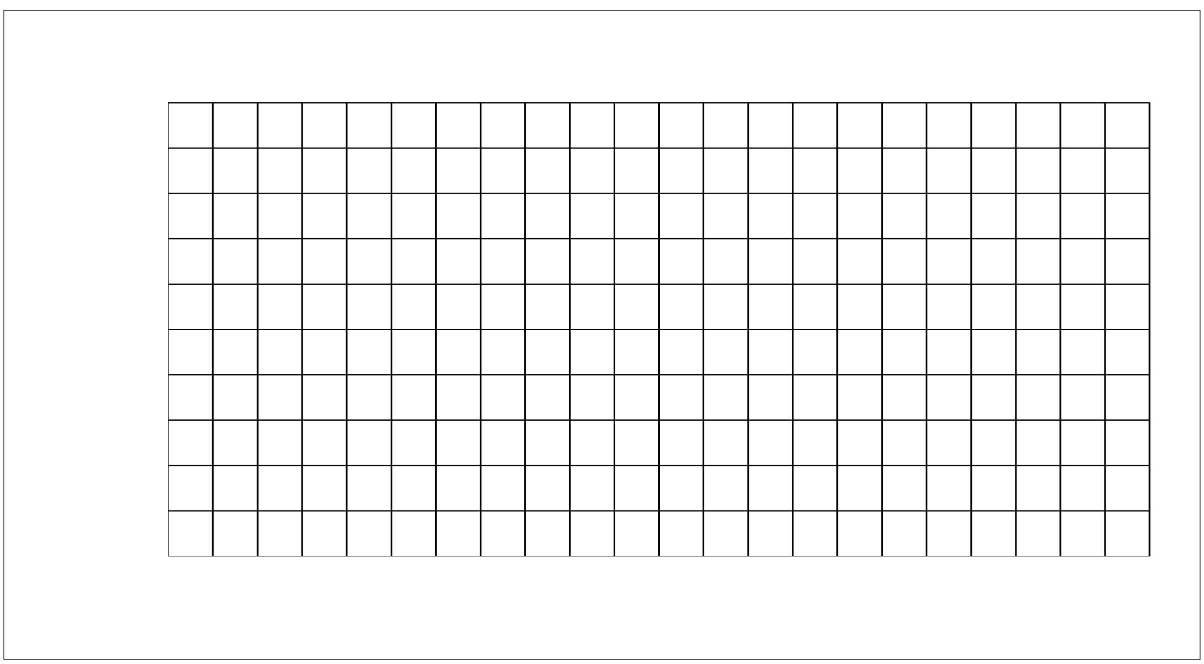
If unsure about an answer, check it after each part. Answers are at the end of the lesson.

1. In an experiment, a hollow glass container is filled with dry air and attached to a pressure gauge. The system is then sealed so that no air can enter or escape. The glass container is placed into beakers of ice water, boiling water, and liquid water at various temperatures. The temperature of the water baths and the pressure exerted by the air in the container is reported in Table 5.

Table 5

Sealed Fixed Volume of air: Temp. vs. Pressure	
ºC	kPa
0.0	202
100.	276
58	248
82	262
20	250
12	210
44	230

   a. Based on PV=nRT, what are the symbols for the variables held constant in this experiment?
   b. What are the symbols for the two variables that can change in this experiment?
   c. In this experiment, which is the independent variable?
   d. Assuming we will plot the independent variable on *x*, write a range chart for the data. For this graph, do not modify the ranges to add zero.
   e. Based on the grid below, calculate the scale minor unit for each scale.
   f. Adjust the ranges in the range chart based on the scale major unit that is double the minor unit.
   g. Plot the graph for Table 5. If you use your own graph paper, use the same number of grid lines as the grid below. Add the function to the graph.
   h. Are there any *outlier* points in which errors in measurement may have been made?



ANSWERS

1a. The variables V and n are held constant. (R is a constant, not a variable, in PV=nRT.)

1b. P and T vary in this experiment (though temperature is *recorded* in degrees Celsius instead of kelvins.)

1c. Temperature is the more controlled variable, and is therefore the more *independent*, because if we place the cylinder into ice water, or boiling water at standard pressure, we know and control what the temperature will be. Once the system is sealed, we do not control what the pressure readings will be: the pressure is *dependent* on the controlled temperature.

1d. The problem says to plot the independent variable on *x*.

<u>*x-scale*</u>: Low #: **0.0** ºC High #: **100.** ºC Minor unit: ______ Major: ______

<u>*y-scale*</u>: Low #: **202 kPa** High #: **276 kPa** Minor unit: ______ Major: ______

1e. *x*-scale minor unit = $\frac{100 - 0}{22 \text{ grid lines } - 2} = \frac{100}{20} =$ **5** Don't round.

If the division results in a round number that is easy to count by, use it.

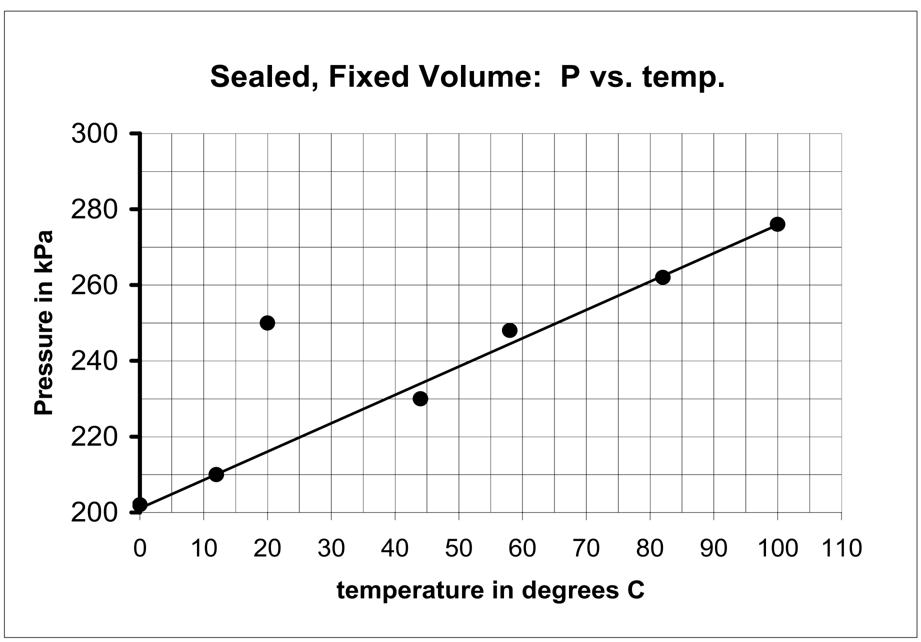
y-scale spacing = $\frac{276 - 202}{10 \text{ grid lines } - 2} = \frac{74}{8} = 9.25$ Round up to **10.**

1f. <u>*x-scale*</u>: Low #: **0.0** ºC High #: **100.** ºC Minor unit: **5** Major: **10**

<u>*y-scale*</u>: Low #: ~~202~~ **kPa 200** High #: ~~276~~ **kPa 280** Minor unit: **10** Major: **20**

The *x*-scale numbers are divisible by **10**. Do not change them. For the *y-scale*, expand the range at both ends. Make both ends divisible by 20, making the low number lower and high number higher.

1g. The graph should result in a smooth curve or a straight line. The data does not fit exactly on a line, but *most* of the points fit close to a straight line. The function should be a best estimate of where the data would be without experimental error.



1h. The point at 20 ºC would seem to be an outlier (a point not consistent with the other data). This often represents an error in measurement, but on occasion may be accurate -- and interesting.

* * * * *

Lesson 20D: Deriving Equations From Linear Data

Timing: Do this lesson *if* you are asked to *derive an equation* from a graph in which experimental data points fall close to a straight line.

Pretest: If you can do the last problem in this lesson, you may skip the lesson.

* * * * *

Developing Equations From Graphs of Straight Lines

In investigating two related quantities in science, a key goal is to develop a mathematical equation that accurately predicts how one quantity will change when the other is changed. With such an equation, we can answer "what if" questions by solving an equation, without having to experiment to determine every result.

If two variables graph as a straight line, the equation relating the variables can be determined using

$$y = mx + b$$

Reading the Coordinates of Points on a Line

In determining the equation for a line, a key step is determining the slope.

When graphing experimental data, often the points will be "roughly linear," falling close to, but not exactly on, a straight line. For roughly linear data, we try to draw the line "through the middle" of the points: where we think the data points would be if there were no error.

To describe and predict the relationship between two variables, the slope of a line "through the middle" of the points should result in a more accurate equation than a slope calculated between two points that are not exactly on the line. For this reason, calculating the slope from experimental data requires reading the coordinates of two points on the line, rather than the values of data points that are not exactly on the line.

Determining the coordinates of points on a line requires estimation.

The graph below represents the Table 4 relationship found in the previous lesson.

For each of the cases below, knowing the value for one variable, *estimate* the value for the other variable on the graphed line.

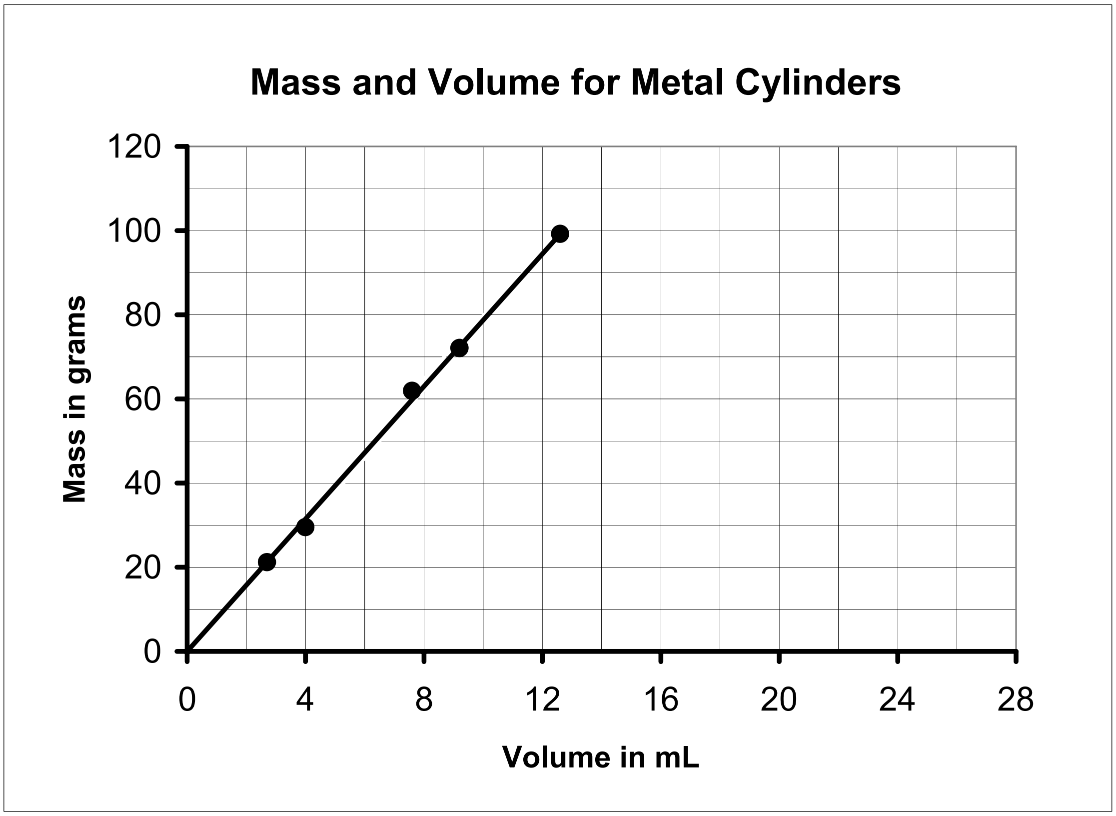
a. At mass = 20.0 grams, volume ≈ _______________.

* * * * *

> Your answer should be close to 2.5 mL. You may want to use an index card or sticky note as a "T-square" to line up with the line and the two scales. Putting a "tick mark" on scale with the WANTED unit may help in estimating the answer.

b. Try this one. At volume = 8.0 mL, mass ≈ __________________

* * * * *



Your answer should be close to 63 grams.

c. At volume = 4.0 mL, estimate the mass based on the graph = ___________________

In Table 4 at the beginning of the prior lesson, the mass at 4.0 mL = ________ , but the line is slightly above the point.

Do the following by estimating, based on the graphed line above.

d. At volume = 12.0 mL, mass ≈ ___________________

e. If mass = 40.0 grams, volume ≈ ________________.

f. At mass = 90.0 grams, volume ≈ ________________.

* * * * *

Your answers should be *close* to these.

d. At volume = 12.0 mL, mass ≈ 94 g

e. If mass = 40.0 grams, volume ≈ 5.1 mL

f. At mass = 90.0 grams, volume ≈ 11.4 mL

How can you improve your estimates? Making a larger graph, using graph paper with finer lines, using a numbering scale that allows for easy estimation, and using widely separated points all help. However, in reading the coordinates of a point on a line, your answers will have uncertainty.

Calculating the Slope of a Line

When calculating the slope of a line graphed from experimental data, the two points chosen on the line should be *widely separated* to reduce the impact of uncertainty in reading the point coordinates.

Try the following example, then check your answer below.

Q. For the line in the graph of the metal cylinders data above, calculate the slope of the line between the two points where mass = 80.0 grams and mass = 0 grams.

* * * * *

WANTED: **m**

DATA:

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$$

* * * * *

At y = 80.0 g , reading the graph, the x value is about 10.2 mL.

At y = 0 g , since this line passes through the origin, assume x = 0 mL.

Solve for slope. Include units.

* * * * *

m = ? x_1 = 0 mL y_1 = 0 g (assumption) x_2 = 10.2 mL (estimate) y_2 = 80.0 g

* * * * *

$$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{80.0\text{ g} - 0\text{ g}}{10.2\text{ mL} - 0\text{ mL}} = \mathbf{7.85\ \frac{g}{mL}}$$

In calculations based on a line through the origin, (0,0) is considered to be exact. The 0 values for the origin therefore have infinite *sf* and do not limit the *sf* in an answer.

Because points on the line that are not the origin can be difficult to read accurately, the slope should be calculated a *second* time using two *different* widely separated points.

On the graph above, estimate the values for grams at mL = 2.0 and mL = 12.0 . Then calculate the slope between those two points, and check your answer below.

* * * * *

Your estimates for y values should be *close* to these.

x_1 = 2.0 mL y_1 = **15.6 g** (estimate) x_2 = 12.0 mL y_2 = **94.3 g** (estimate)

* * * * *

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } y}{\text{change in } x} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{94.3\text{ g} - 15.6\text{ g}}{12.0\text{ mL} - 2.0\text{ mL}} = \mathbf{7.87\ \frac{g}{mL}}$$

The slope between any two points on a line should be the same. In practice, two slope calculations based on coordinate estimates must be *close*, but due to uncertainty often will not be exactly the same. What value should we use for the slope of the line? *If* the two slopes are close, *average* the two answers.

If your two slope calculations are not close, check your reading of the point coordinates and/or your slope calculation.

Based on the calculations above, write the value for the slope. Include units.

* * * * *

The calculated slope is the average of the two calculations: **m = 7.86 g/mL**.

Practice A

1. Calculate the slope between these points. Include units.

   a. Points (10. cm, –40. ºC) and (–20. cm, 50 ºC)   b. (–20 ºF, –50 cm) and the origin.

2. Using the graph of P vs. ºC at the end of the *answers* to the previous lesson,

   a. Estimate ºC at 270 pKa   b. Estimate P at 5 ºC.   c. Using those two points, find **m**.

Writing the Specific Equation For A Line Through the Origin

The equation y = mx + b has two variables (y and x) and two constants (**m** and **b**).

For the math graphs in Lesson 20B, to write the *specific* equation for a line on a graph, we left the variables as y and x, but substituted values for **m** and **b**.

For graphs in science, when writing the *specific* equation for a straight line on a graph,

- In place of y and x, write the quantities and units plotted on the scale (such as *mass in grams* or *volume in mL*) and
- In place of **m** and **b**, write the numeric value of each constant with its *units*.

Based the calculations above for the metal-cylinders graph above, do those substitutions into y = mx + b and write the specific equation for the line.

* * * * *

For this graph,

y = **mass in grams** (the quantity plotted on the y-axis)

m = 7.86 g/mL calculated above

x = **volume in mL** (the quantity plotted on the x-axis)

b = 0 , since the line goes through the origin.

Substitute those terms in y = mx +b.

* * * * *

The specific equation for the graphed line and the data is

$$\textbf{(mass in grams)} = 7.86\,\frac{\textbf{g}}{\textbf{mL}} \bullet \textbf{(volume in mL)} + 0$$

Testing the Equation

Once a specific equation is written, it should be tested.

- Pick a point in the data table that plotted *close* to the line.
- Substitute *one* of the two coordinates for that point into the specific equation and *calculate* a value for the other coordinate.
- Compare the calculated to the table value.

If the two values are close, the equation works.

Try it. For this example, use V = 9.2 mL, plug it into the specific equation, and calculate a predicted mass. Compare the prediction to Table 4 at that volume.

Table 4

Mass and Volume for Metal Cylinders

Volume in mL	Mass in grams
2.7	21.2
4.0	29.5
9.2	72.4
12.6	99.2
7.6	61.9

* * * * *

Specific equation: (mass in grams) = $7.86 \frac{g}{mL}$ • (volume in mL) + 0

Substituting V = 9.2 mL: (mass in grams) = **$7.86 \frac{g}{mL}$ • (9.2 mL)** = **72 grams**

The table value at 9.2 mL is **72.4 g**. The equation predicts, allowing for uncertainty, the measured value.

Allowing for uncertainty, the equation should *predict* the volume given any mass in the table, and predict the mass given *any* volume in the table.

The specific equation describes the line on the graph, and

The specific equation predicts the relationship between the two variables plotted on the graph and written in the equation.

Extrapolating From the Data

If an equation correctly predicts the experimental results for several small samples, it be able to be used for predictions on larger samples. Use the specific equation above to answer this question.

Q. If a cylinder of the same material had a volume of 1,000. mL, what would be its mass?

* * * * *

(mass in grams) = 7.86 g/mL • (1,000 mL) = **7,860 g**

However, though that is the predicted value, the prediction may not be accurate. For experimental data, predicting values on the line *between* data points (interpolating) is usually safe, but predicting results *beyond* the range of the measured data (extrapolating) is risky. For example, predictions for real gases based on ideal gas law break down as gases approach conditions at which they condense into liquids or solids. When evaluating experimental results, conclusions should be drawn cautiously and limited to the conditions under which the experiment was run.

Explaining the Specific Equation In Words

The specific equation for the metal cylinders data above can be written as

(mass in grams) = **7.86 g/mL** • (volume in mL)

This equation matches the form *y* = (constant) • *x* which is the general equation for a direct proportion.

In Lesson 18A, we discussed statements and equations that can be written when a graph results in a straight line through the origin. Review those rules if needed, then express the meaning of the data in Table 4, and the graph of that data, in words and in equations.

* * * * *

If a graph results in a straight line through the origin, the variables plotted on *y* and *x* are *directly proportional*. In general, for a graph of a line through the origin,

y = (slope) • *x* which means *y* = (constant) • *x* which means *y/x* is constant.

Based on the specific equation that fits the data for the cylinders,

- **Mass** in grams for these cylinders (*y*) is directly proportional to their **volume** (*x*):

  Mass of each cylinder = (constant) (Volume of each cylinder)

- The *ratio* of mass to volume for the cylinders is constant, and is equal to the slope of the line on the graph.

$$\frac{\textbf{Mass of cylinders}}{\textbf{Volume of cylinders}} = \textbf{a constant value} = \textbf{the slope of the graphed line}$$

This last statement can be tested by dividing the second column by the first for any point in the data. Try that calculation for any two points in Table 4. Fill in the table with each result.

* * * * *

Does the mass/volume ratio equal the slope that you found above?

At the end of a laboratory experiment, you are often asked to speculate on the possible meaning of the results. Take a moment to ponder: in this experiment, why would the g/mL ratio be constant?

* * * * *

Table 4

Mass and Volume for Metal Cylinders

Volume in mL	Mass in grams	g/mL
2.7	21.2	
4.0	29.5	
9.2	72.4	
12.6	99.2	
7.6	61.9	

The problem stated that the cylinders were solid (not hollow) and that all were made of the same metal. The data show that these cylinders all had the same mass to volume ratio. The ratio of mass to volume for an object is its density, so all of these cylinders have the same density. The data suggests that solid samples of this metal have a constant density.

Will the density identify the metal? If you are near a computer, try an online search on densities of metals. See if the results allow you to tentatively identify the metal used in the cylinders for this experiment.

* * * * *

To our prior list of steps to use when graphing data, let us add step

10. If the graph is a straight line,

a. write the specific equation for the line.

Calculate the slope using *two* points on the line. Find **b** using x and y values for any *one* point on the line and y = **m**x+**b**. Substitute values for **m** and **b**, and quantities and units for y and x, into y = mx+b.

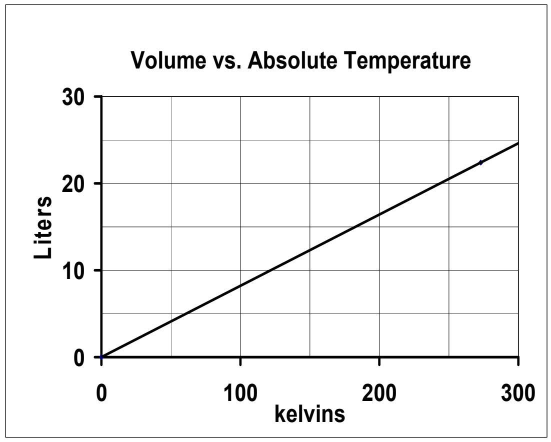
b. Test your specific equation.

For a point in the graph close to the line, pick one coordinate from the data table. Use the specific equation to predict the value for the other coordinate. The calculated and table value should be close.

c. Explain the equation in words.

If the graphed line goes through the origin, write the statements and equations that can be applied to direct proportions.

Practice B



1. For the graph at the right,

   a. If x_1 = 0 K , y_1 = _____

      If x_2 = 300. K , $y_2 \approx$ _____

   b. Calculate the slope.

   c. Read the y-intercept.

   d. Write the specific equation for the line.

   e. Translate the equation into words.

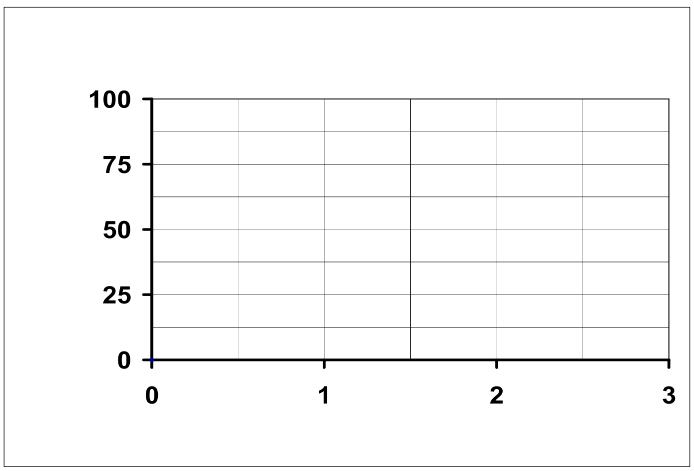
   f. At x = 400. K, y = ?

2. On the grid at the right, complete the graph of distance versus time represented by the following equation.

   Distance in miles

   = (25 miles/hr) (time in hours)

   Draw the function. Add labels for the title and the axes.



3. For a collection of solid metal cylinders, mass and volume measurements are recorded for each cylinder. The data is at the right.

   a. Graph this data using the steps in the summary above. Use your own graph paper or the sample below.

   b. What might explain the difference between the data in Table 6 at the right and the data for the similar experiment in Table 4?

   c. Label the third column g/mL. Calculate a value for that ratio for each data point, including units.

   d. Explain the results in the third column.

Table 6

Mass and Volume for Metal Cylinders – Set 2

Volume in mL	Mass in grams	
3.5	9.5	
7.1	55.9	
4.2	33.1	
5.6	15.1	
13.1	35.4	
2.6	20.5	

ANSWERS

Practice A

1a. Define the lower x as $\mathbf{x_1}$: x_1 = —20. cm y_1 = 50. ºC x_2 = 10. cm y_2 = —40. ºC

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-40.\ ^\circ\text{C} - (50.\ ^\circ\text{C})}{10.\ \text{cm} - (-20.\ \text{cm})} = \frac{-90\ ^\circ\text{C}}{30\ \text{cm}} = \mathbf{-3.0\ \frac{^\circ C}{cm}}$$

1b. If numbers are on the same scale, they must have the same units, so the units of the 0 at the origin match the units of the other numbers on that scale.

m = ? $\boldsymbol{x_1}$ = —20 ºF y_1 = —50 cm x_2 = 0 ºF y_2 = 0 cm

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\text{change in } \boldsymbol{y}}{\text{change in } \boldsymbol{x}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.\ \text{cm} - (-50.\ \text{cm})}{0\ ^\circ\text{F} - (-20.\ ^\circ\text{F})} = \frac{50}{20} = \mathbf{2.5\ \frac{cm}{^\circ F}}$$

2a. At 270 pKa, ºC ≈ **92** ºC b. At 5 ºC, kPa ≈ **206** kPa

2c. WANTED: **m** = slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

DATA: Make the data table using the symbols needed to solve the equation.

Set $\boldsymbol{x_1}$ as the lower x value in the two points.

* * * * *

Since ºC is plotted on x, set $\boldsymbol{x_1}$ = 5 ºC

m = ? $\boldsymbol{x_1}$ = 5 ºC , y_1 = 206 kPa , x_2 = **92 ºC** , y_2 = 270 kPa

SOLVE: $$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{270\ \text{kPa} - 206\ \text{kPa}}{92\ ^\circ\text{C} - 5\ ^\circ\text{C}} = \frac{64\ \text{kPa}}{87\ ^\circ\text{C}} = \mathbf{0.74\ \frac{kPa}{^\circ C}}$$

Practice B

1a. x_1 = 0 K y_1 = **0 L** x_2 = 300. K y_2 = **24.6 L** (your answer should be *close*)

1b. WANT = **m** Using the points above, $$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{24.6\ \text{L} - 0\ \text{L}}{300.\ \text{K} - 0\ \text{K}} = \mathbf{0.0820\ \frac{L}{K}}$$

In practice, it may be difficult to calculate a slope on a graph to more than two *sf*.

1c. For a line through the origin, **b = 0**

1d. Into y = mx+ b , substitute the above values for **m** and **b** and the quantities and units on y and x .

Specific equation: **(Volume in L) = 0.0820 $\mathbf{\frac{L}{K}}$ (temperature in kelvins)**

1e. Possible statements include: For this data,

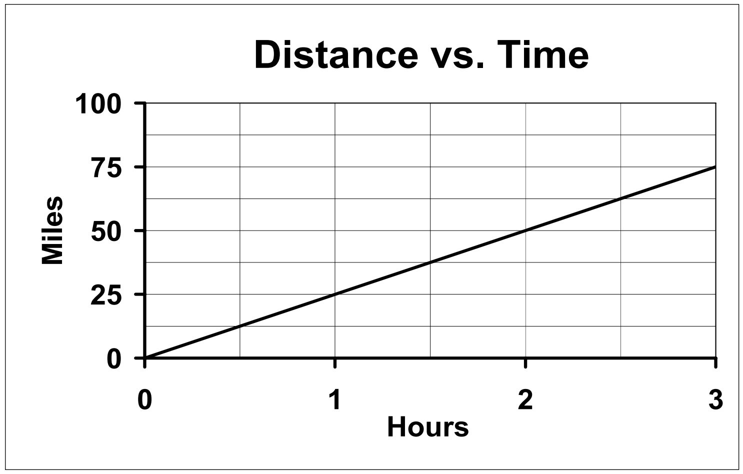
- volume in liters and temperature in kelvins are directly proportional.
- The ratio of volume in liters to temperature in kelvins is 0.0820 L/K .

1f. Substitute the value into the specific equation.

Specific equation: (Volume in L) = 0.0820 $\frac{L}{K}$ **(400. K) = 32.8 L**

2. The equation is in the form y = (constant)(x) , which is the form of a direct proportion. Data that is directly proportional plots as a straight line through the origin.

   Based in the equation, the slope must be 25 mi/hr. You can use that slope to draw the line. The rise is 25 miles when the run is one hour, and the line has a constant slope.



3a. Remember that in science, data often plots as straight lines or smooth curves. Be on the lookout especially for straight lines that extend through the origin. Do not "connect the dots."

   * * * * *

   The graph is shown after the summary below.

3b The data in Table 4 was found to fit one metal with constant density. This table may represent data for two different metals, each with constant density.

3c. All of the values should be *about* 7.9 g/mL or 2.7 g/mL.

3d. The results are consistent with data for two different metals, one with a density of about 7.9 g/mL, the other about 2.7 g/mL. An online search for *metal densities* may identify familiar metals fitting these results.

Summary of Initial Rules: Graphing In Two Dimensions

1. **Decide which variable to plot on *x*.** Often, this is the independent variable.
2. **Write the *range chart* for each scale.**

   x-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___

   y-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___

3. **Consider adding zero to each range, increasing the range.** In most cases, if a range does *not* include zero, change the number to *zero* that *increases* the range.
4. **Mark the boundaries of the plot** on the graph paper.
5. **Calculate the minor unit for each scale. Round UP.** Use this equation:

   $$\textit{Scale minor unit} = \frac{\text{(High \# on scale) minus (Low \# on scale)}}{\text{(The count of the grid lines on the scale)} - \mathbf{2}}$$

   and then ***round UP*** to the next *easy* number to count by and count between.

6. **Decide the major unit for each scale, then make both ranges slightly wider and evenly divisible by the major unit** for that scale. To number every second line on a scale, make the major unit double the minor unit. Add the minor and major units to the range chart.

7. **Number the scales based on the minor unit. Label the scales. Title the graph.**
8. **Plot the points.**
9. **Draw the function:** either a smooth curve or straight line *near* most points.
10. **If the graph is a straight line,**
    a. **write the specific equation for the line.**
       - Calculate **m** using *two* points on the line.
       - Find **b** using $y = mx+b$, x and y for any *one* point on the line, and **m**.
       - Substitute *values* for **m** and **b**, and *quantities* and *units* for y and x, into $y = mx+b$.

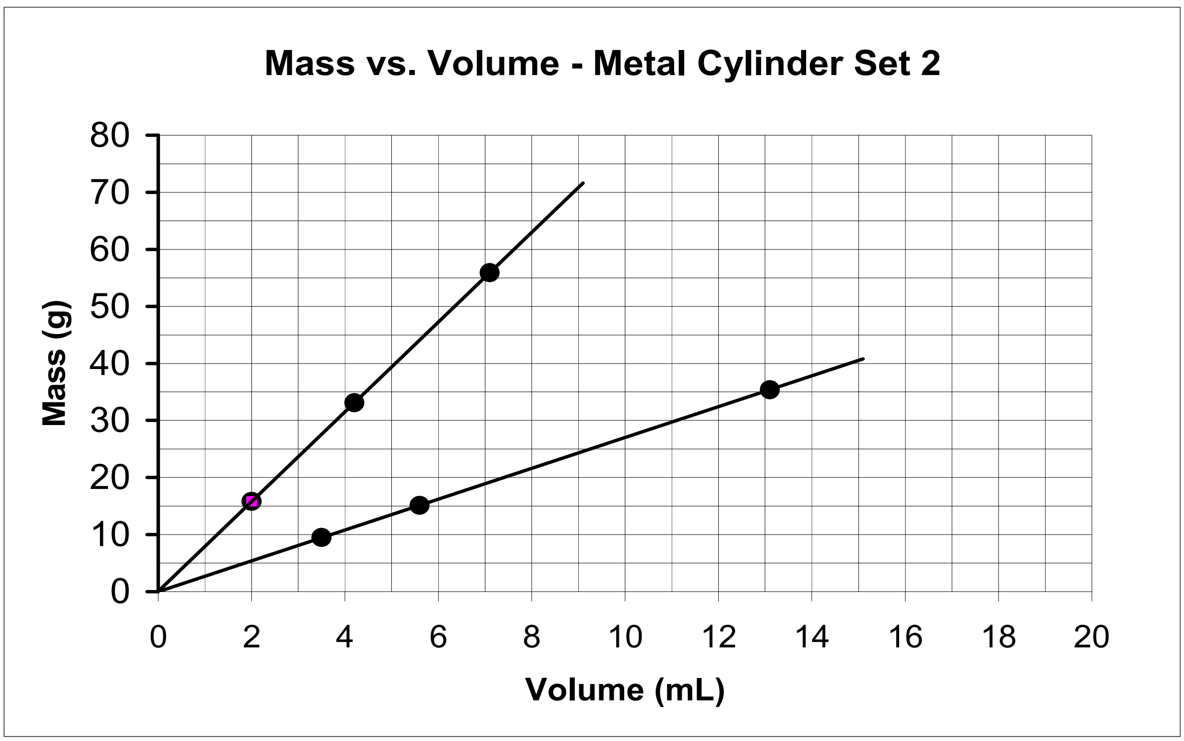
    b. **Test the specific equation.**

       For a point on the graph close to the line, pick one coordinate from the data table. Use the specific equation to *predict* the value for the other coordinate. Compare the predicted and the data table values.

    c. **Explain the equation in words.**

       If the graphed line goes through the origin, explain the relationship using the statements and equations that apply to direct proportions.

* * * * *



Lesson 20E: Linear Data Not Directly Proportional

Pretest: If you can do the last problem at the end of this lesson, you may skip the lesson.

* * * * *

Data Clustered Away from Zero

The following table shows the correlation between temperatures in the Fahrenheit and Kelvin scales.

In this data, the Kelvin temperatures are clustered far from zero. When the numbers on a scale are clustered far from zero, you have a choice. You can number the scale to include zero, or not.

To recognize the type of equation that the data represents, including (0,0) will help. However, if the data on a scale is clustered away from zero, choosing not to include zero will help in calculating an accurate slope for the line.

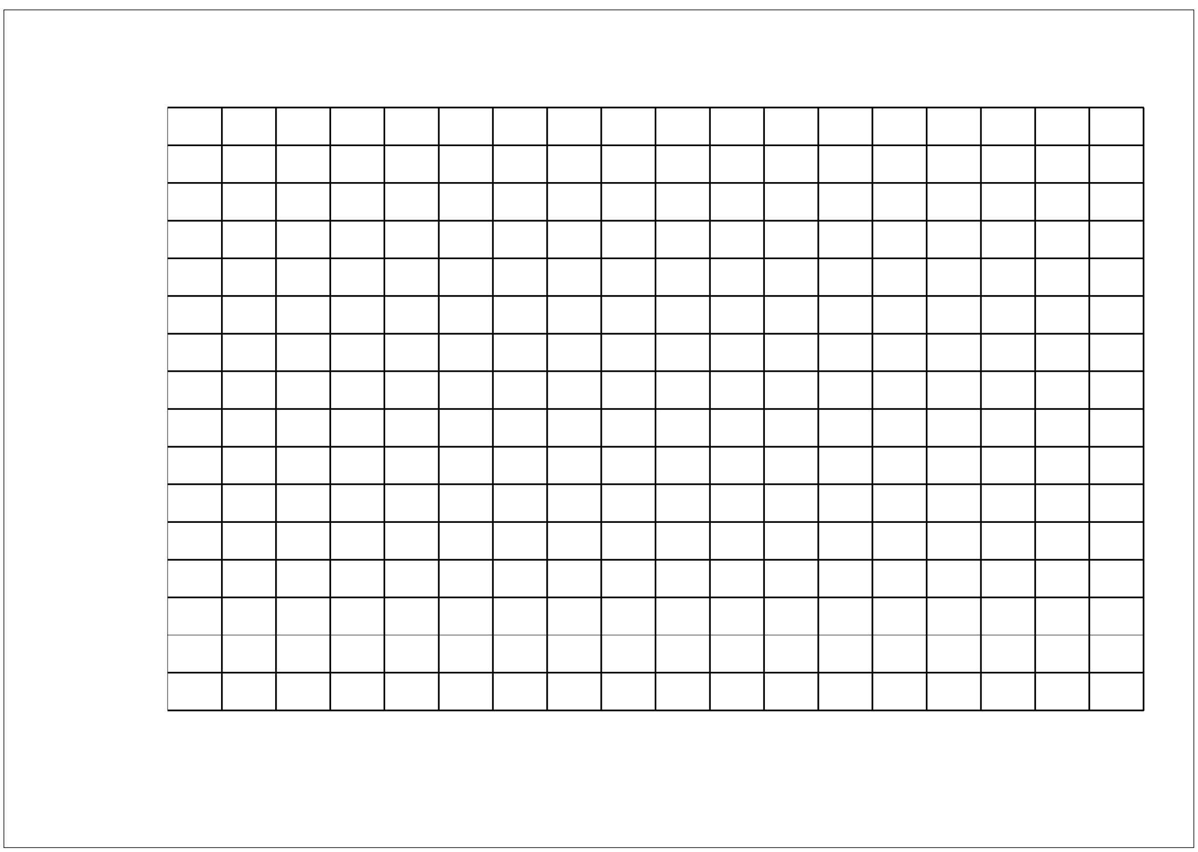
For this graph, do *not* change the range to include zero.

Table 7

Fahrenheit versus Kelvin temperatures	
K	ºF
373	212
293	68
273	32
233	– 40.

For this data, no variable is more controlled than the other. We will arbitrarily choose to graph the Kelvin-scale temperatures on the *x-scale.*

Graph this data using your own half-sheet of graph paper or the sample grid below.



Use the ten graphing steps. If you get stuck, check the answers below.

* * * * *

Step 2. *x-scale*: Low #: **233** High #: **373** Minor Unit: ___ Major: _____

y-scale: Low #: **−40** High #: **212** Minor Unit: ___ Major: _____

* * * * *

Step 5. x-scale minor unit $= \dfrac{373 - 233}{18\ x \text{ grid lines } - 2} = \dfrac{140}{16} = 8.75$ Round up to **10.**

y-scale minor unit $= \dfrac{212 - (-40)}{16\ y \text{ grid lines } - 2} = \dfrac{252}{14} = 18$ Round up to **20.**

* * * * *

Step 6. *x-scale*: Low #: ~~233~~ **220** High #: ~~373~~ **380** Minor Unit: **10** Major: **20**

y-scale: Low #: **−40** High #: ~~212~~ **240** Minor Unit: **20** Major: **40**

In adjusting the ends of the range to be divisible by the major unit, make the lower numbers lower and the higher higher. If a number is already divisible, such as **−40**, no change is needed.

* * * * *

9. See the graph on the following page.

* * * * *

10. ***If* you get a straight line,**

a) **write the specific equation for the line.**

Substitute into y = mx + b the quantities and units for the two variables and the values for the two constants.

For the variables in Table 7,

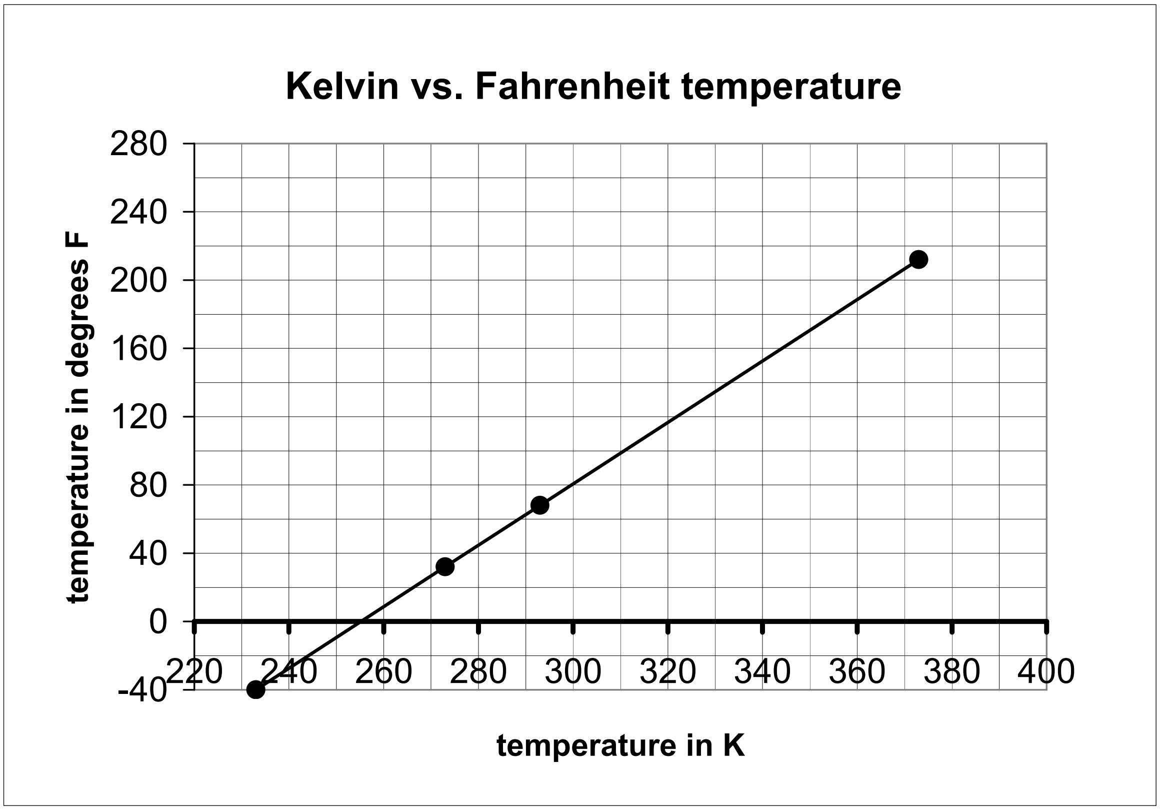
y = temperature in **°F**

x = temperature in **K**

To calculate the constant slope, we must read coordinates of two points on the *line*, not in the data, since the line may not pass through the points. But for this data, which is defined rather than experimental, all of the points fall very close to the line.

Calculate the slope. For this graph, use the highest and lowest x scale values in the data, and then check your answer below.

* * * * *



Using the lowest x (**233**, —40) and highest x (**373**, 212),

x_1 = 233 K , y_1 = —40 ºF , x_2 = 373 K , y_2 = 212 ºF

$$\mathbf{m} = \text{slope} = \frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{212\text{ F} - (-40\text{ F})}{373\text{K} - 233\text{K}} = \frac{252\text{ F}}{140.\text{ K}} = \boxed{\mathbf{1.80\ \frac{{}^{o}F}{K} = m}}$$

Using **m**, calculate b.

* * * * *

b = the y intercept = the value of y where the line crosses the y-axis

= the value of y when x = 0 .

You cannot *read* the y intercept if a graph does not show x = 0, and this graph does not. To find **b**, we can either redo the graph, this time including 0 on both scales, or we can calculate **b** mathematically. Let's try the latter method.

If you have not already solved for **b**, use the data table point at x = 293 K**,** plus the value for **m** above, and solve for y **= m**x **+b** for **b**.

* * * * *

WANT: **b** The only equation we know that uses **b** is

DATA: $y = mx + b$

m = (1.80 °F/K)

At x = 293 K, y = 68 °F

? = b = $y - mx$ = (68 °F) − [(1.80 °F/K)(293 K)]

= (68 °F) − (527 °F) = **− 459 °F = b**

Now write the specific equation for the line.

* * * * *

Substitute the variable quantities and their units in place of y and x, and substitute the numerical constants calculated above for **m** and **b**.

The specific equation for the line is **(temp. in °F) = (1.80 °F/K)(temp in K) − 459 °F**

b. **Test your equation.**

For a point in the data table that is on or very near the line, plug one coordinate (x or y) into the specific equation, and see if the equation correctly predicts the other.

Using the specific equation, solve for the predicted °F at **273** K.

* * * * *

(temp. in °F = (1.80 °F/K)(temp in K) − 459 °F

(temp. in **°F** **=** (1.80 °F/K)(**273 K**) − 459 °F = 491 °F − 459 °F **=** **32 °F**

Compare the calculated to the table value.

* * * * *

The equation predicts the data. Does this answer make sense? Fill in these blanks based on our temperature scale definitions:

273 K = standard temperature for gas measurements = ________ °C = ________ °F

Do **Step 10c**. **Explain the equation in words.**

* * * * *

The specific equation for the line supplies us with an equation that converts between temperature measurements in kelvins and in degrees Fahrenheit.

The data in this problem was based on calculations rather than measurements, and the results are more certain than would be the case for data obtained by experiments. Calculations based on graphs of experimental data must allow for a higher level of uncertainty.

* * * * *

Practice

1. For the graph at the right,

   a. at 0.20 mol/L, s ≈ ______

      At 55 s , mol/L ≈ _______

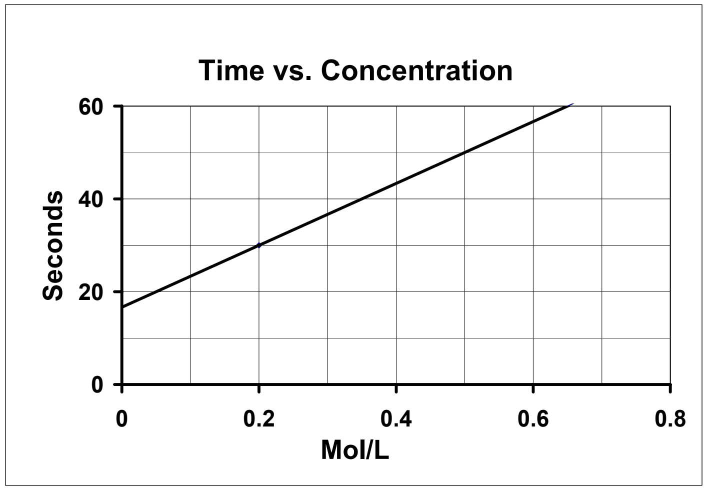
   b. Using those two points, calculate the slope.

   c. Read the y-intercept.

   d. Calculate the y-intercept based on the data at 0.50 mol/L.

   e. Write the specific equation for the line.

   f. At 80. seconds, the predicted mol/L = ?



2. Using the graph of the Table 5 data at the right,

   a. estimate the y-intercept.

   b. If the slope is 0.74 kPa/°C, calculate the y-intercept.

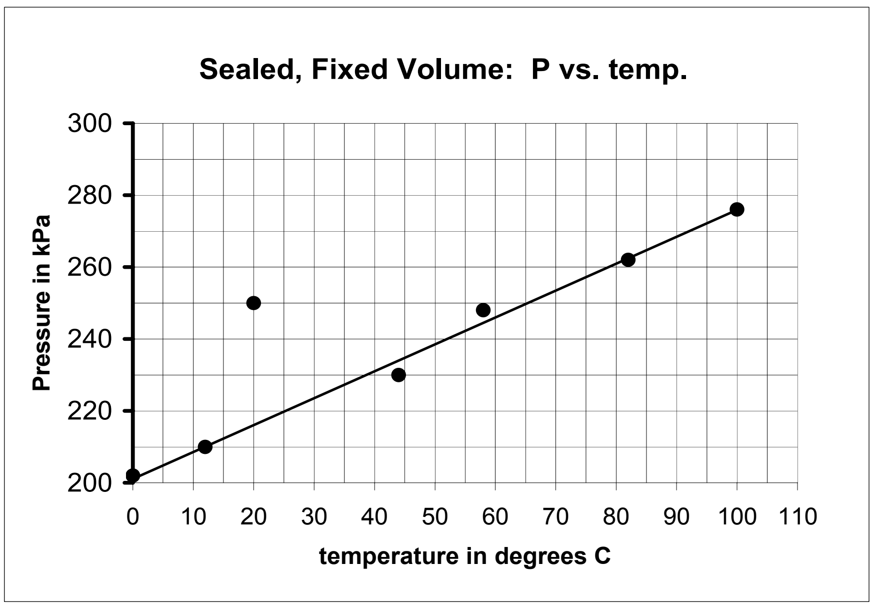
   c. Write the specific equation for the line.

   d. On this line, calculate the °C at 0 kPa.

   e. For an ideal gas, what should be the answer to part d?

   f. Is the graphed relationship a direct proportion? Why or why not?

   g. What change could be made that would result in the graph plotting as a direct proportion?



ANSWERS

1a. *At* 0.20 mol/L , seconds = 30. s   b. At 55 s , mol/L = close to 0.58 M

1b. WANT = **m** = slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

DATA: Set $\mathbf{x_1}$ as the lower x value in the two points.

m = ? $\mathbf{x_1}$ = 0.20 M, y_1 = 30. s , x_2 = 0.58 M , y_2 = 55 s

SOLVE: $\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{55\text{ s} - 30\text{ s}}{0.58\text{ M} - 0.20\text{ M}} = \frac{25\text{ s}}{0.38\text{ M}} = \mathbf{66\ \frac{s}{M}}$

Your answer should be close: 63-70 s/M

1c. Close to 16.5 s.

1d. WANT: **b** The only equation we know that uses **b** is

DATA: $y = \text{m}x + \mathbf{b}$

m = 66 s/M , x = 0.50 mol/L , y = ~ 50. sec (reading graph)

SOLVE: $y = \text{m}x + \mathbf{b}$;

50. s = (66 s/M) (0.50 M) + **b** ;

50. s = 33 s + **b** ;

b = 17 s (your answer should be close: 15-19 seconds)

1e. Specific equation: **(Time in seconds) = (66 s/M)(concentration in mol/L) + 17 seconds**

1f. WANT: mol/L

DATA: 80. s

Equation: **(Time in seconds) = (66 s/M)(concentration in mol/L) + 17 seconds**

or **(s) = (66 s/M)(? mol/L) + 17 s**

(80 s) = (66 s/M)(**? mol/L**) + 17 seconds

63 sec = (66 s/M)(**? mol/L**)

(63 sec)/(66 s/M) = (**? mol/L**)

? Mol/L = 0.95 M Your answer should be *close*. However, this answer is based on extrapolation beyond the data, and extrapolated predictions may not be reliable.

2a. Should be close to the part b answer below.

2b. WANT: **b** The only equation we know that uses **b** is

DATA: $y = \text{m}x + \mathbf{b}$ Make a data table using those symbols.

m = 0.74 kPa/ºC

Knowing **m**, **b** can be calculated from the coordinates of any point on the line.

An easy point to use is the highest point, since it is *on* the line. Using the data in Table 5, x = 100. ºC , y = 276 kPa

SOLVE: y = mx + **b** ;

276 kPa = (0.74 kPa/ºC) (100. ºC) + **b** ;

276 kPa = 74 kPa + **b** ; **b = 202 kPa** (your answer should be *close*)

2c. Pressure in kPa = (0.74 kPa/ºC)(temperature in ºC) + 202 kPa

2d. Substitute 0 kPa into the specific equation:

0 kPa = (0.74 kPa/ºC) (? ºC) + 202 kPa ; —202 kPa/0.74 kPa/ºC = **? ºC = —270 ºC**

A closer answer would be absolute zero (—273 K) , but graphed experimental data has uncertainty.

2e. For an ideal gas, the pressure, measured in any units, should be zero at *absolute* zero.

2f. The graphed relationship is not a direct proportion. A direct proportion must graph through the origin: **b** must equal zero.

2g. If all of the temperatures were converted to an absolute temperature scale, such as the Kelvin scale, a graph of pressure versus temperature values should go through the origin. For ideal gases, P is directly proportional to T (*absolute* temperature).

* * * * *

Lesson 20F: Graphs of Inverse Proportions

Pretest: If you can do the last problem at the end of this lesson, you may skip the lesson.

* * * * *

Graph of Gas Pressure Vs. Volume

In an experiment, a large syringe with a scale marked in mL is filled with air. The tip is sealed, and the syringe is then clamped so that it is vertical, with its moveable plunger pointed upward.

A series of identical chemistry textbooks is placed on top of the plunger one at a time. As the air in the syringe is compressed, the volume of air inside the syringe decreases. The books are then removed one at a time. In Table 8, the data for books and the average volume of air is recorded.

Graph the data in Table 8 on the grid below or on your own paper. Though the more controlled variable is the number of books exerting the pressure, for this graph plot *books* on the ***y***-scale. Include zero on both scales.

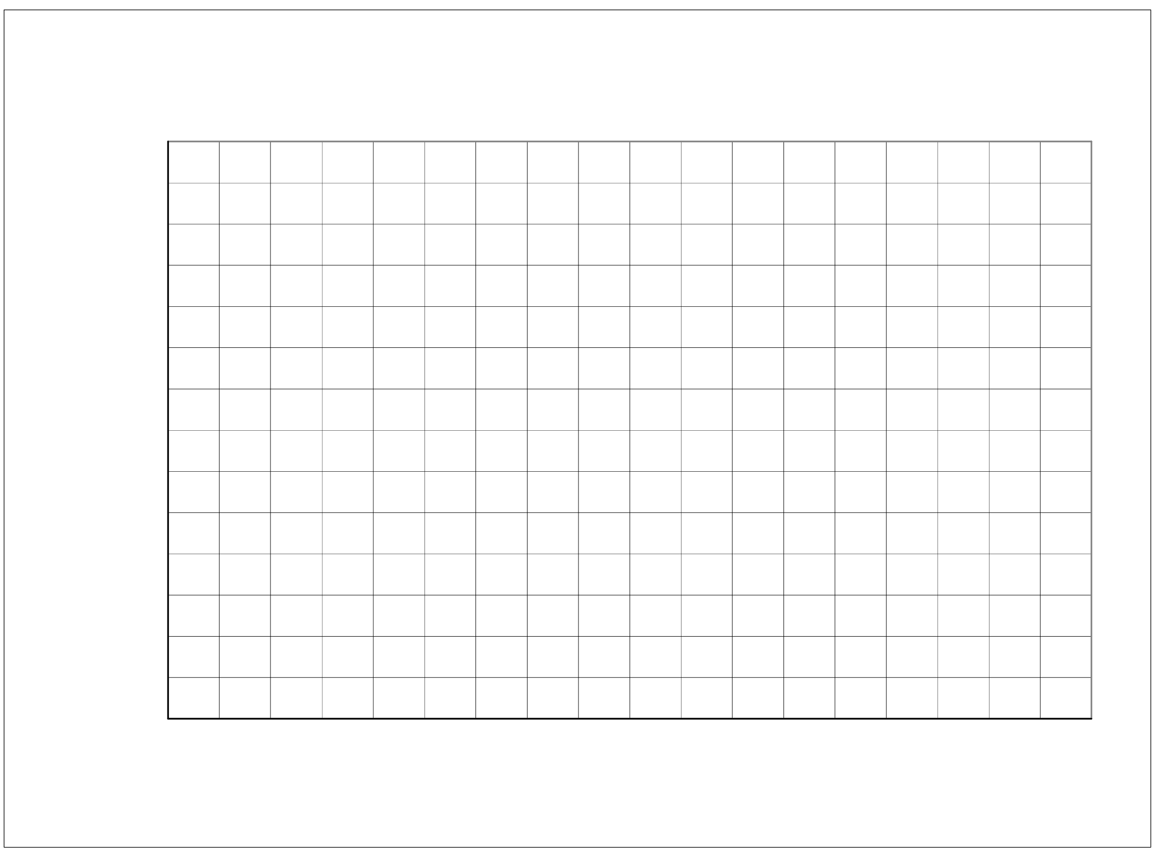
If you get stuck, take a look at the answers below.

* * * * *

Table 8

Books on Syringe With Trapped Air

Pressure In Books	Volume In mL
1	80.0
2	62.5
3	50.6
4	43.0
5	37.3
6	32.9



Step 5. x-scale minor unit = $\frac{80 - 0}{18 \text{ lines} - 2}$ = $\frac{80}{16}$ = 5 Leave at **5.**

y-scale minor unit = $\frac{6 - 0}{14 \text{ lines} - 2}$ = $\frac{6}{12}$ = 0.50 Leave at **0.5.**

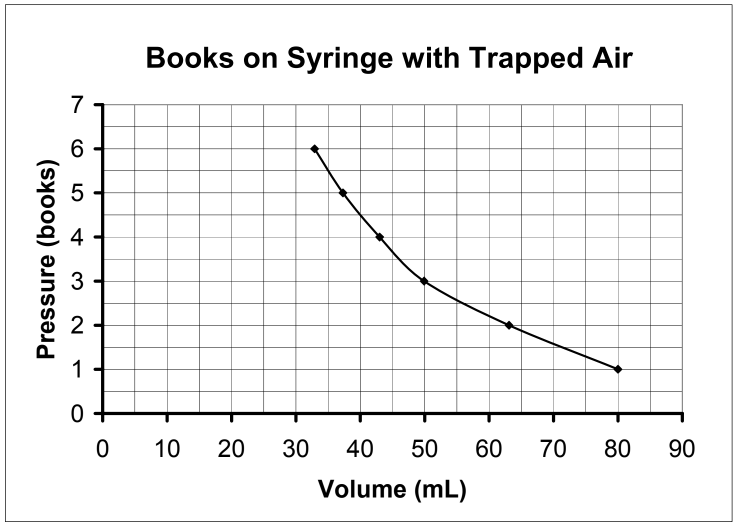
* * * * *

Step 6. _x-scale_: Low #: ~~32.9~~ **0** High #: **80.0** Minor Unit: **5** Major: **10**

y-scale: Low #: ~~1~~ **0** High #: **6** Minor Unit: **0.5** Major: **1**

* * * * *

In this experiment, which ideal gas variables are varied? Which are held constant? What should the relationship be? Which historic gas law is this?



* * * * *

P and V are varied; n and T are held constant. The relationship between these two variables should be Boyle's law:

PV = c at constant n and T

What kind of proportion exists between these two variables, and how should it plot?

* * * * *

P and V should be inversely proportional, and inverse proportions plot as part of a hyperbola.

The shape of the graph appears to be a part of a hyperbola.

When y and x are inversely proportional, y and $\mathbf{1/x}$ are directly proportional, and a plot of y and $\mathbf{1/x}$ should result in a straight line through the origin. The equation for the line is

y = (constant slope)($\mathbf{1/x}$)

Let's test this hypothesis.

In the graph above, mL was plotted on the x-scale. For the data in Table 8B, calculate $1/x$, in this case 1/mL, and enter the result in the last column.

Check your answers on the next page.

<u>Table 8B</u>

Books on Syringe With Trapped Air

Pressure In Books	Volume In mL	1/mL
1	80.0	
2	62.5	
3	50.6	
4	43.0	
5	37.3	
6	32.9	

* * * * *

When 1/mL is calculated, the unit of the answer is written as mL^{-1}.

Now graph pressure on the *y*-scale versus 1/volume on the *x*-scale. Include zero on both scales.

Set the initial range on the *y*-scale from **–6** to **+6** books.

Use the grid below or your own graph paper.

Check your answers below.

* * * * *

Table 8B

Books on Syringe With Trapped Air

Volume In mL	Pressure In Books	1/Volume In mL^{-1}
80.0	1	0.0125
62.5	2	0.0158
50.6	3	0.0200
43.0	4	0.0233
37.3	5	0.0268
32.9	6	0.0304

Step 5. *x*-scale minor unit = $\frac{0.0304 - 0}{18 \text{ lines} - 2} = \frac{0.0304}{16} = 0.0019$ Round up to **0.002**

y-scale minor unit = $\frac{6 - (-6)}{14 \text{ lines} - 2} = \frac{12}{12} = 1$ Leave at **1**

* * * * *

Step 6. x-scale: Low #: ~~0.0089~~ **0** High #: ~~0.0304~~ **0.032** Minor Unit: **0.002** Major: **0.004**

y-scale: Low #: **−6** High #: **6** Minor Unit: **1** Major: **2**

* * * * *

The graph is shown on the next page.

The graph is *not* as predicted. By Boyle's Law, we know that P and V are inversely proportional. The plot of P versus 1/V should therefore plot as a straight line through the origin. In this case, the graph is a straight line, but not through the origin.

Another concern: because by Boyle's law (PV = c) , P times V should equal a constant value. Multiply P times V in the chart at the right for several cases. Is the result close to constant?

In science, relationships tend to be simple, such as a direct proportion that graphs as a straight line through the origin. When a straight line does not go through the origin, it often means that we have defined zero poorly.

Table 8C

Books on Syringe With Trapped Air		
Volume In mL	Pressure In Books	P times V
80.0	1	
62.5	2	
50.6	3	
43.0	4	
37.3	5	
32.9	6	

Consider this data and graph. When the syringe had one book piled on top, was there only one book of pressure on the gas in the syringe? What are we forgetting? Write an answer to that question, then check below.

* * * * *

For the gas in the syringe, the pressure exerted by the gas inside equals the pressure exerted on the plunger outside. What we tend to take for granted is atmospheric pressure. We live on the bottom of an ocean of air. That air exerts a considerable pressure on the surfaces around us that are in contact with the atmosphere.

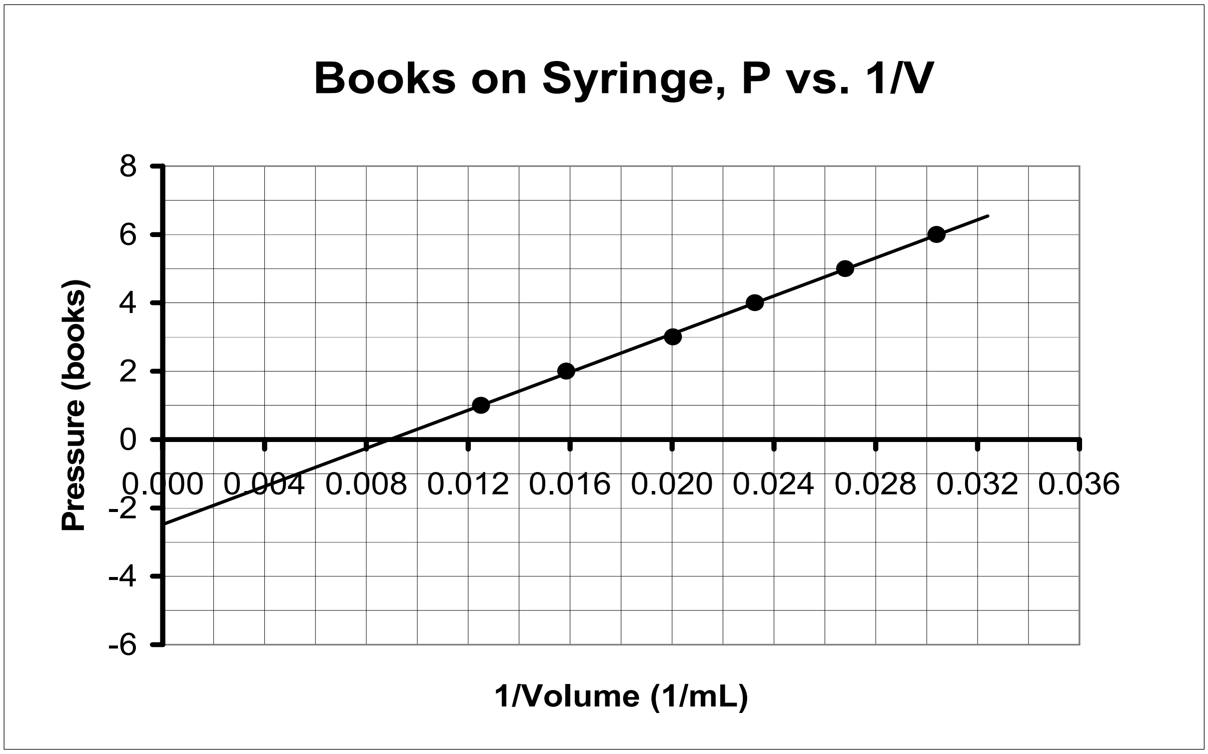
How much pressure is atmospheric pressure? Let's assume that the line on the graph passes through true *zero* pressure. That way, pressure and 1/volume graph as a direct proportion, as they should. Let's keep the same *size unit* of pressure on the *y*-axis: books.

Estimate the *y*-intercept on the graph: ____________ books.

Now take that *y*-intercept number on the axis and make it *true zero*.

Keeping the same size pressure unit, what *number* for P does the old zero become? ______

* * * * *



Answers

The *y*-intercept should be close to − 2.5 books.

Numbering up from the new zero, using the same size P unit, the old zero becomes +2.5 books. The difference between the old zero books pressure and the true zero pressure is the atmospheric pressure: 2.5 books.

At *one* book on the original scale, what is the value for the *true* pressure? _______ books

* * * * *

The original 1 book on the *y*-axis becomes 1 + 2.5: a true pressure of 3.5 books.

Table 8D

Books on Syringe With Trapped Air			
Volume In mL	Pressure In Books	True Pressure	True P times V
80.0	1		
62.5	2		
50.6	3		
43.0	4		
37.3	5		
32.9	6		

To find the true pressure for each number of books piled on the syringe, 2.5 books (atmospheric pressure) is added to each original reading for the pressure.

Do that addition in the chart on the above, then complete the additional columns.

Is *true* pressure times volume constant?

* * * * *

<u>SF</u>: Assuming that the y-intercept is only accurate to tenths, the true pressure is accurate to tenths, and the PV products will have two *sf*.

<u>Conclusion</u>: The experimental data indicate that true pressure times volume is constant for gases, if moles and temperature are held constant.

Table 8D			
Books on Syringe With Trapped Air			
Volume In mL	Pressure In Books	True Pressure	True P times V
80.0	1	3.5	280
62.5	2	4.5	280
50.6	3	5.5	280
43.0	4	6.5	280
37.3	5	7.5	280
32.9	6	8.5	280

Summary

When searching for relationships among two variables, one strategy is to graph the data, and from the shape of the graph to develop a hypothesis about the type of type of equation that the graph represents.

In the example above, the initial graph of the data seemed to be in the shape of a portion of a hyperbola, indicating an inverse proportion between the two variables.

The data is then treated by functions so that, if the hypothesis is correct, the data is in the form $y = mx + b$ and graphs as a straight line. If the data is linear, an equation for a line that predicts the data and relates the variables can be calculated.

If the straight line does not pass through the origin, it is often an indication that a better definition for zero exists for one or both of the variables than the ones chosen in the experiment. In measuring quantities, the most useful scales are those that define zero in a way that results in direct proportions between one quantity and other quantities taken to a power, such as $P = cV^{-1}$, which is one way to write Boyle's law.

Let's add to our list of science graphing steps.

10d. If a straight line does not go through the origin, try to explain why it does not, and/or consider a different definition for zero.

11. **If the graph is a smooth curve but not a straight line,** write the equation that you think fits the general equation for the curve. Then, adjust the equation to fit the format $y = (\text{constant})\ x$, and plot the adjusted variables. If the graph is linear, do step 10. If not, try a different equation to explain the curve.

Practice

1. For the *linear* graph in this lesson , using the *original* y-scale pressure values,
   a. estimate these values by reading the graph.

      At $x = 0.0320\ \text{mL}^{-1}$, y = ____________ At $y = -1$ book, x = ____________

   b. Using those two points, calculate the slope of the line.
   c. Write the specific equation for the line.

d. Change the format of the specific equation so that it matches the format for a direct proportion.

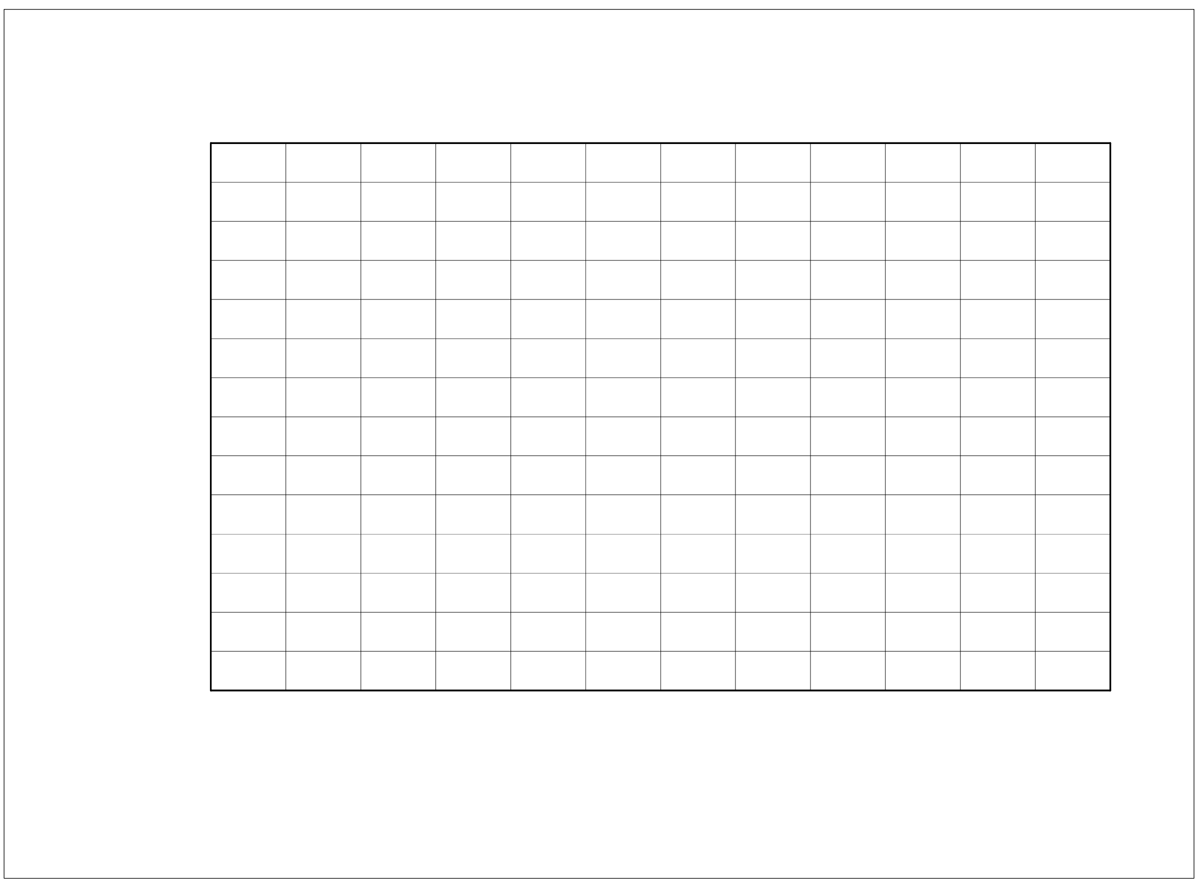
e. Compare the part b answer to the last column in Table 8D. Why are they related?

2. In an experiment, liquids A and B are mixed together and stirred. The reaction mixture changes color after time. To study the reaction rate, the time required after mixing for different mixtures to change color is recorded. [A] is held constant, and [B] is varied. Based on the resulting data in Table 9,

   a. Calculate 1/seconds and fill in the chart.
   b. Graph [B] on the *y*-axis versus 1/seconds. Include the origin on the graph.
   c. Calculate the slope of the line, with units.
   d. Estimate the *y*-intercept by reading the graph.
   e. Write the specific equation for the line.
   f. Explain the results in words.
   g. What is the inherent relationship between the *rate* of a process and *time* it takes?
   h. Write a relationship between [B] and rate.

Table 9

[B] versus Time of Reaction		
[B] in M	seconds	1/seconds
0.100	25	
0.080	30.	
0.050	48	
0.040	57	
0.020	120	



ANSWERS

Your answers are correct if they are *close* to these. When interpreting a graph, allowances must be made for estimation and uncertainty.

1a. At x = 0.0320 mL^{-1}, y should be close to 6.5 books.

At y = — 1 book, x should be close to 0.0054 mL^{-1}.

1b. WANT = **m** = slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

DATA: **m = ?** x_1 = 0.0054 mL^{-1}, y_1 = — 1 book, x_2 = 0.032 mL^{-1}, y_2 = 6.5 books

SOLVE: $$\mathbf{m} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{6.5 \text{ books} - (-1 \text{ book})}{0.0320 \text{ mL}^{-1} - 0.0054 \text{ mL}^{-1}} = \frac{7.5 \text{ books}}{0.0266 \text{ mL}^{-1}} = \mathbf{280 \text{ book} \cdot \text{mL}}$$

1c. (Original P reading in books = (280 book • mL) (1/Volume in mL^{-1}) — 2.5 books

1d. (Original P reading in books **+** 2.5 books) = (280 book • mL) (1/Volume in mL^{-1})

or (True pressure) = (280 book • mL) (1/Volume in mL^{-1})

This equation matches the format $y = mx$, a formula for a direct proportion between y and x. This equation will therefore graph as a straight line through the origin.

By adding the value + 2.5 books to the original pressure, this new form of the equation reflects the adjustment to the original pressure needed to calculate true pressure, so that P = m(1/V) .

1e. Since the line equation is True P = m(1/V) , then (True P)(V) = m .

2a. See table at right. 2b. See next page.

2c. See below, under answer 2h.

2d. b = 0 2e. [B] = (2.4 M • s) (1/s)

2f. As the [B] goes down, the time required for the reaction goes up proportionally.

Based on the graph, [B] = (constant slope)(1/s)

This means that [B] and (1/time) are directly proportional. The above equation can be rewritten as

[B] • (s) = constant = graph slope = 2.4 M • s

This format matches the xy = constant form of an inverse proportion.

Therefore, [B] and the time of the reaction are inversely proportional.

Table 9

[B] versus Time of Reaction

[B] in M	seconds	1/seconds
0.100	25	**0.040**
0.080	30.	**0.033**
0.050	48	**0.021**
0.040	57	**0.018**
0.020	120	**0.0083**

2g. As the rate of a process increases, the time the process takes decreases.

You can write: Rate = c (1/time) and Rate in seconds = c (**1/**time in seconds)

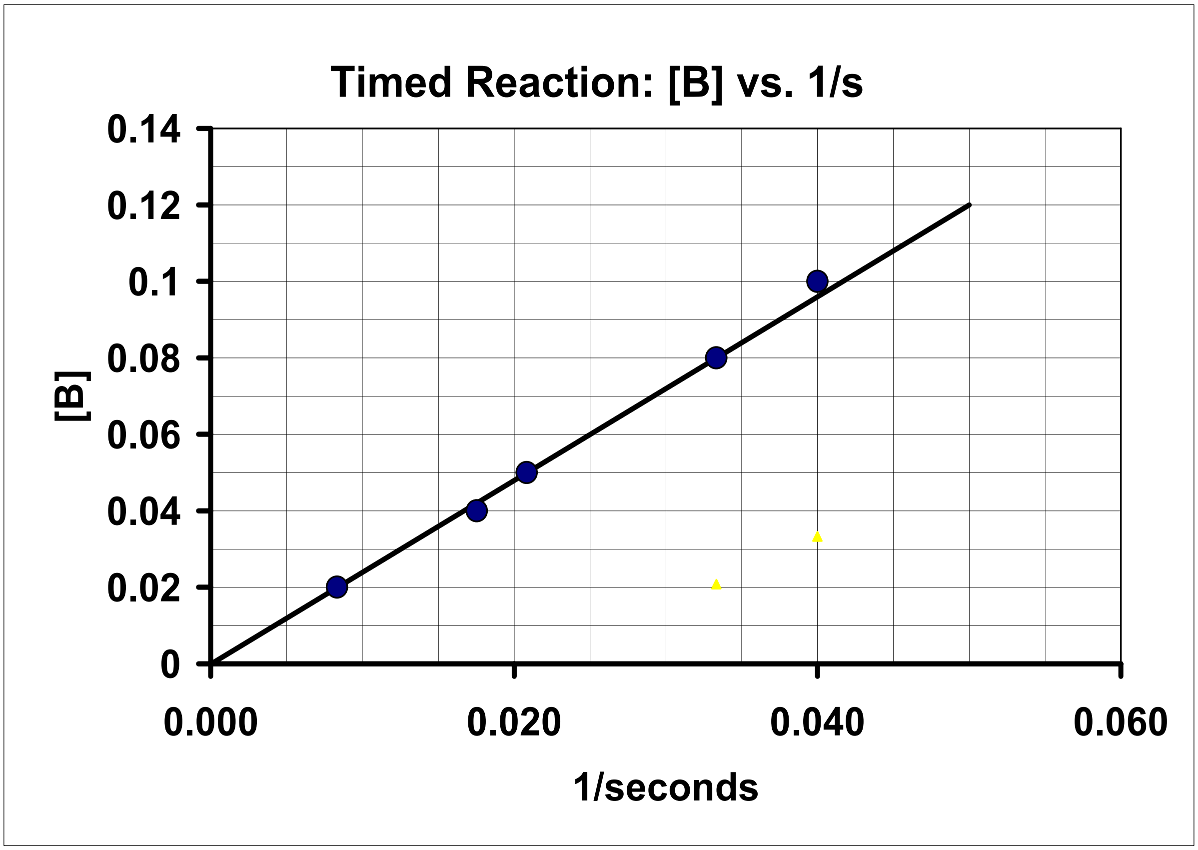
2h. Since [B] = (constant slope)(1/s) above, [B] = (a different constant) (Rate) , or in words: the reaction rate is proportional to [B].

2c. WANT = **m** = slope = $\frac{\text{rise}}{\text{run}} = \frac{\Delta y}{\Delta x} = \frac{y_2 - y_1}{x_2 - x_1}$

Read any two widely separated points on the line. If we estimate

DATA: **m = ?** x_1 = 0.0 s^{-1}, y_1 = 0 M , x_2 = 0.050 s^{-1} , y_2 = 0.120 M

SOLVE: **m** $= \frac{y_2 - y_1}{x_2 - x_1} = \frac{0.120\text{ M} - 0\text{ M}}{0.050\text{ s}^{-1} - 0\text{ s}^{-1}} = \frac{0.120\text{ M}}{0.050\text{ s}^{-1}} =$ **2.4 M • s = m**



SUMMARY: Graphing In Two Dimensions

1. **Decide which variable to plot on *x*.** Often, this is the independent variable.
2. **Write the *range chart* for each scale.**

   x-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___

   y-scale: Low #: _________ High #: __________ Minor Unit: ___ Major: ___
3. **Consider adding zero to each range, increasing the range.** In most cases, if a range does *not* include zero, change the number to *zero* that *increases* the range.
4. **Mark the boundaries of the plot** on the graph paper.
5. **Calculate the minor unit for each scale. Round UP.** Use this equation:

$$\textit{Scale minor unit} = \frac{\text{(High \# on scale) minus (Low \# on scale)}}{\text{(The count of the grid lines on the scale)} - \mathbf{2}}$$

and then ***round UP*** to the next *easy* number to count by and count between.

6. **Make both ranges slightly wider and evenly divisible** by the *major* unit for that scale. To write a number at every second line on a scale, set the major unit as double the minor unit.
7. **Number the scales based on the minor unit. Label the scales. Title the graph.**
8. **Plot the points.**
9. **Draw the function:** either a smooth curve or straight line *near* most points.
10. **If the graph is a straight line,**
    a. **write the specific equation for the line.**
       - Calculate **m** using *two* widely spaced points that are *on* the line.
       - Find **b** using y = mx+b , x and y for any *one* point on the line, and **m**.
       - Substitute *values* for **m** and **b**, and *quantities* and *units* for y and x, into y = mx+b.

    b. **Test the specific equation.**

       For a point on the graph close to the line, pick one coordinate from the data table. Use the specific equation to *predict* the value for the other coordinate. Compare the predicted and the data table values.

    c. **Explain the equation in words.** If the graphed line goes through the origin, write the statements and equations for direct proportions.

    d. If a straight line does not go through the origin, try to explain why it does not do so, and/or consider a different definition for zero.
11. **If the graph is a smooth curve but not a straight line,** write the equation that you think fits the general equation for the curve. Then, adjust the equation to fit the format y = (constant) x , and plot the adjusted variables. If the graph is linear, do step 10. If not, try a different equation to explain the curve.