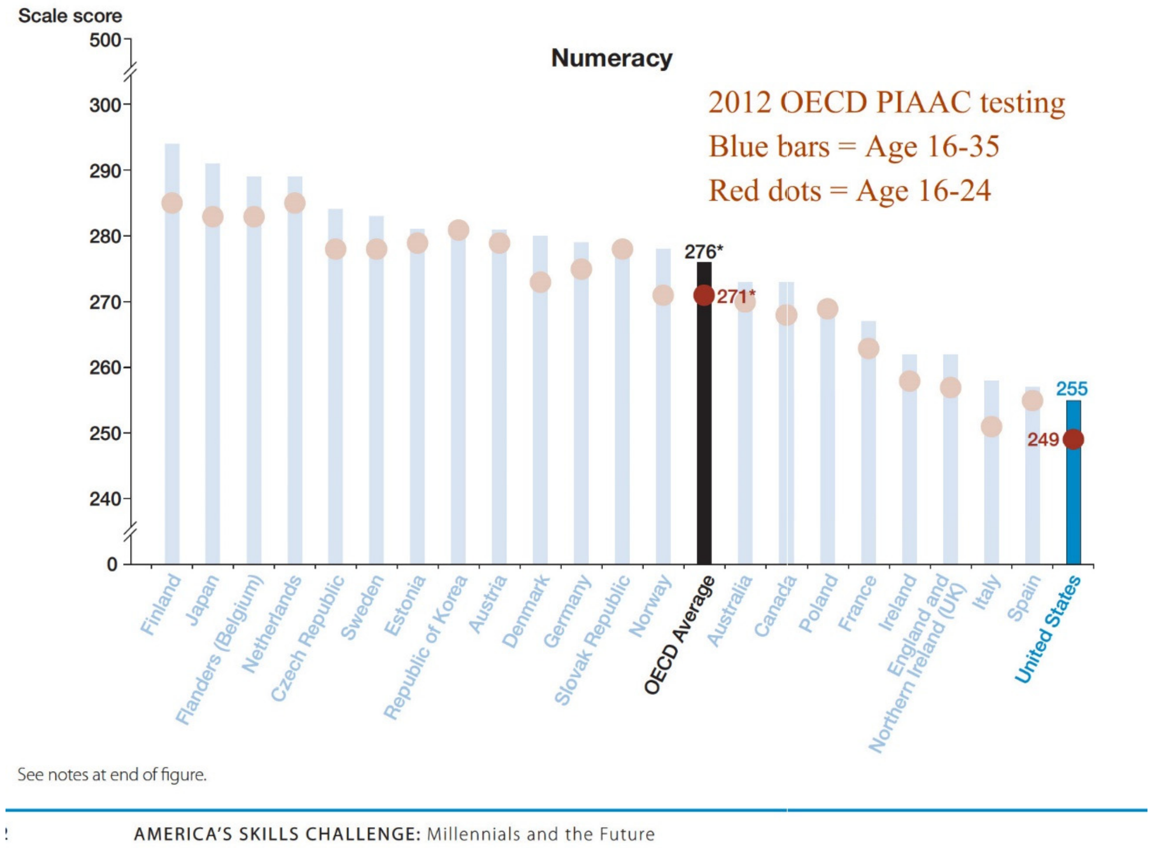
Cognitive Science and the Common Core Math Standards

Overview

In 2012, the Organisation for Economic Co-operation and Development (OECD) tested skills in 22 developed-world nations including the United States. Analysis of the data by the Educational Testing Service (ETS) found that in "numeracy" (solving problems with mathematical content), U. S. 16-24 year olds ranked 22nd out of 22 (Goodman et al. 2015).



Low skills in numeracy have consequences. Between 1984 and 2011, as a percentage of U.S. bachelor's degrees, degrees awarded in the physical sciences and engineering fell 40% (NSB 2012, NSF 2014). Nearly 60% of first-year college students who declared their intention to major in science, technology, engineering, or mathematics (STEM) in 2003 failed to gain STEM degrees within 6 years (Olson and Riordan, 2012).

What Went Wrong?

This paper compares K-12 math standards in most states to recent findings of cognitive research. The evidence indicates that a significant percentage of state math standards, now and for the past two decades, have asked students to solve problems in ways that the human brain measurably cannot do. This suggests an explanation in part for U. S student achievement in mathematics.

The good news? Cognitive studies also suggest ways in which U. S. math standards can be repaired. Let's review the new science.

Cognitive Architecture

The study of how the human brain solves problems is a subdiscipline in the field of cognitive science. Cognitive research divides problems into two types. In "well-structured" problems, experts agree on "right answers" and paths to answers are well-defined. Problems assigned in K-16 mathematics are nearly always well-structured. "Ill-structured" problems include those in which experts may disagree on answers (such as some problems in economics or political science) (Simon 1973).

Until 2009, among cognitive experts, questions involving "rote" learning and "constructivist" instructional strategies were vigorously debated. In the 2009 book *Constructivist Instruction: Success or Failure?,* eighteen articles from leading scientists reviewed the available research. Ways to teach ill-structured problem solving continued to be debated, but substantial agreement was found for well-structured problem solving (Hartman and Nelson 2015).

In *The Structure of Scientific Revolutions*, Thomas Kuhn proposed that the progress of science is a history of models and assumptions that change when they are found to be in conflict with new measurements and discoveries. Formerly debated assertions by experts become "new scientific facts" and "accepted models" when they are not challenged by other domain experts for a number of years (Kuhn 1962).

Between 2009 and 2017 among leading cognitive scientists, for well-structured problems, the areas of agreement noted in *Constructivist Instruction* have continued to be essentially uncontested.

Solving Well-Structured Problems

The points below, numbered for ease of reference, are a simplified description of the cognitive science model for well-structured problem solving. For recent findings, references will be supplied that, when possible, are reviews for educators written by experts that include primary sources.

1. Human children are biologically programmed to learn and fluently apply the words and rules of the dialect spoken around them. Academic learning in other fields, including reading and math, is not powerfully instinctive, requires effort, and benefits from instruction (Pinker 1994, Geary 2002, Sweller 2008).
2. The brain instinctively decomposes acquired knowledge into small "elements" such as a simple image, a word sound, or a simple arithmetic relationship (Anderson et al. 2000).
3. The brain solves problems using *long-term* memory (LTM) where elements of knowledge are stored and *working* memory (WM) where elements are processed.
4. LTM is composed of tens of billions of "neurons" (one type of brain cell) and their connections. Each neuron can store an element (or "chunk" of associated elements).

5. LTM has enormous capacity. Most U. S. 18-year olds can generally define about 60,000 “dictionary words” (Biemiller 2001), name images, odors, places and faces, apply math facts and procedures, and fluently apply rules from unconscious memory in topics including motor skills and speech.
6. In WM, multiple elements are temporarily stored during processing. Elements may enter WM from the senses, by recall from LTM, or by cognitive processing in WM (Willingham 2006).
7. Elements processed by WM that are not yet in LTM tend to be stored temporarily in LTM neurons. Thought about the element or effort to recall its relationships lengthens LTM retention and promotes element recall (Willingham 2003, 2008).
8. An element entering WM that can serve as a “cue” that causes an LTM neuron containing a matching element to “activate” and “fire,” sending out an electrical impulse (Anderson et al. 2004, Willingham 2008, Koedinger et al. 2012).
9. Each LTM neuron can grow hundreds of physiological “wires” that carry impulses to and from other neurons across connections at “synapses.” Neurons connected to a firing neuron may also fire and cause other linked neurons to fire.
10. Connections between neurons tend to form, or to speed and/or strengthen, when neurons fire at close to the same time during problem solving (Koedinger et al. 2012). Large networks of linked elements are said to form conceptual frameworks or “schema.”
11. Solving begins with a goal and initial data. Data are processed in WM in steps that may include recalling a relationship, calculating an answer, or storing data from a reference.
12. If a cue is part of a relationship that has been “memorized to automaticity” (is easily and quickly recalled), the cue can cause activation in its LTM network. Activated elements can be recalled into WM and applied to solve the problem (Anderson et al. 2004, Kirschner et al. 2006).
13. Elements entering WM that have not previously been stored in LTM or are not activated by cues in the problem are termed “novel” elements. These may include input from the senses (such as problem data), or a “middle step answer” determined during processing, or a “looked up” or calculator answer (Kirschner et al. 2006).
14. **The Duration Bottleneck:** Unless elements are rehearsed (repeated to keep in memory), WM can generally retain a novel element for only 3 to 30 seconds. (Peterson and Peterson 1959, Cowan 2010).
15. **The Capacity Bottleneck:** WM has an essentially unlimited ability to retrieve and process elements and relationships activated in LTM by cues (Ericsson and Kintsch 1995), but in adults, WM can hold and process only about 3 to 5 novel elements at a time (Cowan 2001, 2010).
16. If the limited slots in novel WM are full, a new novel element moved into WM will bump out a stored novel element. If the bumped-out element is needed to solve the problem, mental confusion tends to result (Kirschner et al. 2006).

17. Extensive practice using automated knowledge to solve a variety of problems tends to construct *fluency*: an intuitive sense of what facts and rules to apply when (Clark 2006).
18. Memorized stepwise procedures (algorithms) can work around the WM bottleneck by limiting how many elements must be retained in novel WM at any one time during processing (Willingham 2009).

Multiple well-regarded models for problem-solving have been proposed that vary somewhat in terminology and components from the description above, but all recent models include a WM for processing that is large for elements recallable with automaticity but minimal for novel elements.

To summarize: When solving math problems of any complexity, due to WM limits, students must rely almost entirely on well-memorized facts and algorithms.

Example

For those who know their times tables, the strengths, limits, and a work-around for WM limits can be experienced with an experiment (of a type suggested by Willingham): Try multiplying 93 times 72 "in your head." Do not use fingers or pencil and paper. Take a moment to try.

Did a "middle step answer" bump out of WM an element you needed to remember from a previous step? Now try 93 times 72 with pencil and paper.

Did you use an algorithm automated long ago? Which method was easier? Why?

Standards

Cognitive science suggests an explanation in part for current levels of U.S. math achievement.

Before 1975, young people memorized math facts and algorithms as a foundation for careers including building trades, mechanics, accounting, business, science, and engineering.

Inexpensive calculators became available in the 1970's, and by 1996, a majority of U. S. 8th graders reported daily use of calculators in math classes (Waits and Demana 2000). Between 1990 and 2000, over 40 states adopted "state math standards" aligned with 1989 recommendations by the National Council of Teachers of Mathematics (NCTM). The NCTM advocated calculator use "at all grade levels" (Klein 2002, Loveless 2002, 2003, Schoenfeld 2004).

Additionally, in Grades 5-8, the 1989 NCTM standards called for "increased attention" to "reasoning" and "decreased attention" to "memorizing rules and algorithms," "manipulating symbols," and "rote practice" (NCTM 1989). NCTM-type standards remained in place in most U. S. states until 2009 (Carmichael et al. 2010).

The cognitive science of 2017 suggests that with "decreased attention" to "memorizing rules and algorithms," skill in calculations would decline, and between 1995 and 2002, available data from states using NCTM-type standards show a sharp decline in student test scores in math computation (Hartman and Nelson 2016). For the few states with standards that emphasized "computational facility," test scores were relatively high (Schmid 2000, Loveless 2003), but by

2012, overall U. S. young-adult skills in numeracy ranked dead last among 22 nations in OECD testing.

CCMS Adoption

Federal legislation in 2002 incentivized testing based on state standards, but with nearly 50 different sets of standards and tests, comparisons between states, or to national or international norms, were difficult at best.

In 2009, the National Governors Association and the Council of Chief State School Officers sponsored the drafting of a single set of standards in English and mathematics. Between 2010 and 2012, over 40 states adopted the Common Core Math Standards (CCMS) (National Governors Association 2010). A number of states rescinded CCMS adoption or revised the standards between 2012 and 2016, but in most, standards remain similar to the CCMS (Heitin 2015, Norton et al. 2017). By 2015, most local districts were gradually purchasing textbooks advertised as aligned with the CCMS and most states based annual testing on CCMS-type standards.

The impact of new standards cannot be measured until teaching is based upon them for a number of years, but one important role of science is to identify rules that allow accurate prediction of the outcomes of procedures and processes. What results do cognitive science predict for the CCMS?

Subtraction and Division

The knowledge taught in mathematics is generally divided into facts, procedures, and concepts (Willingham 2009, Siegler and Lortie-Forgues 2015).

In the 2008 *Report of the National Mathematics Advisory Panel*, a presidential commission, five of the nation's leading cognitive scientists (David Geary, Valerie Reyna, Wade Boykin, Susan Embretson, and Robert Siegler) advised that the "central" strategy to improve student problem solving is

> "[T]he achievement of automaticity, that is, the fast, implicit, and automatic retrieval of a fact or a procedure from long-term memory …. [A]rithmetic facts … should be thoroughly mastered, and indeed, over-learned."

"Overlearning" is defined as repeated practice to perfection in recalling knowledge.

Cognitive scientist Richard Clark (2006) notes that researchers using computer programming to model information processing by the brain make:

> "… a very compelling case that all effective applied knowledge must be proceduralized and automated in order to circumvent the limits on working memory…. Most [other researchers] reach a similar conclusion about the importance of the automaticity process."

Citing extensive studies, cognitive scientist Daniel Willingham (2009) writes:

> "{A]nswers must be well learned so that when a simple arithmetic problem is encountered…, the answer is *not calculated* but simply retrieved from memory." (emphasis added)

But the CCMS ask students to know "from memory" only half of the arithmetic facts, and to calculate the other half. The 1st grade CCMS includes:

> **1.OA.6.** Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten (e.g., 13 – 4 = 13 – 3 – 1 = 10 – 1 = 9); …..

The related 2nd grade standard is:

> **2.OA.2.** Fluently add and subtract within 20 using mental strategies (from 1.OA.6). By the end of Grade 2, know from memory all sums of two one-digit numbers.

In a 2016 interview (Fordham 2016) of Dr. Jason Zimba, one of three "lead writers" of the CCMS, the Fordham Foundation asked if in the standards, "'being fluent in' also means 'know from memory.'" Zimba answered:

> JZ: They aren't the same thing, and the language of the standards makes this clear…. [In both 2.OA.2 and 3.OA.7]: If being fluent were the same thing as knowing from memory, the second sentence would not have been necessary. …. In every case, fluency pertains to an act of calculation."

By this definition of fluency, the CCMS never ask that students "know from memory" the subtraction or division facts, calling for calculation instead. Cognitive science calls for retrieval from memory, and not calculation, for all fundamental facts.

Recall vs. Calculation

Why must all arithmetic facts must be "memorized to automaticity?" According to science:

- When 56/8 must be done on a calculator, mentally storing the answer for transfer to paper will occupy a novel WM slot which, if novel WM is already full, will bump out an element that may be needed during subsequent processing.
- Even a quick mental calculation of 56/8 requires information to be placed into novel WM. Automated retrieval does not.
- Any calculation takes more time than automated recall, and the multiple elements in novel WM begin to drop out after only a few seconds.
- Calculated answers are more likely to have errors than recalled facts (Willingham 2009).
- Automated recall helps to free slots in novel WM for "context cues" that distinguish different types of calculations. Processing those cues is a key step in building the conceptual frameworks that promote recall of appropriate facts and algorithms (Willingham 2003, 2008, 2015).

Given the time and effort required to automate recall, it would seem logical to take the CCMS advice: memorize simple addition and multiplication facts, then calculate the subtraction and division. But standards must work both mathematically and cognitively. Willingham (2009) notes:

> "[A]utomatic retrieval of basic math facts is critical to solving complex problems…. [B]efore they are learned to automaticity, calculating simple arithmetic facts does indeed require working memory."

Arithmetic is the foundation for mathematics. By asking that only half of the fundamental arithmetic facts be automated, the CCMS creates what science predicts will be a "critical" barrier to subsequent learning.

Learning Facts: Sequence and Timing

For all of 1st grade and part of 2nd, the CCMS ask pupils to "fluently" solve multi-step addition and subtraction before they know addition facts (see 1.OA.6 and 2.OA.2 above). In several 3rd grade 3.OA standards, the CCMS ask students to multiply and divide before knowing their multiplication facts.

The 2010 CCMS assumes the brain can work effectively with facts before they have been well-memorized. Science in 2017 says the student brain cannot.

Age Appropriate?

On average, 1st graders have a smaller novel WM capacity than 4th graders, who have a smaller capacity than adults (Cowan 2001). Gathercole (2006, 2008) notes, "Poor working memory skills are relatively commonplace in childhood" and advises to "avoid working memory overload in structured learning activities." Asking 1st graders to perform multi-step addition and subtraction before knowing addition facts would seem by definition to raise concerns of WM overload.

Required Time

Counting members of commutative pairs separately, there are more than 160 math facts that involve two single digits for addition and multiplication, and over 160 more for subtraction and division. Learning mathematics requires extensive "verbatim" recall in addition to less precise "gist" (summary) recall. Geary et al. (2008) write:

> "Verbatim recall of math knowledge is an essential feature of math education, and it requires a great deal of time, effort, and practice."

Achieving automaticity generally involves strategies such as overlearning spaced over days and weeks, repeated self-testing, and interleaved practice (Willingham 2003, Brown et al. 2015). These strategies are time-consuming, but according to science, they efficiently promote the automaticity in recall that is required for math facts and procedures.

CCMS goals include "know from memory" the addition facts at "the end of Grade 2" and multiplication facts at "the end of Grade 3," but the CCMS do not state when learning of these math facts is to begin. The CCMS do not call for subtraction and division facts to be known from memory at all. Given the large number of standards at each grade in Grades 1-3 before mastery of component facts is mentioned, it is not clear that the CCMS provide reasonable time for a goal that all students automate recall of all arithmetic fundamentals.

Standard Algorithms

For millennia, to address poorly understood but quite evident cognitive limitations, students were taught to solve multi-step problems by applying apply standard, stepwise procedures (algorithms). Willingham summarizes:

> "certain procedures are used again and again. Those procedures must be learned to the point of automaticity…" (Willingham 2004).

Geary et al. (2008) write that

> "fundamental algorithms should be thoroughly mastered, and indeed, over-learned…."

The CCMS call for students to fluently apply the standard algorithms for arithmetic operations, but in multiple topics, science predicts the CCMS instructional sequence is likely to interfere with applying those algorithms fluently. For multiplication, the 4th grade standard is:

> **4.NBT.5**. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The corresponding 5th grade standard is:

> **5.NBT.5**. Fluently multiply multi-digit whole numbers using the standard algorithm.

For each of the four multi-digit operations, the CCMS ask students to practice multiple strategies for one year or two before learning the standard algorithm. Cognitive studies have found that if two processes are learned that are similar, steps from one can "interfere" with recalling the other (Dewar et al. 2007).   Practicing multiple "strategies" for a year will move individual steps and sequences into memory that will be errors in the standard algorithm.

After practicing multiple different multiplication procedures over a two year period, will students know "with automaticity" after Grade 5 which procedure steps to apply?

Standard algorithms are exact (verbatim) procedures. Learning four procedures takes time. For each procedure, is it a wise use of limited instructional time to require students to spend a year or more practicing multiple non-standard procedures that presumably will not be used when the standard algorithm becomes standard?

To align the CCMS with science, each standard algorithm would be practiced until it is automated. The underlying concepts would then be explored in a variety of ways to promote understanding, which could include the "strategies" in the CCMS.

Theories of Learning

Reasoning versus Algorithms

The 1989 NCTM standards emphasized reasoning over memorization. The CCMS, for facts and procedures, ask students to begin each topic by practicing general reasoning strategies for a year or more. Cognitive science has found that for students in math, using "general reasoning strategies" in the absence of well-memorized facts and algorithms is nearly always ineffective as a strategy to solve problems (Sweller et al. 2010). Summarizing research, Clark et al. (2012) explain why a "reason first, then memorize" sequence is not likely to work:

> "If the learner has no relevant concepts and procedures in long-term memory…., searching for a solution overburdens limited working memory…. As a consequence, novices can engage in problem-solving activities for extended periods and learn almost nothing."

Discovery and Misconceptions

Schwartz and his colleagues (2011) report that "guided inquiry to introduce a topic can prepare students to see deeper conceptual principles." However, if extensive discovery is encouraged before instruction identifies what is correct, there is substantial risk students will move misconceptions into memory, and misconceptions are very difficult to root out (Willingham 2003, Rosenshine 2012).

Except for a small amount of initial "discovery," science supports learning facts and standard procedures first. Clark et al. (2012) summarize:

> "Decades of research clearly demonstrate that for novices (comprising virtually all students)…, teachers are more effective when they provide explicit guidance accompanied by practice and feedback, not when they require students to discover many aspects of what they must learn…."

"Discovery" may have a strong philosophical appeal, but when students are asked to spend extensive time doing what science says their brains cannot reliably do, is it likely to discourage and frustrate?

To align standards with science, "discovery" could briefly introduce a topic and its context. Next, students would practice the retrieval of facts, followed by problems applying procedures and activities focused on deeper understanding.

Activities and Conceptual Understanding

Science emphasizes that conceptual understanding is vitally important in mathematics and it needs to be taught (Siegler and Lortie-Forgues 2015). Geary et al. (2008) write:

> "[T]he cognitive processes that facilitate rote retention (e.g., of over-learned arithmetic facts), such as repeated practice, can differ from the processes that facilitate transfer and long-term retention, such as conceptual understanding."

At the right time, Clark et al. (2012) note that a variety of problem-solving activities can be useful:

> "Independent problems and projects can be effective – not as vehicles for making discoveries, but as a means of practicing recently learned content and skills."

The CCMS "strategies" have the potential to help students organize conceptual frameworks, but when should those activities should occur? The introduction section of the CCMS quotes Schmidt, Houang, and Cogan (2002):

> "[T]o be coherent, a set of content standards must evolve *from* particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures…) *to* deeper structures inherent in the discipline…." (emphasis added)

Schmidt et al. align with cognitive science, but the CCMS for topics noted above move in the opposite sequence: *from* multiple activities aimed at "deeper structures" *to* "simple math facts and procedures."

Physiologically, "deeper" connections cannot grow until elements are stored in LTM neurons (memorized) and fundamental relationships are established (Anderson et al. 2000).

Conclusion

The CCMS are an improvement over the 1989 NCTM-type standards in place in most states prior to 2010. Many activities advocated by the CCMS support the construction of conceptual frameworks if they are scheduled properly, representing an improvement over many "traditional" math curricula.

However, the repeated CCMS omissions and delays in memorization result in learning strategies that in the student brain simply do not work. When processing information that is not well memorized, the limits of WM are now verified science. Whenever new science is confirmed, existing theories related to that science must be re-evaluated.

The finding of a critical role for initial memorization may be counter-intuitive, surprising, and/or disappointing, but that is how new science often tends to be.

For many topics in addition to those addressed above, the 2010 CCMS is at odds with 2017 science. However, given the weaknesses identified in arithmetic, the foundation for mathematics, the author would submit that a sufficient case has been made for a detailed review of the current CCMS and similar standards by a panel that includes both cognitive experts and educators.

When science reaches a new consensus on methodologies likely to benefit students, educators are morally obligated to adjust and align their instruction with scientific best practices. But teachers are also required to direct their instruction toward meeting state standards. For the system to work, these two obligations must align.

Because public education is taxpayer funded, elected officials must hold final responsibility for public schools, but if state officials adopt or revise math standards without consideration for the recommendations of science, instruction will be far less effective than it needs to be.

Until standards are repaired, to what extent should instructors be held accountable for levels of student achievement? Schools need more than effective standards, but when standards that direct instruction, according to science, in significant measure do not work, students and teachers cannot be successful.

Among developed-world nations, U.S. 16-24 year olds rank dead last in numeracy. Going forward, if the standards that guide instruction deny science, our society will not prosper.

Improved understanding of how the brain solves problems is tremendous scientific progress. Applying those findings to math standards would benefit both students and our nation.

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