# Cognitive Science and the Common Core Mathematics Standards

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## Abstract

Between 1995 and 2010, most U.S. states adopted K-12 math standards which discouraged memorization of math facts and procedures.  Since 2010, most states have revised standards to align with the K-12 Common Core Mathematics Standards (CCMS).  The CCMS do not ask students to memorize facts and procedures for some key topics and delay work with memorized fundamentals in others.

Recent research in cognitive science has found that the brain has only minimal ability to reason with knowledge that has not previously been well-memorized.  This science predicts that students taught under math standards that discouraged initial memorization for math topics will have significant difficulty solving numeric problems in mathematics, science, and engineering.  As one test of this prediction, in a recent OECD assessment of numeracy skills among 22 developed-world nations, U.S. 16-24 year olds ranked dead last.  Discussion will include steps that can be taken to align K-12 state standards with practices supported by cognitive research.

## Introduction

### U.S. Math and Science Achievement

In 2012, the Organisation for Economic Co-operation and Development (OECD) tested skills in 22 developed-world nations including the United States. In "numeracy" (solving problems with mathematical content), U. S. 16-34 year olds and 16-24 year olds ranked dead last (OECD 2013). Figure 1 displays an Educational Testing Service (ETS) rendering of these data from the OECD's Survey of Adult Skills (Goodman, Sands, Coley, p. 12).

**Figure 1. Average scores in numeracy for age 16-34 (millennials) and age 16-24, by participating country/region: 2012**
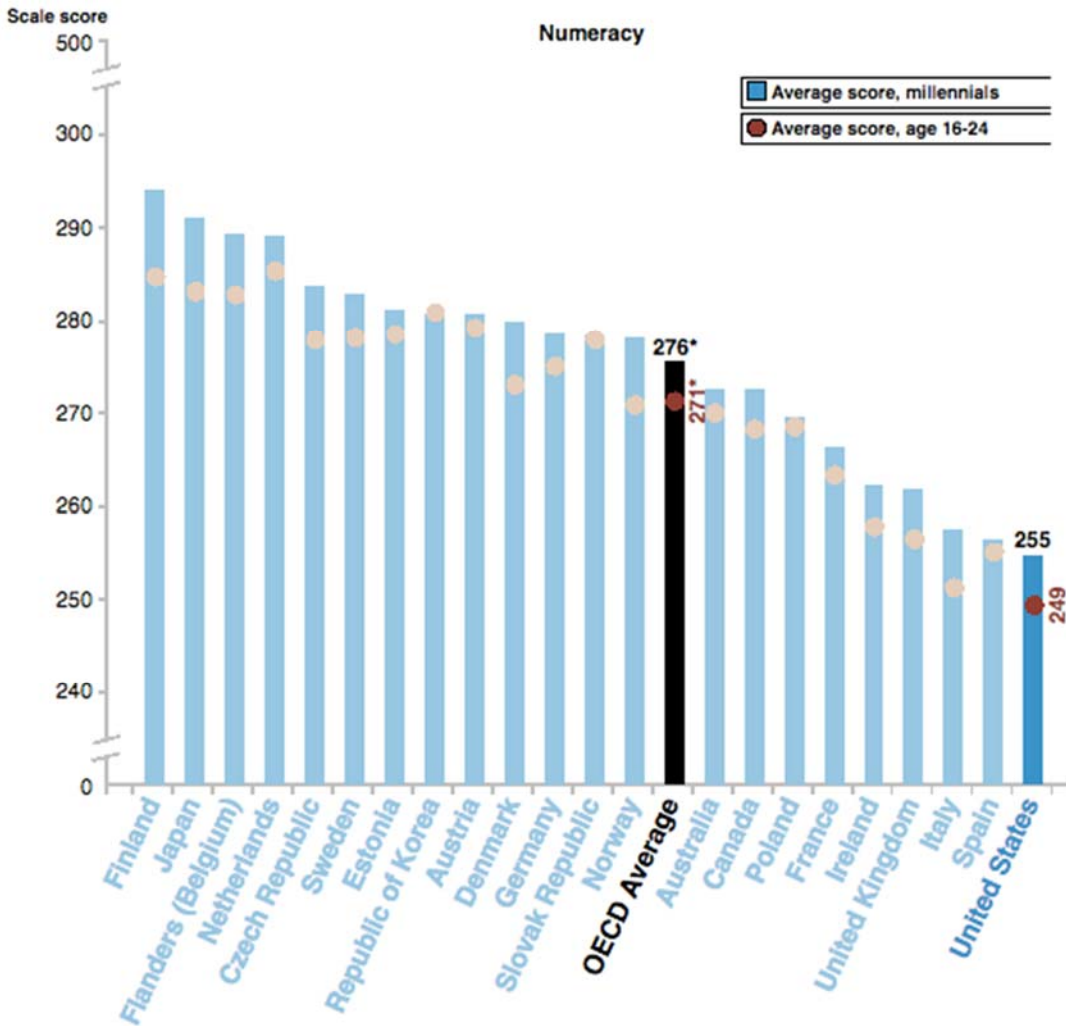


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Low skills in numeracy have consequences. Between 1984 and 2011, as a percentage of U.S. bachelor's degrees, degrees awarded in the physical sciences and engineering fell 40% (NSB 2012, NSF 2014). Nearly 60% of first-year college students who declared their intention to major in science, technology, engineering, or mathematics (STEM fields) in 2003 failed to gain STEM degrees within 6 years (Olson and Riordan, 2012).

U.S. students who are U.S. citizens have disproportionate difficulty in achieving degrees in fields that require substantial preparation in mathematics. According to the National Science Foundation (NSF), of the doctorates awarded by U.S. universities between 2002 and 2006, non-U.S. citizens received 44% in chemistry, 57% in mathematics, 58% in physics, and 65% in

computer sciences.  Of U.S. Ph. D's awarded in engineering, 33% went to U.S. citizens, while 44% went to citizens of China, India, and Korea.  Of U. S. doctorates in electrical engineering, 23% went to U.S. citizens (Falkenheim 2007).

A high percentage of foreign students remain in the US after schooling (Finn 2012) and it is well-documented that they make a disproportionate contribution to the American economy. One wonders, however, how long these "best and brightest" from around the globe will remain in a nation where schools fail to prepare their children to follow in their footsteps.

**What Went Wrong?**

Comparing K-12 math standards in most U.S. states to findings of recent cognitive research, the evidence indicates that a significant percentage of state math standards, now and for the past two decades, have asked students to solve problems in ways that a student (non-expert) brain measurably cannot do.  This suggests an explanation in part for U. S. student achievement in mathematics.

Cognitive studies also suggest ways in which U. S. math standards and achievement can be repaired.

# Cognitive Architecture

The study of how the human brain solves problems is a sub-discipline in the field of cognitive science.  Cognitive research divides problems into two types.  In a "well-structured" problem, experts agree on a "right answer" and paths to the answer are well-defined.  The calculation problems explained and assigned in "traditional" math textbooks are nearly always well-structured.  "Ill-structured" problems include those in which experts may disagree on answers (as in many problems in economics or political science) (Simon 1973).

Between 1990 and 2009, as new technologies and research increased scientific knowledge of how the brain works, the importance of memorization and "constructivist" strategies in instruction was debated among cognitive experts.  In the 2009 book *Constructivist Instruction: Success or Failure?,* methods of teaching ill-structured problem solving continued to be contested among leading cognitive scientists, but substantial agreement was noted on methods to teach well-structured problem solving (Tobias and Duffy, 2009).

Thomas Kuhn proposed in *The Structure of Scientific Revolutions* (1962) that the progress of science is a history of models and assumptions that change when they are found to be in conflict with new measurements and discoveries.  Formerly debated assertions by experts become "new scientific facts" and "accepted models" when they are not challenged by other domain experts for a number of years.

Between 2009 and 2017, among leading cognitive scientists, the agreement noted in *Constructivist Instruction* for well-structured problems has continued to be uncontested.

**Solving Well-Structured Problems**

Below is a simplified description of the cognitive science model for well-structured problem solving. When available, references cited will be relatively jargon-free reviews for educators written by cognitive experts that include extensive citation of peer-reviewed research.

Learned knowledge can be divided into two types. "Primary" knowledge is that which humans are biologically programmed to learn instinctively during "window periods" of development. Examples during childhood include learning to distinguish faces, comprehend the local spoken dialect, and create fluent speech in the dialect.

"Secondary" knowledge is what we are not programmed by evolution to acquire automatically, but is beneficial to know for functioning in our society. Examples include learning to read, write, and solve math problems. Primary learning occurs without effort by repeated exposure to primary knowledge, but learning secondary knowledge generally requires cognitive effort. The purpose of schooling is to move secondary knowledge into memory (Pinker 1994, Geary 2002, Sweller 2008).

Two types of memory relied upon in problem-solving are *long-term* memory (LTM) where elements of knowledge are stored and *working* memory (WM) where elements are processed. If a stimulus from the senses becomes a focus of attention, the brain decomposes this newly acquired knowledge into small "elements" such as a simple image, sound, or relationship, and moves the elements into working memory (Anderson et al. 2000).

Long-term memory is composed of tens of billions of "neurons" (one type of brain cell) and their connections. Each neuron can store a knowledge element. Elements processed in working memory that are not yet stored in long-term memory tend to be added to LTM.

Human LTM has enormous capacity. Most U. S. 18-year olds can generally define about 60,000 "dictionary words" (Biemiller 2001). In addition, entering adulthood, humans can identify tens of thousands of images, odors, places and faces, apply math facts and procedures, and fluently apply many thousand rules and procedures in areas including motor skills and creating speech.

Each LTM neuron can grow hundreds of physiological connections to other neurons. This "wiring" can carry electrical impulses to and from other neurons, with linkages between cells connecting at "synapses."

If an element entering WM has previously been stored in LTM, the element can serve as a "cue" that causes the LTM neuron storing the matching element to "activate" and "fire," sending out an electrical impulse. The neurons connected to a firing neuron may also fire and cause other linked neurons to fire (Anderson et al. 2004, Willingham 2008, Koedinger et al. 2012). A large network of linked elements is said to form a conceptual framework or "schema" (plural "schemata").

Connections between neurons tend to form, or to speed and/or strengthen, when neurons fire at or close to the same time during problem solving (Koedinger et al. 2012). In general, "memory is the residue of thought" (Willingham (2003, 2008). Relationships between the elements stored in neurons are more likely to be retrievable if the elements are repeated processed together in WM. Repeated thought that involves a relationship among stored elements tends to strengthen

their wired connection so that when the neuron containing one is activated, the others are more likely to activate.

**Strengths and Limitations of Working Memory**

Working memory is where the brain solves problems.  Problem solving begins with a goal and initial data.  Data are processed in WM in steps that may include storing data from the senses, recalling a relationship from LTM, or calculating a middle-step answer.

If a cue is part of a relationship that has been "memorized to automaticity" (is easily and quickly recalled), the cue can cause activation of related elements in its LTM network.  Working memory has an essentially unlimited ability to recall elements from relationships that have been substantially activated and apply those elements to solve a problem (Ericsson and Kintsch 1995, Anderson et al. 2004, Kirschner et al. 2006).

Elements entering WM that have not previously been stored in LTM or do not tend to be substantially activated by cues in the problem are termed "novel" elements.  These may include novel input from the senses (such as problem data), or a "middle step answer" determined during the steps of problem solving, or a "looked up" or a calculator answer (Kirschner et al. 2006).

For these novel elements, working memory is limited in both duration and capacity.  Unless novel elements are rehearsed (repeated to keep in memory), WM can generally retain an element for only 3 to 30 seconds. (Peterson and Peterson 1959, Cowan 2010).  For most adults, WM is generally estimated to be able to hold and process only 3 to 5 novel elements at each step during problem solving (Cowan 2001, 2010).

**Working Memory Implications for Learning**

Perhaps the most significant discovery of cognitive research on learning in the past two decades is that working memory has an essentially unlimited ability to hold and process elements of knowledge that can be activated by cues and quickly recalled from long-term memory, but can hold only a few elements at each point in processing which cannot quickly be recalled.  An individual's problem solving ability depends on how much information about the topic is stored in the individual's LTM and the quantity and quality of the linkages that identify the relationships among memorized elements.

If information must be calculated on a calculator or looked up on the internet, instead of recalled from memory, the resulting novel element or elements are difficult to retain in memory for a length of time during subsequent steps of problem solving.

Further, at each step of problem solving, if too much information from any source is novel, the limited slots in novel WM may be full, and a new novel element moved into WM will bump out a stored novel element. If the bumped-out element is needed to solve the problem, mental confusion tends to result (Kirschner et al. 2006).

Cognitive scientists have identified three major ways to "work around" the bottleneck in working memory: Automaticity, algorithms, and chunking.

- If the facts, rules, and procedures governing a topic are "automated" (stored in LTM and well linked), information can be recalled via cues into WM and processed when needed. Extensive practice using "automated" knowledge to solve a variety of problems gradually tends to construct *fluency*: an intuitive sense of what facts and rules to apply when (Clark 2006).

- Memorized stepwise procedures (algorithms) can work around the WM bottleneck by limiting how many elements must be retained in novel WM at any one time during processing (Willingham 2009).

- If elements that frequently occur together are memorized to form a single "chunk" (such as a familiar area code or acronym), the chunk of problem data can be remembered as a single element of unique data during processing.

Multiple well-regarded models for problem-solving have been proposed that vary somewhat in terminology and components from the description above, but all recent models include a WM for processing that is large for elements recallable with automaticity but minimal for novel elements.

Summarizing the implications for learning of these new scientific discoveries:

- When solving math problems of any complexity, due to WM limits, students must rely almost entirely on well-memorized facts and algorithms.
- Solving problems involving secondary knowledge requires study: first effort to move new information into long-term memory, and then problem solving that builds conceptual frameworks by processing new information in a variety of distinctive contexts.
- To improve problem-solving skills, the goal of study must be to increase the content and improve the organization of an individual's long-term memory.

**Experiencing (and Working Around) WM Limits**

For those who were required to learn their times tables and practice the "standard algorithms" of arithmetic, the strengths, limits, and a work-around for WM limits can be experienced with an experiment (of a type suggested by Willingham): Multiply 93 times 72 "in your head." Do not use fingers or pencil and paper. Take a moment to try.

Did a "middle step answer" bump out of WM an element you needed to remember from a previous step? Now try 93 times 72 with pencil and paper.

Did you use an algorithm automated long ago? Which method was easier? Why?

# Standards

The recent findings of cognitive research suggest an explanation in part for current levels of U.S. math achievement.

Before 1975, as a part of learning math, young people memorized math facts and algorithms. That knowledge served as a foundation for careers including science, engineering, building trades, mechanics, accounting, and business.

Inexpensive calculators became available in the 1970's, and by 1996, a majority of U. S. 8th graders reported daily use of calculators in math classes (Waits and Demana 2000). Between 1990 and 2000, over 40 states adopted "state math standards" aligned with 1989 recommendations by the National Council of Teachers of Mathematics (NCTM) (Klein 2002, Loveless 2002, 2003, Schoenfeld 2004). The NCTM advocated calculator use "at all grade levels."

Additionally, in Grades 5-8, the 1989 NCTM standards called for "increased attention" to "reasoning" and "decreased attention" to "memorizing rules and algorithms," "manipulating symbols," and "rote practice" (NCTM 1989). NCTM-type standards remained in place in most U. S. states until 2010 (Carmichael et al. 2010).

The cognitive science of 2017 suggests that with "decreased attention" to "memorizing rules and algorithms," skill in calculations would decline, and between 1995 and 2002, available data from states using NCTM-type standards show a sharp decline in student test scores in math computation (Hartman and Nelson 2016). For the few states with standards that emphasized "computational facility," test scores were relatively high (Schmid 2000, Loveless 2003), but by 2012, overall U. S. young-adult skills in numeracy ranked dead last among the 22 nations in OECD testing.

**CCMS Adoption**

Federal "No Child Left Behind" legislation in 2002 incentivized testing based on state standards, but with nearly 50 different sets of standards and tests, comparisons between states, or to national or international norms, were difficult at best. Despite sizable state spending and school time devoted to testing, it was difficult to tell how students and states were doing compared to students in other states and nations.

In 2009, the National Governors Association and the Council of Chief State School Officers sponsored the drafting of a single set of "Common Core" standards in English and mathematics. Between 2010 and 2012, over 40 states adopted the resulting Common Core Math Standards (CCMS) (National Governors Association 2010). A number of states rescinded CCMS adoption or revised the standards between 2012 and 2016, but in most, standards remain similar to the CCMS (Heitin 2015, Norton et al. 2017). By 2015, most local districts were gradually purchasing textbooks advertised as aligned with the CCMS and most states based annual testing on CCMS-type standards.

The impact of new standards cannot be measured until teaching is based upon them for a number of years, but one important role of science is to identify rules that allow accurate prediction of the outcomes of procedures and processes.

By applying what cognitive science knows about how the brain solves problems, can we predict the likely outcomes of instruction based on the standards of the CCMS?

# Subtraction and Division

The knowledge taught in mathematics is generally divided into facts, procedures, and concepts (Willingham 2009, Siegler and Lortie-Forgues 2015).

In the 2008 *Report of the National Mathematics Advisory Panel*, a presidential commission, five of the nation's leading cognitive scientists (David Geary, Valerie Reyna, Wade Boykin, Susan Embretson, and Robert Siegler) advised, to avoid working memory limits, that the "central" strategy to improve student problem solving is

> "[T]he achievement of automaticity, that is, the fast, implicit, and automatic retrieval of a fact or a procedure from long-term memory …. [A]rithmetic facts … should be thoroughly mastered, and indeed, over-learned."

In cognitive studies, "overlearning" is defined as repeated practice in recalling new knowledge.

Cognitive scientist Richard Clark (2006) notes that researchers who use computers to model information processing by the brain make:

> "… a very compelling case that all effective applied knowledge must be proceduralized and automated in order to circumvent the limits on working memory…. Most [other researchers] reach a similar conclusion about the importance of the automaticity process."

Citing extensive studies, cognitive scientist Daniel Willingham (2009) writes:

> "{A}nswers must be well learned so that when a simple arithmetic problem is encountered…, the answer is *not calculated* but simply retrieved from memory." (emphasis added)

The CCMS, however, ask students to know "from memory" only half of the fundamental arithmetic facts, and to calculate the other half.  The 1st grade CCMS includes:

> **1.OA.6.**  Add and subtract within 20, demonstrating fluency for addition and subtraction within 10. Use strategies such as counting on; making ten
> (e.g., 8 + 6 = 8 + 2 + 4 = 10 + 4 = 14); decomposing a number leading to a ten
> (e.g., 13 − 4 = 13 − 3 − 1 = 10 − 1 = 9); …..

The related 2nd grade standard is:

> **2.OA.2.**  Fluently add and subtract within 20 using mental strategies (from 1.OA.6).
> By the end of Grade 2, know from memory all sums of two one-digit numbers.

In a 2016 interview (Fordham 2016) of Dr. Jason Zimba, one of three "lead writers" of the CCMS, the Fordham Foundation asked if in the standards, "'being fluent in' also means 'know from memory.'"  Zimba answered:

> JZ: They aren't the same thing, and the language of the standards makes this clear…. [In both 2.OA.2 and 3.OA.7]: If being fluent were the same thing as knowing from memory, the second sentence would not have been necessary. ….  In every case, fluency pertains to an act of calculation."

By this definition of fluency, the CCMS never ask that students "know from memory" the subtraction or division facts, calling for calculation instead. Cognitive science emphasizes the importance of retrieval from memory, and the disadvantages of calculation, for all fundamental facts.

**Recall vs. Calculation**

Why must all arithmetic facts must be "memorized to automaticity?" According to science:

- When 56/8 must be done on a calculator, mentally storing the answer for transfer to paper will occupy a novel WM slot which, if novel WM is already full, will bump out an element that may be needed during subsequent processing.

- Even a quick mental calculation of 56/8 requires information to be placed into novel WM. Automated retrieval does not.

- Any calculation takes more time than automated recall, and each of the other elements stored in novel WM during prior problem steps begin drop out after only a few seconds.

- Calculated answers are more likely to have errors than recalled facts (Willingham 2009).

- Automated recall helps to free slots in novel WM for "context cues" that distinguish different types of calculations. Processing those cues is a key step in building the conceptual frameworks that promote recall of appropriate facts and algorithms (Willingham 2003, 2008, 2015).

Given the time and considerable effort required to automate recall, it would seem logical to take the CCMS advice to memorize simple addition and multiplication facts, then calculate the subtraction and division. But math standards must work both mathematically *and* cognitively. Willingham (2009) notes:

> "[A]utomatic retrieval of basic math facts is critical to solving complex problems…. [B]efore they are learned to automaticity, calculating simple arithmetic facts does indeed require working memory."

Arithmetic is the foundation for mathematics. By asking that only half of the fundamental arithmetic facts be automated, the CCMS creates what science predicts will be a "critical" barrier to subsequent learning.

## Learning Facts:  Sequence and Timing

For all of 1st grade and part of 2nd, the CCMS ask pupils to "fluently" solve multi-step addition and subtraction before they know addition facts (see 1.OA.6 and 2.OA.2 above). In several 3rd grade 3.OA standards, the CCMS ask students to multiply and divide before knowing their multiplication facts.

The 2010 CCMS assumes the brain can work effectively with facts before they have been well-memorized. Science in 2017 says the student brain cannot.

**Age Appropriate?**

On average, 1st graders have a smaller novel WM capacity than 4th graders, who have a smaller capacity than adults (Cowan 2001). Gathercole (2006, 2008) notes, "Poor working memory skills are relatively commonplace in childhood" and advises to "avoid working memory overload in structured learning activities." Asking 1$^{st}$ graders to perform multi-step addition and subtraction (see 1.OA.6) before knowing addition facts would seem by definition to raise concerns of WM overload.

**Required Time**

Counting members of commutative pairs separately, there are more than 150 math facts that involve two single digits for addition and subtraction, and over 150 more for multiplication and division. Learning mathematics requires extensive "verbatim" recall, which is generally more difficult to achieve than less precise "gist" (summary) recall. Geary et al. (2008) write:

> "Verbatim recall of math knowledge is an essential feature of math education, and it requires a great deal of time, effort, and practice."

Achieving automaticity generally involves strategies such as overlearning spaced over days and weeks, repeated self-testing, and interleaved practice (Willingham 2003, Brown et al. 2015). These strategies are time-consuming, but according to science they efficiently promote the automaticity in recall that is required for math facts and procedures.

CCMS goals include "know from memory" the addition facts at "the end of Grade 2" and multiplication facts at "the end of Grade 3," but the CCMS do not state when learning of these math facts is to begin. The CCMS do not call for subtraction and division facts to be known from memory at all. Given the large number of standards at each grade in Grades 1-3 before mastery of component facts is mentioned, it is not clear that the CCMS provide reasonable time during instruction for a goal that all students automate recall of all arithmetic fundamentals.

# Standard Algorithms

For millennia, to address poorly understood but quite evident cognitive limitations, students were taught to solve multi-step problems by applying standard, stepwise procedures (algorithms). Willingham summarizes:

> "certain procedures are used again and again. Those procedures must be learned to the point of automaticity…" (Willingham 2004).

Geary et al. (2008) write that

> "fundamental algorithms should be thoroughly mastered, and indeed, over-learned…."

The CCMS call for students to fluently apply the standard algorithms for arithmetic operations, but in multiple topics, science predicts the CCMS instructional sequence is likely to interfere with applying those algorithms fluently. For multiplication, the 4th grade standard is:

> **4.NBT.5**. Multiply a whole number of up to four digits by a one-digit whole number, and multiply two two-digit numbers, using strategies based on place value and the

properties of operations. Illustrate and explain the calculation by using equations, rectangular arrays, and/or area models.

The corresponding 5th grade standard is:

**5.NBT.5**. Fluently multiply multi-digit whole numbers using the standard algorithm.

For each of the four multi-digit operations, the CCMS ask students to practice multiple strategies for one year or two before learning the standard algorithm.  Cognitive studies have found that if two processes are learned that are similar, steps from one can "interfere" with recalling the other (Dewar et al. 2007).   Practicing multiple "strategies" for a year will move individual steps and sequences into memory that will be errors in the standard algorithm.

After practicing multiple different multiplication procedures over a two year period, will students know "with automaticity" after Grade 5 which procedure steps to apply?

Standard algorithms are exact (verbatim) procedures. Learning four exact procedures takes effort over time.  Is it a wise use of precious instructional time to require students to spend a year or more practicing multiple non-standard procedures that presumably will not be used after the point that the standard algorithm becomes the standard that students are encouraged to use?

To align the CCMS with science, for each arithmetic operation, each standard algorithm would be the first procedure taught, and it would be practiced until its use is automated.  The underlying concepts would then be explored in a variety of ways to promote understanding, which could include the "strategies" in the CCMS.

# Theories of Learning

**Reasoning versus Algorithms**

The 1989 NCTM standards emphasized reasoning over memorization.  The CCMS, for facts and procedures, ask students to begin each topic by practicing general reasoning strategies for a year or more.  Cognitive science has found that for students in math, using "general reasoning strategies" in the absence of well-memorized facts and algorithms is nearly always ineffective as a strategy to solve problems (Sweller et al. 2010).  Summarizing research, Clark et al. (2012) explain why a "reason first, then memorize" sequence is not likely to work:

"If the learner has no relevant concepts and procedures in long-term memory…., searching for a solution overburdens limited working memory….  As a consequence, novices can engage in problem-solving activities for extended periods and learn almost nothing."

**Discovery and Misconceptions**

Schwartz and his colleagues (2011) report that "guided inquiry to introduce a topic can prepare students to see deeper conceptual principles."  However, if extensive "inquiry" or "discovery" is encouraged before instruction identifies what is correct, there is substantial risk students will move misconceptions into memory, and those misconceptions can be very difficult to root out (Willingham 2003, Rosenshine 2012).

Except for a small amount of initial "discovery," science supports learning facts and standard procedures first.  Clark et al. (2012) summarize:

> "Decades of research clearly demonstrate that for novices (comprising virtually all students)…, teachers are more effective when  they provide explicit guidance accompanied by practice and feedback, not  when they require students to discover many aspects of what they must learn…."

To align standards with science, "discovery" could briefly introduce a topic and its context. Next, students would practice the retrieval of facts, followed by problems applying procedures and activities focused on deeper understanding.

**Activities and Conceptual Understanding**

Science emphasizes that conceptual understanding is vitally important in mathematics and it needs to be taught (Siegler and Lortie-Forgues 2015).  Geary et al. (2008) write:

> "[T]he cognitive processes that facilitate rote retention (e.g., of over-learned arithmetic facts), such as repeated practice, can differ from the processes that facilitate transfer and long-term retention, such as conceptual understanding."

At the right time, Clark et al. (2012) note that a variety of problem-solving activities can be useful:

> "Independent problems and projects can be effective – not as vehicles for making discoveries, but as a means of practicing recently learned content and skills."

The CCMS "strategies" have the potential to help students organize conceptual frameworks, but when should those activities should occur?  The introduction section of the CCMS quotes Schmidt, Houang, and Cogan (2002):

> "[T]o be coherent, a set of content standards must evolve *from* particulars (e.g., the meaning and operations of whole numbers, including simple math facts and routine computational procedures…) *to* deeper structures inherent in the discipline…." (emphasis added)

Schmidt et al. align with cognitive science, but the CCMS for topics noted above move in the opposite sequence:  *from* multiple activities aimed at "deeper structures" *to* "simple math facts and procedures."

Physiologically, "deeper" connections cannot grow until elements are stored in LTM neurons (memorized) and one or more fundamental relationships are wired that support neuron activation (Anderson et al. 2000).

## Conclusion

From the viewpoint of cognitive research, while the CCMS are superior to the "1989 NCTM-type standards" in place in most states prior to 2010, the repeated omissions and delays in initial memorization in the 2010 CCMS result in learning strategies that simply do not work in the brain of non-experts (students).  When processing information that is not well memorized,

the limits of WM are new but verified science. Whenever new science is confirmed, existing theories related to that science must be re-evaluated.

Teachers do not decide K-12 state standards. When state standards at each grade level are extensive, as in the CCMS, teachers will be unlikely to have time to add additional topics that fill in what is missing and is needed to make the standards work.

Given the weaknesses identified by cognitive studies in the CCMS approach to arithmetic, the foundation for mathematics, the author would submit that a sufficient case has been made for a detailed review of the current CCMS and similar state standards by a panel that includes both cognitive experts and educators.

Thanks to the progress of scientific research, most of it funded by U.S. taxpayers, we now know how to help students learn math and science far more efficiently and effectively, but U.S. policy makers whose views and power decide K-12 math standards are failing to put that knowledge to good use. Reasons for this inaction, justified and not, include:

- The areas of scientific consensus on how the brain solves problems are complex, recent, and not widely disseminated in non-technical terminology.

- Institutions within education may require time to become familiar with the new science.

- Traditional policy-makers in education are not accustomed to having their philosophical preferences constrained by science.

- The many education public-policy organizations that endorsed the CCMS as an improvement over past standards may not find it easy to call for a review of 2010 standards only recently implemented, even if change is supported by new scientific research.

- Since 1995, standards in most states were debated and frequently changed without much evidence of positive impact, tending to promote "reform fatigue."

Given those circumstances, unless there is substantial pressure from U. S. economic and political leaders who respect science and understand the importance of preparing our nation's students for mathematics, science, and engineering, math standards that do not work are likely to remain in place in most states for quite some time.

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**References:**

Anderson, J. R., Bothell, D., Byrne, M. D., Douglass, S., Lebiere, C., Qin, Y.: An integrated theory of the mind. Psychological Review 111(4) 1036 (2004)

Anderson, J., Reder, L., Simon, H.: Applications and misapplications of cognitive psychology to mathematics education. Tex. Educ. Rev. 1(2) 29-49 (2000)

Biemiller A.: Teaching vocabulary. Am. Educator 25: 24-28 (2001)

Brown, P., Roediger, H., McDaniel, M.: Make It Stick: The Science of Successful Learning. Harvard University Press, Cambridge, MA (2014)

Carmichael, S., Wilson, W., Porter-Magee, K., Martino, G.:  The State of the State Math Standards 2010.  Fordham Foundation, Washington, DC (2010)

Clark, R.: Not knowing what we don't know: reframing the importance of automated knowledge for educational research. In: G. Clareboutand and J. Elen (Eds.) Avoiding Simplicity, Confronting Complexity.  Sense Publishers, Rotterdam (2006)

Clark, R., Sweller, J., Kirschner, P.:  Putting students on the path to learning:  The case for fully guided instruction.  Am. Educ., 36(1), 6-11 (2012)

Cowan, N.: The magical number 4 in short-term memory: A reconsideration of mental storage capacity. Behav. and Brain Sciences 24, 87-185. (2001)

Cowan, N.: The magical mystery four:  How is working memory capacity limited, and why?  Current Directions in Psychological Science 19(1), 51-57 (2010)

Dewar, M., Cowan, N., Della Sala. S.: Forgetting due to retroactive interference. Cortex 43.5 616 (2007)

Ericsson, K. A., Kintsch, W.: Long-term working memory. Psychological Rev. 102, 211-245 (1995)

Falkenheim, J.: NSF Infobrief 08-301, National Science Foundation. (2007) https://www.scribd.com/document/1006889/National-Science-Foundation-nsf08301

Finn, M.:  Stay Rates of Foreign Doctorate Recipients from U.S. Universities, 2009, (2012) https://orise.orau.gov/stem/reports/stay-rates-foreign-doctorate-recipients-2009.pdf

Fordham Institute:  Does common core math expect memorization? https://edexcellence.net/articles/does-common-core-math-expect-memorization-a-candid-conversation-with-jason-zimba  (2016)

Gathercole, S., Lamont, E., Alloway, T.: Working memory in the classroom. Working memory and education: 219-240  (2006)

Gathercole, S.: https://thepsychologist.bps.org.uk/volume-21/edition-5/working-memory-classroom  (2008)  or  https://thepsychologist.bps.org.uk/getfile/1946

Geary D.: Principles of evolutionary educational psychology. Learning and Individual Differences 12: 317-345 (2002)

Geary, D., Reyna, V., Boykin, W., Embretson, S., Siegler, R.:  Chapter 4 - Report of the Task Group on Learning Processes.  In: Foundations for Success: The Final Report of the National Mathematics Advisory Panel, U.S. Dept. of Education, Washington, DC (2008)

Goodman, M., Sands, A., Coley, R.:  America's skills challenge: Millennials and the future. Educational Testing Service, Princeton, NJ (2015)

Hartman, J., Nelson E.: "Do we need to memorize that?" or cognitive science for chemists. Foundations of Chemistry, 17(3), 263-274  (2015)

Hartman, J., Nelson E.: Automaticity in Computation and Student Success in Introductory Physical Science Courses. http://arxiv.org/abs/1608.05006 (2016)

Heitin, L.: Nebraska's new math standards. Curriculum Matters Blog, Education Week, (9/15/2015)

Kirschner, P., Sweller, J., Clark, R.: Why minimal guidance during instruction does not work: An analysis of the failure of constructivist, discovery, problem based, experiential and inquiry-based teaching. Educational Psychologist 41, 75-86 (2006)

Klein, D.: Brief history of American K-12 mathematics education in the 20th century. In: J. Royer (Ed.). Mathematical Cognition: Current Perspectives on Cognition, Learning, and Instruction. Information Age Publishing, Greenwich, CT (2002)

Koedinger, K., Corbett, A., Perfetti, C.: The Knowledge-Learning-Instruction (KLI) framework: Bridging the science-practice chasm to enhance robust student learning. Cognitive Science, 36 (5), 757-798 (2012)

Kuhn, T. S.: The Structure of Scientific Revolutions. University of Chicago Press: Chicago, (1962)

Loveless, T.: The 2002 Brown Center Report on American Education. Brookings Institution Press, Washington, DC (2002)

Loveless, T.: The regulation of teaching and learning. In: Hallinan, M., Gamoran, A., Kubitschek, W., Loveless, T. (Eds.). Stability and Change in American Education: Structure, Process, and Outcomes, pp 171-192 Eliot Werner Publications, Clinton Corners, NY (2003)

National Council of Teachers of Mathematics: Curriculum and Evaluation Standards for School Mathematics, NCTM, Reston, VA (1989)

National Governors Association and the Council of Chief State School Officers. Common Core State Standards Initiative: Common Core State Standards for Mathematics., Washington, DC (2010)

National Science Board: Science and Engineering Indicators 2012. Appendix Table 2-12 (2012) http://www.nsf.gov/statistics/seind12/c2/c2s2.htm. Accessed 20 September 2016

National Science Foundation: Science and Engineering Indicators 2014, Appendix Table 2-17 (2014) http://www.nsf.gov/statistics/seind14/index.cfm/appendix/tables.htm Accessed 20 September 2016

National Science Foundation: NCSES Survey of Earned Doctorates 2015, Table 22 (2016) https://www.nsf.gov/statistics/2017/nsf17306/data.cfm Accessed 17 January 2017

Norton, J., Ash J., Ballinger, S.: Common Core revisions: What are states really changing? Abt Associates, Cambridge, MA (2017)

OECD: Literacy, Numeracy, and Problem Solving in Technology-Rich Environments – Framework for the OECD Survey of Adult Skills. OECD Publishing, Paris (2013).

Olson, S., Riordan, D.: Engage to Excel: Producing One Million Additional College Graduates with Degrees in Science, Technology, Engineering, and Mathematics, Table C-6. Report to the President. Executive Office of the President, Washington, DC (2012)

Peterson, L., Peterson, M.: Short-term retention of individual verbal items. J. of Experimental Psychol. 58, 193-198 (1959)

Pinker, S.: The Language Instinct: How the Mind Creates Language. Wm. Morrow, New York (1994)

Rosenshine, B.: Principles of instruction: Research-based strategies that all teachers should know. Am. Educ. 36.1 12 (2012)

Schmid, W.: New battles in the math wars. Harvard Crimson, May 4 (2000)

Schmidt, W., Houang, R., Cogan, L.: A coherent curriculum. Am. Educ., Summer, 1 (2002)

Schoenfeld, A.: The math wars. Educational Policy 18, 253-286 (2004)

Schwartz, D., Chase, C., Oppezzo, M., Chin, D.: Practicing versus inventing with contrasting cases: The effects of telling first on learning and transfer. Journal of Educational Psychology, 103(4), 759-777 (2011)

Siegler, R., Lortie-Forgues, H.: Conceptual knowledge of fraction arithmetic. Journal of Educational Psychology 107.3 909 (2015)

Simon, H.: The structure of ill-structured problems. Artificial Intelligence 4, 181-201 (1973)

Sweller, J.: Instructional implications of David C. Geary's evolutionary educational psychology. Educational Psychologist 43, 214-216 (2008)

Sweller, J.: What human cognitive architecture tells us about constructivism. In: Tobias S. and Duffy T. (Eds.) Constructivist Instruction: Success or Failure? pp. 127-143. Routledge, New York (2009)

Sweller, J., Clark, R., Kirschner, P.: Teaching general problem-solving skills is not a substitute for, or a viable addition to, teaching mathematics. Notices of the Am. Mathematical Society 57(10), 1303-1304 (2010)

Tobias, S., Duffy. T., (Eds.): Constructivist Instruction: Success or Failure? Routledge, New York (2009)

Waits, B., Demana, F.: Calculators in mathematics teaching and learning. In: Burke, M. and Curcio, F (Eds.) Learning mathematics for a new century. National Council of Teachers of Mathematics, Reston, VA (2000)

Willingham, D.: Allocating student study time: Massed versus distributed practice. Am. Educ. 26(2), 37-39 (2002)

Willingham, D.: Students remember what they think about. Am. Educ., 27(2), 37-41 (2003)

Willingham, D.: Practice makes perfect—but only if you practice beyond the point of perfection. Am. Educ. 28(1), 31-33 (2004)

Willingham, D.: How knowledge helps. Am. Educ. 30(1), 30-37 (2006)

Willingham, D.: What will improve a student's memory? Am. Educ. 32(4), 17-25 (2008)

Willingham, D.: Is it true that some people just can't do math? Am. Educ. 33(4), 14-19 (2009)

Willingham, D.: Do students remember what they learn in school? Am. Educ. (Summer) 7 (2015)

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11/15/18